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Endurance, Perdurance and Category Theory

Abstract:

In this paper I argue that the concepts of «temporal parts», «temporal extension» and «thing which perdures over time» are special cases of «spatial parts», «spatial extension» and «spatially extended thing» correspondingly while there is a sense in which the concept of endurance and that of time are identical. Using the formal apparatus of the category theory I provide an ontology which allows an enduring object and the perduring «history» of this object to be different descriptions of the same thing - very like the intension and the extension of a predicate. As far as both of the descriptions are obtained simultaneously within the same formal construction there is no reason to believe that one of them is ontologically prior. Besides I show that an enduring object may have an additional «relational» extension which in certain cases is interpreted as a minimal time interval through which the object can endure. Categorical definitions of the concepts of «temporal part», «spatial part», «point» and «moment» are provided.

1. Introduction

Think about two different ways of how a thing may persist in time [1]. Say that a thing *endures through* time when it - the whole thing - happens to be in different times (possibly changing its properties and spatial location). Say that a thing *perdures over* time when it has a certain duration (which comprises some temporal stages, i.e. «temporal parts»). For example of enduring thing you might take your own body - it exists now, it existed yesterday and most probably it will exist tomorrow. For example of perduring thing you might take your life - this event takes a certain time but it does not «move», i.e. does not change its location in time.

Given any enduring thing T_e we may consider an «adjoint» perduring thing T_p which is the «trajectory» or «life» of T_e . Particularly for a body moving in 3D space we may consider its trajectory («world line») in 4D space-time. The converse apparently is not true: there are many perduring things, i.e. a processes - such as burnings and rainings - which seemingly has no certain adjoint enduring thing. This suggests the view that enduring things ontologically supervenes on perduring ones. Accordingly to this view, there exist only perduring things and there is some special class of perduring things which are describable in terms of endurance. As far as I know this class was never strictly distinguished; rather proponents of the ontological priority of the perdurance consider the concept of endurance as vague. David Lewis [1], for example, uses the following analogy.

Consider a long road which goes through many small villages. Although inhabitants of every village routinely call «the road» only a part of the whole road which lays inside their village, they may still agree as a result of equivocity that «the road» which all the people refer to is the same. Similarly, Lewis argues, we confusingly call, say, «John», a guy as if he would wholly exist here and now and still believe that John now and John an hour ago is one and the same person. But an ontologist should realize that «John now» is only a temporal part of the «whole» John extended in time as well as in space. Allegedly it is no more than a habit in certain cases to distinguish spatial parts of things but neglect their temporal parts. Thus the very concepts of endurance and identity through time are allegedly dismissed.

2. Can the notions of endurance and perdurance be generalized?

Let me start with giving more precise definitions of the notions of endurance and perdurance. When speaking about the endurance and/or perdurance one refers to «different times», one usually means different *moments* of time. However the notion of the moment of time is not necessary to define the endurance and the perdurance: we may think only about periods of time being neutral to the question whether a period of time comprises indivisible «moments» or not. However in this case the reference to «different times» in the definitions of endurance and perdurance should be restricted; what we need for the definitions is *mutually exclusive* times.

Def.1: a thing A endures through time iff there are times T_1, T_2 such as A happens to be at T_1 not being at T_2 and A happens to be at T_2 not being at T_1 .

A formalization of this definition by means of the propositional logic is problematic because it is not clear how «happens to be» is to be formalized. If we just interpret «happens to be» as «is» then we immediately come to contradiction: a thing is and is not at the same time T_1 (similarly for T_2). A formalization of the given definitions demands different logical tools. In the second part of the paper I suggest a tool for it.

Def.2: a thing A perdures over time iff A has parts B, C and there are times T_1, T_2 such as B is at T_1 not being at T_2 while C is at T_2 not being at T_1 .

The latter definition unlike the former can be straightforwardly written down by means of the propositional logic. Denote «V is a part of W» as $P(V,W)$ and «V is (temporally located) at W» as $TL(V,W)$. Then we have:

Def.2': a thing A perdures over time iff

$\exists B,C,X,Y(P(B,A)\&P(C,A)\&TL(B,X)\&\neg TL(B,Y)\&TL(C,Y)\&\neg TL(C,X))$

Though this definition as an element of a formal theory is useless unless it is not provided by some axiomatic of the predicates of the parthood and of the temporal location, it shows that a formalization of Def.2 by means of propositional logic does not involve such principal difficulties as that of Def.1. I think that this is an important reason why many philosophers try to dismiss temporal endurance in favor of (or reduce it to) temporal perdurance.

Tough the endurance and the perdurance were introduced as concepts about existence in time, they seemingly are also applicable to existence in space. Say that a thing *endures through space* when the thing happens to be (wholly) in different places. Say that a thing *perdures over space* when the thing has parts which have different spatial locations. A moving body and a movement of a body are examples of things correspondingly enduring through and perduring over space (as well as time). Actually the endurance through and the perdurance over space are new names for well-known concepts: a thing enduring through space is just a moving thing (given the movement is understood as a change of spatial location) while a thing perduring over space is just a spatially extended thing. However such a renaming is useful for our purposes. There is a strong tradition to apply spatial terms to temporal concepts: think about a «long story» and «short vacations».

Applying temporal terms to spatial concepts I am doing otherwise.

In the spatial case similar restrictions about «different places» should be made.

Def.3: a thing A endures through space iff there exist places X, Y such as A happens to be in X not being in Y and A happens to be in Y not being in X.

Def.4: a thing A perdures over space iff A has parts B, C and there exist places X, Y such as B is in X not being in Y and C is at Y not being in X.

Thus we extended the concepts of endurance and perdurance to the spatial case. We can think however about a further generalization. Consider different *generic properties*, for

example, «to be colored», «to have a mass», «to have an electrical charge», etc. Every generic property «comprises» a number of specific properties, particularly, the generic property «to be colored» comprises such properties as «to be red» and «to be green», the generic property «to have a mass» comprises «to weight 5 kg» and «to weight 2 kg», etc. Formally speaking, properties S_1, \dots, S_n are *specific* for a property G (which is called *generic* for S_1, \dots, S_n) iff whenever a thing has a property S_i it has the property G and whenever a thing has the property G it has some property S_j :

Def.5: S_1, \dots, S_n are specific for G iff $\forall A (\forall i (S_i(A) \Rightarrow G(A)) \& (G(A) \Rightarrow \exists j S_j(A)))$

Now we may define the endurance through and the perdurance over a generic property:

Def.6: Consider a generic property G and its specific properties S_1, \dots, S_n . Say that a thing A *endures through* G iff 1) A has G ; 2) there are i, j such as A happens to have S_i not having S_j and A happens to have S_j not having S_i .

Def.7: a thing A *perdures over* G iff 1) A has G ; 2) A has parts B, C ; 3) there are i, j such as B has S_i not having S_j and C has S_j not having S_i .

For example consider an apple on a tree; while the apple grows its color changes from green to yellow. We say that the apple endures through the color (i.e. the generic property «to be colored»). For example of a thing perduring over the color consider another apple such as one of its side is green while the other is yellow. Taking «to have a temporal location» and «to have a spatial location» as generic properties (for the properties «to be at such-and-such time» and «to be in so-and-so place» correspondingly), we may consider the endurance through and the perdurance over time and space as special cases of the endurance through and the perdurance over a generic property.

Now the following question arises: what differs the endurance through and the perdurance over time and space from the endurance through and the perdurance over other properties?

Firstly let me show that (1) if a thing endures through any property then this thing also endures through time. Suppose a thing A endures through a property G . Then A happens to have a property S_1 not having a property S_2 and otherwise happens to have a property S_2 not having a property S_1 (Def.6). Hence A happens to have a property S_1 and happens

not to have S_1 . But it is impossible to have and not to have a property at the same time. Hence there are times T_1, T_2 such as A happens to have a property S_1 at T_1 not being at T_2 and A happens not to have a property S_1 at T_2 not being at T_1 .

Particularly we proved that if A endures through space, i.e. moves, then A endures through time.

Consider the following proposition (2): if a thing endures through time then it endures through some other property G. Prima facie (2) seems to be false: it seems to be (at least logically) possible that a thing has exactly the same properties during all the time when it exist. However this argument is invalid. Imagine a perfectly static world W where no change ever occurs. Could things in this world endure through time? No. For to say that a thing A in W endures through time we should somehow distinguish between two different times T_1 and T_2 (see Def.1). Any distinction between two different times is a change, for example a change of positions of arrows at some clocks or a change of visible position of the Sun. Particularly a distinction between two different times may be a change of attention. Suppose I want to observe the world twice. For this purpose I could look at the world, then close my eyes, then open eyes and look at the world again. But in W no such a change is possible. Hence nothing endures through time in W. Moreover the fact that no interval of time in W can be distinguished makes a reason to say that in W there is no time at all.

Thus if A endures through time in W then W changes and some change of W is the distinction between times T_1 and T_2 supposed by the condition of A's endurance through time (Def.1). Consider the following two options: 1) A changes having at T_1 and at T_2 different intrinsic properties; 2) A does not change, i.e. always has the same intrinsic properties while something else in the world W changes. Understanding the term «relational property» wide enough we can say that in the latter case A changes some of its *relational* properties. Thus we have: if a thing endures through time then some A's property changes and this change is the distinction between T_1 and T_2 . Suppose A changes a property S_1 for a different property S_2 . Then time T_1 can be defined as «the time when A has property S_1 » while time T_2 is defined as «the time when A has property S_2 » (or otherwise). By Def.1 A happens to be at T_1 not being at T_2 and happens to be at T_2

not being at T_1 , hence A happens to have property S_1 not having property S_2 and happens to have property S_2 not having property S_1 (see Def.6).

To prove (2) we have to do two things: (a) to find a property G which is generic for S_1 , S_2 and (b) to show that «to endure through G» differs from «to endure through time». We hardly can point a property G which is generic universally, i.e. which is such as any thing A has this property no matter how A changes. However we can argue that for every change such a generic property exists. Actually when we speak that a thing A changes its properties we not only suppose that A survives through the change but we also suppose that the thing has a generic property relevant to the change. For example a thing may change its color - then the color, i.e. the property «to be colored», is the generic property relevant to this change. Or a thing may change its spatial position, its temperature, or whatever but in any case when a thing changes we suppose something «what» changes in this thing. Actually we should not grant generic properties ontologically: my argument does not demand this. We can take the notion of generic property as an instrument to describe changes. Suppose that a dry thing becomes wet. We can say that its wetness changes from zero to a certain amount. The «zero wetness» apparently does not exist. But this concept is useful to describe a dry thing which becomes wet. I use the notion of generic property exactly in this sense. Particularly when I say that «to exist in time» and «to exist in space» are generic properties for the properties «to be at time T» and «to be in place P» correspondingly I do not suppose that «time» and «space» are something else but certain concepts.

Thus if a thing A endures through time then it also endures through *some* generic property G. To prove (2) we need to make the last step (b): to prove that this property G always differs from the property «to exist in time», i.e. that A's endurance through G necessarily differs from A's endurance through time. But this is obviously false! When we say that a thing endures through time *and* somehow change its properties this means two things: 1) that the thing changes some (usually relational) property which is chosen as a measure of time and 2) the thing changes some other properties. When we say that a thing endures through time but as far as we can observe it does not change its properties, this means that the change of a property of the thing which is chosen as a measure of time

is the only change we can observe. (I use here the words «A changes its property P» for «A endures through P» as defined by Def.6) Often we use the words «a thing does not change» even looser ignoring many customary, expected changes, particularly periodic and reversible changes. Consider an example. Suppose, I observe a building during a day - from the sunrise to the sunset - and report that I have observed no changes. Suppose, I have no watches and determine the time of the beginning and of the end of the observation just looking at the sky. Saying that there were no observed changes I ignore the fact that the building has been illuminated differently early morning and in the middle of the day. I do not count the change of illumination because in the given context this is just the way how «the time goes». Consider another example: I observe a room from inside during half an hour looking at watch every 5 minutes. Suppose I do not observe any change again. But saying that no change has occurred during the period I ignore a change of position of arrows at my watches which is a change of the same kind as, say, a change of the arrangement of furniture in the room. (I Leave aside the question about changes of myself without which I could not distinguish between different times.) Thus «A's endurance through time» is always *identified* with A's endurance through a certain property (which has other *name* than «time») and hence the proposition (2) is false: although this is true that if a thing A endures it endures through a certain property G, it may be still the case that G and «time» in the given context are identical. Hence this is possible that «to exist in time» is the only generic property which a thing endures through, though «to exist in time» in every case has some other name such as «to be colored», «to have a position in space», etc.

Speaking more precisely we should distinguish between two close but different meanings of the words «a thing A endures through time»: one special and the other general. In the *special* sense «to endure through time» means to endure through a certain generic property chosen as a measure of time. It is a matter of physic and technology but not of metaphysic to decide what generic property must be chosen as a measure of time. A position of a thing with the respect to the Sun and/or Moon served for this purpose for years but in our days this is no more universally acceptable. There are both theoretical and practical needs of new gauges of time which may not be discussed here. In the *general*

sense (which is more relevant for a present discussion) the words «a thing A endures through time» means simply «A endures through some property» or simply «A endures». The upshot is that we failed to generalize the notion of endurance through time but discovered that in its *general* sense (which I found more relevant) the term «endurance through time» was *pleonastic*: it meant the same as «endurance through a property» or «endurance» *simpliciter*.

Let me stress the difference between the two cases of the endurance mentioned above: the first case when an enduring thing changes its intrinsic properties and the second case when an enduring thing changes only its relational properties. Suppose I observe a thing A which does not change its intrinsic properties. During the observation I myself experience changes of my intrinsic properties. These changes might be changes of perceptions caused by changes of a timer or, if I use no timer, these might be some voluntary and/or involuntary changes in my organism. Suppose again that I look at A, then close my eyes, then open and look again. This allows me to say that A endures: looking at A twice I see one and the same thing (let me for now avoid the question why I believe that both times I see one and the same thing but not two similar but different parts of the same thing). Note that the sense in which I say that A endures is purely subjective. Closing and opening my eyes I make A to endure! We may say, of course, that A changes its relational property of being observed by me: it is observed by me, then not observed, then observed again. But since A does not change its *intrinsic* properties my observation makes no influence on A and hence the fact that A endures is not, so to speak, a fact about A but rather a fact about myself. If some changes of my intrinsic properties are caused by some other changes, for example by changes of the configuration of the Solar System, then the endurance which I «prescribe» to A is in a sense objective, because in this case the relevant changes of intrinsic properties of myself exactly correspond to changes of intrinsic properties of another thing, namely the Solar System. However since A does not change its intrinsic properties the change of Solar System does not influence A and again the fact that A endures is about me and Solar System but not about A. When A *does* change its intrinsic properties the situation is completely different. Suppose A changes color. Then I might observe A and count only changes of my perceptions caused

by changes of A's color. This again allows me to say that A endures. But unlike the former the latter fact is exactly about A! Thus after the distinction between relational and intrinsic properties we can distinguish between relational and intrinsic sorts of endurance: a thing endures *relationally* if it changes its relational properties and a thing changes *intrinsically* if it changes its intrinsic properties.

Now return to the perdurance. Is the concept of *perdurance over time* pleonastic like the concept of the endurance through time? We can distinguish three senses the term «time» here. The first is the most general and the most relevant to a philosophical discussion: in the first sense «time» simply means «endurance». If a thing happens to be in different situations it exists in time in this sense of the term. In the second, special sense «time» means «endurance through a certain (generic) property T». We use the term «time» in the second sense when we study *how* a thing changes in time. Consider a diagram showing how the temperature of a body changes in time. Suppose the time is measured by astronomical means, namely by the position of the Sun. Then the body endures through at least two properties: one is its temperature and the other is its position with the respect to the Sun. In this case «time» means the endurance through the latter property. Finally «time» may be just identified with a generic property T such as the «endurance through T» means «time» in the special sense. In the above example «time» in this sense is just «a position with the respect to the Sun». We use the term «time» in this third sense when we ask «what is the time?» and are answered, say, «5 p.m.». Let me call this third sense of the term «time» «technical».

The meaning of the term «perdurance over time» depends on the meaning of the term «time». It is clear that with the first and the second senses of «time» the expression «perdurance over time» makes no sense at all. (For «perdurance over endurance» is an absurd.) But with the term «time» in the *technical* sense the expression «perdurance over time» makes a sense - this is the perdurance over a generic property G in the sense of Def.7. However we should accurately define which thing endures through, which perdures over time and what is G in every particular case. Think about a moving body B. Suppose that we have no clocks and can only claim that B happens to be in different spatial positions. This is enough to claim that B endures, that is, (with the initial

pleonastic terminology) endures through time. Then «B's spatial position» is what we call «time in the technical sense». Consider the event M of B's movement. This event takes a certain time and a certain place, it perdures over time and over space. But since «time» in the given context means (in the technical sense) nothing but B's position in space, the place and the time of the event M are identical. (We can say that in this case space and time dimensions «glue» with each other and make a one-dimensional space-time.) Then spatial parts of M are its temporal parts. For M this is the same thing to perdure over space (i.e. a spatial position) and to perdure over time. But obviously this does not mean that that in the given example there is no difference between space and time at all. Particularly B might be a spatially extended body, but this does not make it to perdure over time: by the given conditions B is an enduring body. This puzzle resolves when we remember that «a spatial position» means «time in the technical» sense only for B, i.e. «time in the technical sense» is «a spatial position of B» but not of another thing, particularly not of a part of B. The thing which perdures over the property «to be a spatial position of B», i.e. perdures over time, is the *event* M but not B itself. (This sounds not very well and in the end of the paper I give a sense in which we can consider B and M as one and the same thing.)

Suppose however that we have some clocks to measure how B moves. Then, generally speaking, we do not need B to define things perduring over time. For time in this case means (in the technical sense) a position of the clock's fingers. Then every thing perduring over time should perdure over a position of the fingers. Besides it may perdure over many other things including «a position of B». With clocksⁱ the event of B's movement becomes two-dimensional: its one dimension is a position of the clock's fingers, i.e. the time, while its another dimension is a position of B. What I say completely complies with what physicists say about the «time dimension»: principally it does not differ from any «space dimension», this is a matter of mere agreement which dimension we call temporal and which we call spatialⁱⁱ. Another question is that the notion of time is not reducible to «time dimension»: the concept of «time dimension» is consistent only if «time» is understood in the technical sense but to define what is time in

the technical sense we need the concept of endurance, that is, the concept of time in the general sense.

The above examples of the perdurance over time were special cases of the perdurance over space, when time in the technical sense was identified with a spatial position of a certain thing. Can time be measured by something but a spatial movement? Can time be measured by, say, a change of color?

To answer this question we need to rethink the concept of space in a similar way as we have already rethought the concept of time. In the beginning we supposed that «to be at time T» is one property among others such as «to be red», «to be pleasant», etc. This gave us the idea to generalize the concepts of the endurance through and the perdurance over time speaking about the endurance through and the perdurance over an arbitrary generic property. But later we discovered that time is not a «usual» property: in the technical sense it is a name which is given to a property P such as the endurance through P of a thing B is chosen as the «gauge» of the endurance, while in the general sense of the term «time» is identical with «endurance». Let me now show that «to be in place P» also is not an «usual» property but «space» in the general sense of the term is identical with «perdurance» (and hence «perdurance over space» is *not* a particular case of «perdurance over a property»).

The reasons are similar. Firstly note, that (3) if a thing A perdures over a property G (Def.7) then A perdures over space (Def.4). For if A perdures over G then A have at least two parts B, C such that B has a property S_i not having a property S_j while C has S_j not having S_i . But what might be the parts B,C if not *spatial* parts? If, for example, there is a thing such as one its part is green and another yellow (or one is hot and another cold) these two parts differs spatially. I find it to be obvious at the same extent as the fact that if a thing as a whole happens to have and not to have a certain property then the thing happens to be at different timesⁱⁱⁱ. The fact that B,C are spatial parts means that they have different mutually exclusive spatial locations, i.e. that there are places P_i, P_j such as B is located at P_i not being located at P_j while C is located at P_j not being located at P_i , which means (Def.4) that A perdures over space. (Def.4 is, of course, pleonastic: it demands that a thing have parts *such as* they occupy different (mutually exclusive) places. But we

showed that *any* parts of a thing occupy different mutually exclusive places. Thus the second condition is redundant.)

Consider another proposition (the analogue of the proposition (2) about time): (4) if a thing perdures over space then it perdures over a certain property. That is, if a thing have different spatial parts those parts differ by some properties (other than «to have such and such spatial locations»). Actually, given an empty perfectly homogeneous space it is impossible to distinguish any part of it. To distinguish, for example, between the right and the left halves of the space I need at least my body as a region of space which have different properties than the surrounding regions of space: my body is dense while the surrounding is thin. The intuitive concept of «real space» is mostly based on such a distinction between the dense and the thin. However any other difference of properties, for example, a difference of color, will also do. (Imagine a world where living organisms including humans are structured bubbles living in a dense environment!)^{iv}. When we say that two things have different spatial locations but still are exactly similar, i.e. have exactly the same properties - think about two halves of a ball - we do not count those properties which makes the distinction between their spatial locations possible. It may be the case that the two halves of the ball have the same intrinsic properties but they certainly have different relational properties as far as they cause different perceptions - otherwise they could not be observed as two at once. This is very similar to the case of time. In both cases a distinction between different specific properties of some generic property - color, density, etc. - gives a reason to make another distinction which is the distinction between different times in one case and the distinction between different places in the other case. But what differs the two cases? In the case of space as well as in the case of time certain mutually exclusive properties (i.e. properties which cannot be possessed by one and the same thing at once) are nevertheless «combined» with one and the same thing. This is possible in two different ways: in the way of endurance and in the way of perdurance. Mutually exclusive properties are combined in the way of endurance when a thing *happens* to have each of these properties (having no two of them *at once*). Then we say that the thing endures and have the mutually exclusive properties *at different times*. Mutually exclusive properties are combined in the way of perdurance when a thing

has parts and different parts of the thing has different mutually exclusive properties. Then we say that the thing perdures and the difference between the parts is a spatial difference. Thus we may say that in the general senses of the terms «space» and «perdurance» are identical as well as «time» and «endurance»; of course the term «extension» would be more convenient for what we call here «perdurance» but the question of terminology is not essential. The term «perdurance over space» is pleonastic like the term «endurance through time»: any possible endurance is «over space» though «space» may be determined by different properties in different cases (and in a special case which is specified below we can say «time» instead of «space»).

Note also that the concept of part appears to be purely spatial: to have different parts and to experience different «happenings» are two alternatives which equally allows to combine mutually exclusive properties.

Thus any perduring thing is a spatially extended thing. Particularly if a thing A changes its color then a perduring thing B which is «A's change of color» may be nothing but a spatially extended thing. If A's change of color is identified with time (in the technical sense) then B is called «perduring over time» and its parts are called «temporal parts». Correspondingly, «A's change of color» in this case may be called a «movement» in the «color space».

Note that trying to generalize the notions of endurance through time and perdurance over time we have got different results. While the endurance through time appeared to be the endurance in general, the perdurance over time appeared to be a special case of the perdurance over *space*, that is of the spatial extension.

3. The category of things

Consider definitions 6,7. Though Def.7 is pleonastic as well as Def.4 those two definitions comply with all our reasoning. Both of the definitions use the notion of property and particularly of generic property defined by Def.5. However with the notion of generic property we have a problem which is analogous with the problem of endurance vs. perdurance. This is the problem of intension vs. extension of a predicate. Actually we can understand a generic property differently. If properties are understood extensionally,

i.e. as classes of things which have those properties, then a class of things having a generic property G is the union of classes of things having corresponding specific properties S_i . Thus extensionally S_i can be thought of as parts of G , for example, the property «red» as a part of the property «colored». From this point of view every particular color, for example, «red» represents only a part of the color in general; unless we do not know all the particular colors we have only a partial knowledge of the color. If however properties are thought of intensionally, i.e. as certain concepts, then a specific property S_i is a specification of its generic property G , that is S_i has the whole G in its «core». For example, a red thing in this case represents the whole property «to be colored» *plus* a differentia which makes this color the red color. Thus a generic property G is wholly presented in every its specific property S_i . From this point of view it is not necessary to know all the particular colors to know what is the color - it is enough to know only one or two particular color(s) and to distinguish between what is specific for this particular color(s) and what relates to the color in general.

From the extensional point of view a generic property G is a sum of its specific properties while from the intensional point of view G «survives through» its specific properties, that is «happens to be» in different modifications. The analogy with the endurance vs. perdurance controversy is obvious.

Thus defining the endurance and the perdurance via generic properties we only hide the problem (or at least get another very similar problem). We can make things easier however refusing from the notion of property at all. All we need for ontology are «things» and a primitive «placement».

The idea is that things are «placed» at each other like a billiard ball on a table. The crucial point is that one thing may be placed at another in different ways. Particularly a ball may be placed on a table differently. In this case we say that one thing has a number of different placements at another. The fact that a thing A has different placements at a thing B is enough to say that A endures (through B)!

Def.8: a thing A *endures through* a thing B iff there are more than one placement of A at B .

Def.9: a thing A *rests* at a thing B iff there is one and only one placement of A at B .

How to deal with cases when a thing changes some its intrinsic property, for example its color, I explain below. I would like only to say in advance that we do not need to stipulate such a thing as the «color» for it.

Note that the fact that A has one placement at B and another placement at C does not entail that A endures. If A is placed at D which is a part of both B and C then A does not endure. If A endures through B then B certainly perdures - it should have an extension to allow different placements. If a billiard table would have no extension, no ball could move on it. However this is obviously too strong to define the perdurance: a ball cannot move without a space but a space can exist without any moving body. The fact that there are different placements at A - no matter whether those placements are placements of the same or of different things - is enough to say that A perdures.

Def.10: a thing A *perdures over* things B,C,... iff B,C,... are placed at A (provided that there is more than one placement at A).

Def.8,10 entail: If A endures through B then B perdures over A. But the converse is not true. For B can rest at A (Def.9).

Note that with the notion of placement we have defined the perdurance without the notion of part. We will define this important notion later.

To build a formal theory let me use the mathematical notion of *category* [4]. I will not reproduce here the standard definition of category but define the «category of things» which is of our interest here. Note that I use the term «thing» where in the category theory is used the term «object». The reason is that the term «object» which is plainly neutral in mathematics is not such in metaphysics.

Let an arrow \rightarrow mean «placement». This is a primitive concept of this theory.

Let different arrows be combined in the following way: the beginning or the end of one arrow is attached to the beginning or to the end of another arrow. For example two different arrows may be combined in the following three ways: $\rightarrow\leftarrow$, $\leftarrow\rightarrow$, $\rightarrow\rightarrow$.

Besides the end of an arrow can be attached to the beginning of the same arrow:

E1

Let me call such an arrow «circular».

Then suppose that if the end of an arrow a is attached to the beginning of another arrow b then there is the third «composition» arrow ba^y such as its beginning is attached to the beginning of a and its end is attached to the end of b :

E2

Besides we suppose that the composition of arrows is associative, that is, $(cb)a = c(ba)$, that is, if there are three arrows a, b, c making a chain then

E3

no matter in which order we combine the arrows a, b, c , we obtain the same arrow cba .

This entails that a chain of any number of arrows has the only composition arrow which begins in the beginning of the first arrow and ends at the end of the last arrow.

We shall call a «thing» any point of contact of arrows. This explains the above suppositions. The supposition of the composition of arrows, i.e. of placements, means that the placement is transitive: if a thing P is placed at a thing R and R is placed at S then P is placed at S (see the former picture). The associativity of the composition means only that P has a certain position at T (see the latter picture).

Suppose that the beginning and the end of every arrow contacts with the end or with the beginning of another arrow (possibly of itself), i.e. that every arrow goes from one thing to another (possibly from a thing to the same thing). That is to say that the placement is the placement of one thing at another thing (possibly the placement of a thing in itself).

Finally suppose that every thing A is a point of contact with itself of a circular arrow 1_A (called the *identity* placement) such as any incoming arrow f combined with 1_A gives f back and 1_A combined with every outgoing arrow g gives g back: $1_A f = f$ and $g 1_A = g$

E4

This latter agreement means that every thing is placed at itself. It is easy to prove that a thing has the only identity placement.

What we have got is the general notion of category without any specification^{vi}. Let me show how it works.

This is a diagram showing a thing A which endures through a thing B :

E5

A perduring thing is just a thing which has more than one incoming arrow:

E6

It is of course possible that a thing endures and perdures:

E7

The parthood is defined as follows:

Def.11: A placement f at a thing A is a *part* of A iff it is left cancelable, that is for any placements g, h $fg = fh$ entails $g = h$.

An arrow which satisfies the above condition in the category theory is called *monic* arrow or *mono*. This is the categorial version of the notion of *mapping into*. A monic arrow coming to an object A is called a *subobject* of A . This is the categorial version of the notion of subset. Note that by Def.11 a part is not a thing but a placement. A placement in A is a part of A iff it does not «confuse» placements which it «follows» by composition, i.e. iff $g \neq h$ entails $fg \neq fh$. It is easy to see that the parthood so defined is transitive: the composition of parts is a part (the compositions of monics is monic)^{vii}.

Under which conditions we can say that a perduring thing perdures *over time*? Consider a ball B moving on a table T . Suppose that the only measure of time is the movement of B , i.e. - using the terminology introduced above - time in the technical sense is identified with a position of B on T . This gives no reason to say that the table T perdures over time because the time is identified with a position of the ball B , not just with a certain place on the table which might be occupied by any other thing. To perdure over time a thing should be an event, for example the event of B 's movement on T . Think about the B 's movement as its trajectory on the table. Since time is identified with B 's position we need no additional «time dimension» to describe this event. Suppose that B leaves a trace on the table when moves. Can the event and the trace be identified? No, because another ball might follow the first one exactly repeating its trajectory. The events of movements of the second and the first balls are different while the trace which they leave is the same. The event E of B 's movement is «exclusive» in the following sense: if another thing C is

placed at E, then C is placed at B (it may either endure through or rest at B). In the categorial terms:

E8

(every thing C which is placed at E is placed at B)

Def.12: a thing E *temporally* perdures over B iff every thing C which is placed at E is placed at B.

If an arrow f from B to E is a part of E (Def.11) than f is *temporal part* of E.

It might be argued against this definition that every event takes (=perdures over) time but not every event is a «story» of a certain thing. For example the killing of Caesar involves at least two things - Caesar and Brutus. The answer is that every event - as far as we speak about *particular* events - is a «story» of some sort of *clocks*, i.e. a thing which endures through all the event in question and whose endurance is identified with time in the *special* sense. This enduring thing might be a witness of a story: this is the case of a «subjective» story. This is not the only possibility however. We can think about Rome as a changing (enduring) thing and suppose that the change of Rome which is considered as the change of time is the change of certain elements of the city which are called «clocks» and «calendars». Besides people living in the city change their beliefs about the current time and the change of the beliefs mostly comply with changes of the clocks and the calendars and can also be considered as changes of time. The place P where Caesar was murdered was a part of the city and we may suppose that this place endured through time^{viii} in the same way as the rest of the city. Then we may say that the killing of Caesar is an episode (i.e. a temporal part) of the history of P (i.e. that every participant of this event is placed in P.)

Above we spoke about the endurance through time without specifying any period of time which a thing endures through. The concept of the endurance through a certain *period* of time involves additional presuppositions. Firstly to speak about the endurance through a period of time we should suppose a temporally perduring thing - the period of time. Now we can get rid with this: we know after Def.8,10 how a thing can endure through a perduring thing and know after Def.12 when a perduring thing perdures over time. There

is however another problem to discuss. Usually the perdurance through a period of time is thought of in this way:

Def.13: a thing *A* *pointwise* endures through a period of time *T* iff *A* happens to exist wholly at every moment of *T*.

(I gave this the name «pointwise endurance through a period» but this is the way how the endurance through a period is usually understood *simpliciter*.)

A weaker variant is this:

Def.14: a thing *A* *strongly* endures through a period of time *T* iff *A* happens to be at every part of *T*.

(I show below that this weaker variant is still very strong.)

Def.14 immediately entails that (5) if a thing *A* strongly endures through *T* then *A* strongly endures through every part of *T*. Look again at the above diagram. Provided that the arrows *BE* are monic, they are temporal parts. Then Def.14 seems somewhat trivial: only arrows from *B* to *E* are temporal parts here by definition (Def.12). If a thing *C* has a monic placement *p* at *E* there is no reason to call *p* a temporal part. Think however about the following example. Suppose a car moving along a road from a point *A* to a point *B*. Suppose for simplicity that the road is of the same width as the car. The car happens to be on different parts of the road between *A* and *B*. But this is *not* the case that the car happens to be *in the whole* on *every* part of this piece of the road. For the car have a certain size, it perdures over the space as well as the road. However if we define parts of the road as monic placements of the car on the road we can certainly say that car does happen to be on *every* part of the road. For with such a definition of a part we just cannot distinguish any part of the road which is smaller than the car. This does not mean of course that the smaller parts cannot be distinguished at all. As far as we have established that the «temporal extension» is a special case of the «spatial extension», (or as physicists say, the time dimension does not principally differ from any space dimension) we might expect that a similar situation is possible with «moving through time» in spite of the limitation of the Def.12. Let us see.

Let the road be *R*, the car be *C* and a part of the car be *P*. Then the diagram $P \rightarrow C \rightarrow R$, provided the arrows are monic, shows how a smaller part of *R* might be distinguished: the

composition arrow PR missed on the diagram is a smaller part of R (remind that the composition of parts is a part). We can see that Def.12 allows a similar construction:
E9

Here B temporally perdures over A (which endures through B) while A in its turn temporally perdures over D (which endures through A). By the composition D itself endures through B. Thus an enduring thing A has in this example a certain «temporal size» as well as a car on a road has a spatial (non-temporal) size. The two cases however are different.

Provided that the arrows DA and the arrows AB are monic the arrows DB are also monic, i.e. they are parts of B, however they are *not temporal* parts of B. For D does not comply with the condition of the Def.12: B perdures over D but does not temporally perdure over D.

Thus if to understand «a period of time» strictly in the sense of Def.12 of the temporal perdurance (i.e. with the respect to a particular thing A which endures through this interval) then we may say:

(6) if a thing A endures through a period of time then A *strongly* endures (Def.14) through this period of time.

(However I am not sure that the words «a period of time are *always* understood this way.)

To illustrate the above formal results take as an example of enduring thing yourself. As far as you identify the time with the change of your states - say, distinguishing between times when you sleep and times when you wake - you may be sure that within any period of time (so defined) you wholly exist: you wholly exist when you sleep and you wholly exist when you wake. Then suppose that you determine the time in the usual way by looking at your watch. Suppose, you look at the watch and see that the time is 5p.m. It means that the watch is in a certain state which may be identified as 5h and *you* are in a state of seeing that your watch shows 5p.m. which can be without a risk of confusion also identified as 5h. Then you look at the watch another time and see that it shows 6p.m, then you look and see 7 p.m. There are two enduring things here: you and your watch; each of these things happens to be in the states which might be uniformly identified as 5h, 6h and 7h; for each thing by Def.12 we can suppose a temporally perduring thing, that is a

period of time from 5 to 7 p.m. (for you and for your watch these periods are different) and claim that you and your watch endure each through its period of time. To «combine» the two periods into one consider the *sum* S of you and your watch. Every relevant state of you (that is, your belief about the current time) corresponds to a state of the watch (but not otherwise if you do not look at the watch permanently). When you change your belief about the time, a state of the watch correspondingly changes (suppose that you see the watch in a right way and believe that the time is T iff the watch shows T). This allows to say that S in the whole also happens to be in the states 5h, 6h, 7h. S also has a period of time T (from 5 to 7 p.m.) and you can identify a «common» time of yourself and your watch with the endurance of S through T.

If you follow your watch more attentively you can distinguish more times within the same period, say 5h34', 5h37', etc. Suppose however that you have a chronometer which shows milliseconds. The situation is different because you cannot follow milliseconds as you can follow hours, minutes and seconds. You cannot look at the chronometer and say or at least to build a belief that «now is 5h34'48.236''» and a few milliseconds later - «now is already 5h34'48.239''». For to build such a belief and moreover to say that takes much more than one millisecond. Hence though the chronometer certainly endures through the period of time from 5h, 34',48.236'' to 5h, 34',48.239'' there is no reason to say that you do. But possibly we could say that the sum S' of you and the chronometer endures through such a period?

Generally, does a thing change if one its part change but another does not? On the one hand it seems that if a part changes the whole changes also. If anything changes in the world then the world changes. On the other hand the idea that a whole changes iff *all* of its parts somehow change seems also reasonable. If a part changes but this change causes no changes of other parts, i.e. the change of the part is «causally isolated» then it is hardly can be called a change of the whole. Consider for example the sum of the Sun and the Earth. Suppose a flash on the Sun occurs, then the Sun changes. The flash causes certain changes on the Earth but no earlier than 8 minutes after the time when it occurs on the Sun: to reach the Earth from the Sun the light takes about this period of time. Hence a minimal interval of the time which is *common* for the Sun and the Earth, i.e. which is the

local time of the system Sun-Earth is about 8 minutes measured by a local clocks on the Earth or on the Sun. Notice that I use the term «local time» not in the usual sense of the local time of a material point but in the sense of the local time of a certain extended thing. If a thing is extended, a common time for the whole thing should be «rough» enough to neglect the time which the light takes to reach one edge of the thing from another. In the local «rough» time a thing endures strongly but in «thin» times of its parts it do not. Particularly the system Sun-Earth wholly exists only within an interval which is no shorter than 8 min. Remind that if we determine the time *by* certain changes of the system Sun-Earth we can never «notice» an interval of time shorter than 8 min. We can notice a shorter interval only comparing changes of the system Sun-Earth with changes of its smaller part such as usual clocks. Similarly we can notice that a car on a road has a certain size only comparing the car with its parts. Thus though the system Sun-Earth wholly exists this morning and this evening it does not wholly exists now and a minute later! Notice that this does not entail that the system develops discreetly as well as the fact that the car has a certain size and cannot be wholly located in every part of the road does not entail that the car moves on the road discreetly.

The limitation of the finiteness of the speed of light is ultimate: no two things can interact faster than with the speed of light. However things can interact much slower. Particularly you interact with the chronometer slower and different parts of you interact with each other also slower. When the chronometer changes its indications your reaction on this change takes much more time than the time which light takes to reach your eyes after the reflection from the chronometer. Thus though the system «a human observer with a chronometer» is relatively small it is still relatively «slow», particularly it is much slower than can be the chronometer itself.

Look again at the above diagram. The fact that D has different placements in A does not entail that A has corresponding different placements in B. Suppose D rests in A, then every change of D corresponds one-to-one to a certain change of the whole:

E10

But if D endures through A then A can perdure over D temporally. Generally, it is reasonable to say that for every thing A there exists a certain minimal time Δ such as any change of A takes a time which is no less than Δ . This means that if A happens to be wholly in some period of time T then $T \geq \Delta$. We can also say that for every thing A there is a certain frequency $1/\Delta$ with which A «reproduces itself»^{ix}. Remember however that (unlike the case of non-temporal parts) temporal parts of Δ of a thing A are not temporal parts of a period of time through which A endures. I might stay near a road and wait while a long vehicle moves along fixing moments of time when it reaches me and goes away but I cannot fix the time when my pen «reproduces itself». Such a time of course does not exist. What I say is that though I can reasonably speak about something what happens with a certain elementary particle, which is a part of my pen, within a nanosecond, it is senseless to speak about what going on with the whole pen within a nanosecond. For the pen does not wholly exist within a nanosecond. The pen does not «begin» and does not «end» during its existence but experiences certain changes, all of which take much more time than a nanosecond. The pen wholly exists in each of its different states but no state of the pen is definable for the time interval less than a certain Δ .

Let us return to the Def.13 of pointwise endurance. This definition uses the notion of *moment* of time which we have not yet defined formally. Let us do it. For this purpose we need one auxiliary (but very important) concept.

Def.15: a thing A of a category is a *photo* iff all things of the category rest in A.

In the usual categorial terminology such a thing (i.e. an object A such as for any object B of the category there is one and only one arrow from B to A) is called *terminal object*.

The reason why I call it photo is clear from the above definition but notice that this is a *global* photo which comprises all things of the category. It is a basic fact that if a category

has a terminal object, the terminal object is *unique*; in our terms, (7) a category of things has no more than one photo. (For the proof see [4]. Actually the question of identity of an object of a category involves some problems which can be relevant for this discussion but I leave this aside.)

The photo of things of a category is not «made at a certain moment of time» but it allows to define *what* is a moment of time. In advance note that

(8) If the photo endures through a thing A then A *temporally* perdures over the photo.

(That is, every thing through which the photo endures is its «history».)

To prove (8) just check the Def.12, use the Def.15, and see the diagram below.

E11

Now consider

Def.16: a *moment* of a thing A is an arrow from the photo to A.

As a matter of terminological agreement we may speak «a point of A» instead of «a moment of A» in contexts where A is not considered as a temporally perduring thing.

With the Def.16 we can clarify the informal Def.13. The words «A exists at a moment M of a period of time T» can be interpreted as

E12

i.e. as a situation when for a moment M of T which temporally perdures over A there is a placement of A in T which is the composition of the placement of A in the photo with M. This saves the intuition which inclines us to take the Def.13 rather as the general definition of the endurance through a period of time than the definition of the special case of *pointwise* endurance through a period of time:

(9) If a thing endures through a period of time then it pointwise endures through the period of time.

Actually, if T has a moment M then M *always* can be composed with the arrow A-Photo (which exists and is unique by the Def.15) to give a placement of A in T

However the sense of the Def.13 which is saved is actually unusual. This is true that if A endures through B then A cannot «miss» any moment of B. However the fact that B temporally perdures over A does not entail that B has any moment at all! B may have no moment or have very few moments. B may have more temporal parts than moments. (The question whether a given moment of B belongs to a given part of B leads to the notion of topos and cannot be considered here.) Remind also that not any category of things has the photo - this is a special case of the categories which allows moments. Seemingly the intuition about the endurance which backs the Def.13 by and large is right but it certainly concerns only one very special case leaving aside others.

Thus one and the same thing may endure and perdure, moreover one and the same thing may endure and perdure *temporally*. Above we considered the case when a thing A endured through a thing B while a thing B endured through a thing C provided that B temporally perdured over A and C temporally perdured over B. But could a thing endure through itself? That is, might A and B in the Def.8 be identical? The case of the mechanical movement does not give us a clear answer. Think about a rotating ball. Do we need an outer space to say that the ball rotates? Physics says that the rotation (the angular momentum) is an intrinsic characteristic of a body (or of a system of bodies) like the mass. This might be used as an argument in favor of the absolute space. There is another option however: to say that the rotation is the change of the ball's location at itself. Our formalism easily allows this. We have supposed that every thing A has a special placement at itself which is its identity placement. It is possible however (in the sense that the formalism allows) that there are more than one placement of A at itself.

E13

Such a category with the only object is called *monoid*. Every placement p of the monoid being composed with its identity placement from left or from right gives p back. Besides p can be composed with itself or with another placement s giving again a placement of A in itself. An important particular case of monoid is the case of (algebraic) *group*: for the rotating ball think about the group of the rotations of the ball, particularly about some its

cyclic subgroup. Let me call a monoid *non-trivial* iff it has non-identity placements (that is iff it has more than one placement).

Now notice that a non-trivial monoid A immediately complies with both Def.8,10: it endures through itself and it perdures over itself. Moreover, it trivially complies with Def.12 of the temporal perdurance (since A is the only thing of the category)! This resolves the whole endurance vs. perdurance controversy in the following way: an enduring object and the event which is the «history» of the object can be exactly the same thing which both endures through and perdures over itself - very much like a predicate which has both an intension and an extension. Actually, suppose a thing A changes and we have no way to distinguish different times but distinguishing different states of A . We still can quite reasonable speak about « A when it is such-and-such» and « A when it is so-and-so» as about different times T_1, T_2 and say that A happens to be in T_1 (that is A happens to be such-and-such) and happens to be in T_2 (that is, it happens to be so-and-so). And we need *not* to say that «such-and-such» and «so-and-so» are parts of some other thing. We can say that they are different placements of A at itself. Provided that the placement «such-and-such» is monic it is a *temporal part* of A . Notice that *every* monic placement of a thing at itself is its temporal part. That is, a monoid can have temporal parts but cannot have non-temporal parts. To have non-temporal parts a thing needs some other thing. Since the identity of a monoid is monic it is also a temporal part. (This can be taken as the usual agreement that a whole is its own part.)

Recall the example of the killing of Caesar. Above in order to speak about this event we had to find another thing which endured through the whole event. We said that somewhat like «the place of this killing» could do the job. But this is of course rather artificial. Much more natural way of speaking would be to say that there is only one thing here, namely the event of the killing of Caesar, which has both spatial and temporal extension. As I already said when we speak about temporal extension and temporal parts we should understand that every extension and every part is spatial and only under certain conditions we call a spatial extension temporal extension and a spatial part temporal part. That is why it is better to say: the event in question has its «intension» which is a certain enduring (object-like) thing, and its «extension» which is a certain perduring (extended)

thing; to present this extended thing we usually need 4 dimensions. The analogy with the intension and the extension of a predicate is straightforward. Think about the predicate «red» as about the class of red things. Then, taking the «red» intensionally, say that the property «red» wholly belongs to every thing of the class. In our terminology the red is «placed at itself in many different ways». There is nothing confusing in this fact.

If a category comprises more than one thing but the things still have many placements in themselves, the picture is more complicated. In this case we should strongly distinguish between different extensions, particularly different temporal extensions, which might be assigned to a thing. We may count only placements of a thing A at itself - the corresponding extension can be called the «proper extension» - this is «the history of the endurance of A through itself». Every part of this event corresponds to a certain state of A. But it is also possible that some other thing B is placed at A, moreover it is possible that B is placed at A such a way that A temporally perdures over B:

E14

Then A would have another kind of extension, particularly temporal extension, which can be called *relational*. «The time of reproduction» Δ about which we spoke above was exactly the case of a relational temporal extension. What we call the «size» of an enduring object is also a relational extension - this is an extension with the respect to a certain ruler. Moreover, since every monoid perdures *temporally* (over itself) every non-temporal extension (i.e. what we usually call just spatial extension) is relational.

Notice that the distinction between placements of a thing at itself and in other things is an alternative for the distinction between intrinsic and relational properties (compare what was said in the previous part about the «intrinsic» and the «relational» endurance.)

Speaking formally my suggestion is to use arrows instead of predicates to deal with what we call «properties».

To conclude I would like to make a few notes about possible prospects of this theory.

We have not resolved the main logical problem about the endurance: given that A happens both to be P and not to be P and given that the «time when A is P» and the «time

when A is not P » is the only distinction between two different intervals of time which we can make, how to avoid the contradiction between « A is P » and « A is not P »? It seems very tempting to use the notion of the «local truth» which arises in the categorial logic (namely in the theory of toposes [4]) to solve the problem roughly as this: « A is P » and « A is not P » both are *sometimes* true. However for the moment I do not know how it might be done. However there is a reason to believe that the category theory might be a useful and powerful tool to make theories of time and temporal endurance. The reason is that the category theory itself is an attempt to revive such «dynamical» mathematical notions as function and variable from their allegedly «static» set-theoretic interpretations. That is why many things which are done in the category theory are relevant to philosophical problems about time, change and endurance, and, I believe, might be fruitfully applied to metaphysics.

ⁱ Strictly speaking - with *external* clocks, i.e. clocks which are not B itself.

ⁱⁱ This is not strictly true for the Relativity - though the close analogy between space and time dimensions arises in this theory - however this appears to be *strictly* true within quantum theories of gravitation. See [2].

ⁱⁱⁱ It might be argued that a thing can have *temporal* parts which are not spatial parts. But what then are temporal parts? It would be quite mysterious simply to postulate that there are two kinds of parts. I do not need two kind of parts to explain what are temporal parts. I show that what is called a temporal part is a special case of a spatial part.

^{iv} About the ontology of bubbles, holes and other «superficialities» see [3].

^v I follow the algebraic tradition to write a composition in the reverse order which originates from the habit to write $F(x)$ but not $(x)F$.

^{vi} Of course we can think about many plausible specifications and limitations. I would like however to leave the question open.

^{vii} Proof. Let f and g be monics. Then for any arrows p, s $fp=fs$ entails $p=s$, $g(fp)=g(fs)$ entails $fp=fs$ which again entails $p=s$. But by associativity $g(fp)=(gf)p$ and $g(fs)=(gf)s$. Hence $(gf)p=(gf)s$ entails $p=s$, that is, gf is monic.

^{viii} Here I use the term «part of A » in a looser sense meaning a *thing* which is placed in A by a monic arrow. (The same way in the category theory is used the term «subobject»).

^{ix} This strongly resembles the quantum mechanical views but for the moment I do not know how serious is this analogy.

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