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Rationality and Relativism

1) Big and Bigger

Relativism and its sub-species like Moral Relativism or Cultural Relativism are usually understood as a kind of anything goes attitude. Chris Swoyer (2003) provides a more precise analysis of this notion interpreting propositions "X is relative to Y" in functional terms. He reads Y as an independent variable and X as dependent one. Then this proposition says that when Y is fixed X is uniquely determined but different values of Y correspond to different values of X (so the function is non-constant). If X stands for moral and Y stands for culture the relativism about these things amounts to saying that the former functionally depends on the latter. (Notice that the relativism in question is both moral and cultural but not in the same way.) For the obvious reason this may sound very embarrassing for people looking for definite answers to moral questions. Since the kind of relativism just described seems to imply sceptical views one often wishes to weaken it in one way or another. I shall try to show that the problem about this kind of relativism is exactly the opposite: it is a very weak relativism which doesn't really deserve its name. And stronger versions of relativism don't imply scepticism as we shall shortly see. So a reasonable strategy is to strengthen a weak relativism until it brings non-trivial results rather than try to weaken it. I shall demonstrate this claim first with a toy example and then with real examples from science and mathematics.

Imagine Andrei and Juha in a hard dispute about the question of whether or not Kaliningrad is a big city. Juha argues that it is and Andrei argues that it isn't. Then Jon intervenes and tells Andrei and Juha that their dispute is pointless: one calls a city big or doesn't call it big dependently of his or her personal experience of urban life, and so the claim that Kaliningrad is big (or not big) has no fact of the matter behind it. At this point Andrei and Juha forget about their disagreement and together accuse Jon in relativism. They admit that they don't know yet the definite answer to their question but say that they hope to find it. They say they are open to critical arguments of each other and ready to revise their views when appropriate. They tolerate different opinions but cannot tolerate someone like Jon who doesn't care about knowing the truth and says that everything is relative to anything.

Jon insists that the question whether Kaliningrad is big is ill-posed. Then he points to the fact that thinking about size of a city *relatively* to size of another city allows for questions, which do have precise answers: Kaliningrad is bigger than Reykjavik, Reikjavik is bigger than Turku, etc. Andrei and Juha try to defend their point suggesting the following analogy: unless it is known in advance that both given apples are red it is pointless to ask which one is redder. Similarly, they say, before asking whether or not one city is bigger than another city one should grant that both are big. However they are smart guys and see that the analogy fails. So they accept Jon's proposal and change their ideas about big and bigger.

Notice the two stages of Jon's relativism. At the first step when Jon argued that the dispute between Andrei and Juha was pointless his relativism fitted Swoyer's schema: Jon pointed to the fact that the wanted answer depends on a hidden variable factor. However at the second step, which actually resolved the dispute, Jon no longer meant by relation a kind of dependence. This latter version of relativism was stronger than the former one in the following precise sense. The former sceptical version amounted to saying that the notion of being big splits into a spectrum of different notions (indexed by values of a neglected factor). This diversified the notion of being big but didn't kill it. At the second step John's relativism amounted to the claim that the notion of being big was plainly unsound and needed to be replaced by the relational notion of bigger than. At the first sceptical stage Jon argued - and quite rightly so - that the dispute between Andrei and Juha was pointless. But at the second constructive stage Jon did more than that and proposed a relativistic revision of basic terms of the dispute, which finally produced a reasonable solution. This solution didn't provide a yes-no answer to the discussed question but proposed better questions relevant to the disputed issue. On the contrary to what Andrei and Juha expected the notion of being bigger understood in terms of binary relation "bigger than" didn't make any use of the notion of being big (understood as a predicate or otherwise).

2) Relativism and Relativity

This toy example shows that relativisation of earlier assumed conceptual framework (in particular of one stemming from the common linguistic practice) may be very fruitful. Actually an essential part of evolution of mathematics and physics since early modern times up to now can be described as a progressive relativisation of basic conceptual frameworks. When the idea of relativity of motion in its modern form has been first put forward by Galileo and Descartes it sounded weird for their contemporaries who used to consider the distinction between rest and motion as *the* basic distinction made in the science of physics (like the distinction between good and evil in the science of moral). In the beginning of 20th century relativistic thinking in physics was not only extremely productive for physics itself but also appealing for general public. It is moreover sad that to the end of this century the name of relativism has been strongly associated with the sceptical view, which rightly stresses complicated relational character of studied issues but doesn't provide any interesting solution. The only systematic attempt motivated by Einstein's work to apply relativistic thinking to moral and political issues, about which I know, has been made in 1921by Richard Burdon Haldane (see Haldane 1921), who was a philosophically educated (and philosophically prolific) acting politician but not an academic philosopher. Anyway this attempt remained singular and didn't start any considerable philosophical trend in the Academia.

One might argue that I'm confusing things here and that in fact relativism (cultural, moral, etc.) has nothing to do with the relativistic physics. I agree that these things are different but disagree that they have nothing to do with each other. The relativisation of the notion of being big gives a simplified but still quite adequate picture of how relativisation works in more complicated cases including both Einsten's relativistic theories. The principle idea is always this: given data, which are determined only "relative to" some other data put all the data into a new framework allowing for new well-determined concepts construed out of these data. (The new framework should be specially designed if there is no appropriate one around.) The newly obtained concepts can be then thought of as "absolute" and become a subject of further relativisation.

A more specific point that I need to make before coming back to general issues about rationality concerns relativistic thinking in mathematics. This subject is still waiting for a systematic study. However it is clear that one can do nothing in modern physics without using mathematics and that all breakthroughs in relativistic physics involved (but of course didn't reduce to) new mathematics. I mention this here because I'm going to use some mathematical relativistic frameworks (including one behind Einstein's General relativity) for underpinning general conceptual schemes called *rationalities*. Although I'm quite enthusiastic about using mathematics in social sciences I shall not make here any technical proposal but only suggest some mathematical ideas (presenting them informally) as models for rationality. One might think that this approach is fancy and very modern. In fact it is not. In the next section I show that *Classical* rationality which I identify with the traditional notion of *ratio* also hinges on a simple but very profound mathematical idea.

3) Measure and Ratio (Classical rationality)

The term "ratio" which is the standard Latin translation of Greek "logos" has at least two different meanings (just like its Greek prototype). One is *reason* or *reasoning* with all its further nuances. The other meaning survived in today's English together with the term itself: "ratio" means ratio of numbers or ratio of certain magnitudes (lengths, masses, etc.) I shall not give any historical or etymological argument here but show the relevance of the latter mathematical notion of ratio to the general notion of reasoning.

Consider the notions of measure and measuring first. A very simple setting making measurement possible is this: given a class of pairwise comparable objects a comparison of X and Y has three possible outcomes: $X>Y$, $X<Y$ and $X=Y$. Then measurement of the given objects amounts to the following: one fixes in the given class certain object U called unit and then prescribe to any object X from the given class value (also called *measure*) "big" when $X>U$, value "small" when $X<U$ and a neutral value when $X=U$. Usually one uses a richer setting, which involves natural numbers and possibly numbers of other kinds (so one can ask for 3,5 kg of potatoes in a food market) but these further details are not important here. What is important is that measurement requires fixed units. When units change one may get confused about what is big and what is small and how much he or she should pay for potatoes. True, there are rules of how to switch from one system of units to another one. However this question is less trivial than it might seem. Think about world financial markets. When exchange rates change rapidly the knowledge of proportion is far from being sufficient for understanding what is going on let alone for behaving reasonably. One might think that in the pure mathematics and theoretical physics transformations of units are less problematic but this is plainly wrong. Let me only mention gauge theories in today's physics without talking about them. A simpler example of non-trivial gauge transformation comes from geometry. Euclidean geometry is gauge-invariant in the usual sense: one may think in this case about lengths up to positive constant factor (not forgetting to

square this factor talking about areas). However Lobachevskian geometry is not gauge-invariant in this sense. As people used to say in 19th century Lobachevskian geometry involves absolute units. Greek mathematicians could justify the Euclidean choice and get a plausible proof of Euclid's Fifth Postulate by assuming another postulate according to which geometry must be gauge-invariant. They didn't get this proof (first obtained in 1766 by Lambert; see Bonola 1955) but invented a device for doing geometry in a gauge-independent way, namely the notion of ratio.

A ratio of two given lengths or areas doesn't depend on units used for measuring these lengths or areas, so a geometrical theory formulated in terms of ratios is gauge-invariant. Whether of a given geometrical theory is gauge-invariant in this sense is a different question. Euclidean geometry is. In terms of setting provided in the beginning of this section this means that one may compare given objects pairwise without fixing one term of comparison and still get a coherent structure but not a mess. However one should guess the right way to do this: first produce ratios and then compare the ratios. The notion of comparison of ratios (not to be confused with the comparison of magnitudes) is the most original part of this Greek invention (due to Eudoxus) both mathematically and philosophically.

I don't claim that Greek mathematicians thought about these issues in the same terms in which I present them here. They rather tried to avoid in their theories arbitrary choices justifying this strategy by very general philosophical arguments concerning the priority of necessity over contingency. But the result was anyway the same. When Descartes in his *Geometry* allowed for the arbitrary choice of units this was a significant brake with the earlier tradition. Lambert's work and Lobachevsky's discoveries made it clear that this move was anything but innocent.

Let me now stress a philosophical aspect of the (mathematical) notion of ratio. The concept of measurement is by itself a strong unifying concept allowing for application of the same system of calculus (elementary arithmetic) in various practical situations and in geometry. (Note1) However traditional systems of measurement involve arbitrary choices of units (Note2) and cannot work unless these units are rigidly fixed. Practically speaking, this can hardly be done at the scale of a big community unless it has a strong centralised power. And in any event one such system will be incompatible with another similar system, which uses different units. This makes again a problem unless the community in question is completely closed and self-sustained. One might argue that the uniformity of gauges can be achieved through the invisible hand of the market without any external power. I think that like linguistic conventions gauge conventions may emerge in this natural way only locally (at smaller scales) but not globally. At least this is what actually happened in history.

The notion of ratio suggests another solution, namely a gauge-independent framework. This framework has two functions. First, it allows for thinking about geometrical issues independently of any particular system of measurements. And second, it provides simple and understandable rules of switching from one system of measurement to another. Although the notion of ratio doesn't make the choice of units dispensable in practical issues it allows one to cope effectively with different units.

I qualify such a gauge-independent framework as *relativistic*. I use here the term "relativistic" in a broader sense than the usual one, which refers to relativistic theories in physics. I do this deliberately because I think that relativity is a philosophical idea, which is too important to associate its name only with a particular physical theory. I associate adjective "relativistic" with term "relativism" understanding by this term relativistic thinking but not the sceptical relativism described in the beginning of this paper. This I also do deliberately trying to make relativism more respectful.

This is how the mathematical notion of ratio can be used as a model of universal reason: just like mathematical ratio provides a common ground for different systems of measurement a more general philosophical ratio provides a common ground for different individual and collective ways of thinking. True, ancient thinkers stressed the absolute rather than the relativistic aspect of ratio. But these are two sides of the same coin as we have already seen. And the relativistic aspect of Greek *logos* and Latin *ratio* shouldn't be overlooked in any event. Moreover so since in the following development of this concept its relativistic aspect became more important (at least if we are talking about science and mathematics). I shall use the above relativistic reconstruction of the notion of ratio for my working definition of rationality. By *rationality* I shall understand a relativistic conceptual framework of a very wide use, which in particular can be used for thinking about social and political issues. As we shall see mathematics suggests some models of rationality, which essentially differ from one just described and which are more powerful. I hope that after reading this section the reader doesn't consider the idea of building rationalities using mathematical models as exotic.

4) Viewpoints (Cartesian rationality)

The concept of *coordinate system* or *frame* usually associated with the name of Descartes allowed for considering Euclidean geometry as relativistic in a stronger sense than one mentioned above. A coordinate system is a simple construction that provides a one-one correspondence between points of the plane and pairs of real numbers called *coordinates* of corresponding points. Using algebraic means one may think then of various geometric objects in terms of real numbers (Note 4). Numerical codes (coordinates) of the same object provided by different coordinate systems are different. How to cope then with all these different codes and not get confused?

There are two approaches to this problem. The first approach relies on a generalised version of the ancient idea of ratio. Given different codes, which presumably code one and the same thing, find an algebraic expression such that substitution of coordinates of points gives the same result in all coordinate systems. Such expression cannot be found when only one point is taken into consideration but two points is already enough: the *distance* between two given points doesn't depend on the choice of coordinate system (provided they use the same length unit) and it can be calculated in every coordinate system by the well-known formula based on the Pythagorean theorem. Thus anything that is done in a coordinate system in terms of lengths (distances between points) doesn't depend of the choice of this particular coordinate system (except that in some coordinate systems calculations turn to be easier than in some others). Since in terms of lengths one can do the whole of Euclidean geometry (and even more) the problem is resolved. Unlike the case of Greek mathematics here arbitrary choices are not completely avoided but only "neutralised" by the above arguments.

The second approach amounts to looking for explicit rules of conversion from one numerical code to another, i.e. of transformations of coordinates given in different coordinate system. While the notion of ratio provides rules of conversion of one gauge system into another immediately the formula for calculating distances between given points by their coordinates doesn't make it immediately clear how to switch between different coordinate systems. A straightforward mathematical solution of this latter problem involves a different setting, and in the present context this different setting is more interesting than the problem itself because it suggests a different notion of relativistic framework (and hence a different notion of rationality). Physicists often think about spatio-temporal frames as viewpoints associated with particular observers. This interpretation is very useful in the present discussion. As a particular viewpoint any given coordinate system provides an image of any other coordinate system as well as an image of anything what this other coordinate system represents in its turn (including other coordinate systems). We get here a simplified mathematical model of Leibniz' monadology: think about a class of viewpoints (coordinate systems) every one of which perfectly represents any other. (What makes the difference between the worlds of Cartesian and Leibnizian monads is that Cartesian monads are perfectly transparent for each other while Leibnizian ones "have no windows".) When one puts this metaphysical picture into mathematical terms this gives an immediate solution of the problem of transformation of coordinates. An image of one coordinate system in another coordinate system makes it clear how the latter is transformed into the former.

A combination of both aforementioned approaches results into the fundamental notion of *invariance through transformation* (of viewpoints). In Special Relativity features invariant under such transformations count as objective (and real when one is a realist), i.e., independent from any particular viewpoint. However this independence should be understood with a pitch of salt because it cannot any longer be thought of without the totality of admissible viewpoints and their mutual transformations. This makes the principle difference between the relativistic framework described in this section and Classical rationality described earlier.

The assumption of full transparency and essential equivalence of all viewpoints implies the following non-trivial fact. In order to describe all admissible transformations between all available viewpoints (coordinate systems) it is sufficient to consider just one viewpoint (one coordinate system) and all its transformations into itself. This obvious mathematical fact (I mean the case of Cartesian coordinate systems) sheds a light on the notion of transcendental philosophy and more generally on what may be called the philosophy of Ego. The basic idea behind this kind of philosophy is that an universal collective rationality can be fully recovered through a single viewpoint labelled as Ego or First Person. The coherency between philosophical and mathematical work of Descartes who pioneered both the philosophy of Ego and the method of coordinates in mathematics is fairly striking even if not particularly surprising from a historical viewpoint! To praise Descartes' genius once again I shall call the rationality described in this section *Cartesian*.

5) Local and Global properties of Manifolds (Riemanean rationality)

When the project "Rationality in Local and Global Contexts" was at the stage of preparation its future participants had very different ideas in their minds about the proposed title. The idea I had in mind was that of Riemanean manifold (I wonder if anybody else had the same). This a mathematical concept, which makes the distinction between local and global properties particularly clear. It is known to general public under the appealing name of curved space.

The notion of manifold I am talking about has been introduced by Riemann in his (1854). It provides a mathematical background for Einstein's General Relativity, which until today remains the best theory of physical spacetime. While Special Relativity is based on Cartesian rationality General Relativity involves a different kind of rationality. This can be seen through an analysis of the notion of manifold alone. In this section I show how the notion of Riemanean manifold brings a new relativistic framework, i.e. a new rationality. I shall call this new rationality *Riemanean*. (Note 4)

This new rationality is obtained from Cartesian rationality through weakening the transparency condition. I shall explain this informally without going to mathematical details. To get a hint about how it works mathematically think about the viewpoints as coordinate systems as before. In the Cartesian setting any viewpoint was in the view of any other. In the Riemanean setting a given viewpoint has in its view only a (small) part of other viewpoints. Other viewpoints are behind its *horizon*. The usual notion of horizon (the limit of visibility existing due to the spherical form of the earth) is perfectly relevant here; the globe with a network of human observers on its surface is a sound model of manifold but not only a metaphor. Each observer sees some other observers but none of them can see all the observers at once. This brings the distinction between local and global properties of a given manifold: "local" refers to a neighbourhood of a given observer covered by his or her viewpoint and "global" refers to the whole thing (the globe with observers on it). The picture of a ball floating in outer space is (helpful but) misleading here since it assumes an external observer. We shall see shortly how this global picture can be produced differently.

An image obtained through any particular viewpoint is self-transformable like in the previous case. In the language of physicists these transformations are called local symmetries. However this time symmetries of a given image are not equivalent to transformations between different images. Each image is transformable into its neighbouring images (ones obtained through neighbouring viewpoints); these latter transformations are not local symmetries but they are also very simple (linear). What prevents the collapse of all the viewpoints into one like in the Cartesian case is not the difference of their local symmetries - they are usually supposed to be the same - but the difference of transformations between different neighbouring viewpoint. (Mathematically these later transformations are specified through the notion of *tensor*.) Since different viewpoints have different neighbouring viewpoints (different observers have different horizons) and since transformations are supposed to be composable (a transformation *A* -->*B* followed by another transformations *B* -->*C* produces transformation *A* -->*C*) it may turn out that any given viewpoint can be transformed to any other viewpoint through a number of intermediate viewpoints (although more generally this may be not the case). Coming back to the picture of observers located around the globe imagine that one of them travels taking viewpoints of all other observers met on the way. Using her memory this traveller can arrange for communication of all other observers even when most of these observers are found outside of horizons of each other. (However in the case when each observer is out of the view of any other this wouldn't work because the traveller wouldn't know where to go.) The global communication so established can perform (and does perform unless the given setting reduces to Cartesian one) features, which cannot be possibly detected from any particular viewpoint. In particular the property of Earth of being ball-like can be tested only by a traveller but cannot be detected by an immovable observer. Such global properties of manifolds are called *topological.*

We get here a kind of collective rationality unaccountable by the philosophy of ego in spite of the fact that all the involved egos (viewpoints) may be perfectly the same like in the Cartesian case. What make the difference are not intrinsic differences between egos but the lack of full transparency in their communication. In the Riemanean setting one gets a rich structure of possible *paths* between given viewpoints, which has no counterpart in the Cartesian case. If immovable viewpoints are interpreted as temporal stages of movable ones (as it is usually done in General Relativity) the notion of path turns to be basic.

In my view the notion of manifold provides a reasonable model of global human communication across political, cultural and other boundaries. (Notice that the notion of boundary is topological.) It seems to be more realistic than the Cartesian model, which assumes the full transparency. This makes it reasonable to think of applications of Riemanean geometry and topology in social sciences or at least of using some *ideas* coming from this part of mathematics in social sciences. The main lesson of Riemanean geometry for social sciences seems to be this: Cartesian rationality developed in 17-19 centuries, which assumes the full transparency and the full grasp of the whole world of possible viewpoints by each particular viewpoint, works *only locally* while the global society needs very different principles of its organisation. Riemanean rationality described in this section is a possible global solution. Remarkably it doesn't require any drastic change of local structures but hinges on the idea of bilateral *connection* between neighbouring local structures. But this model of global rationality has its own limits. It can be called *settled* in the sense that it assumes a fixed immutable topology. Given today's rapidly changing political geography and fast economical and cultural processes at the global scale the settled character of Riemanean rationality is a serious shortcoming. Let me now present my favourite rationality, which is more modern and more dynamic.

6) Toposes (translational rationality)

Both Cartesian and Riemanean rationalities involve the notion of transformation, which I also called *translation* in appropriate contexts. Cartesian rationality in addition involves the notion of *invariance* under transformations allowing one to specify a precise sense in which all admissible viewpoints are *equivalent*. In physics this notion of invariance allows for basic epistemic distinction between *objective* physical features and "subjective" features, which are specific to particular viewpoints and count as observational artefacts (see section 4).

Does the notion of invariance with its epistemic implications survive in Riemanean setting? The straightforward answer to this question is in negative: Cartesian invariance holds here only locally (remind the notion of local symmetry). However it seems that neither Einstein himself nor any of Einstein's followers (including Eddington and Weyl) ever seriously considered epistemic implications of this fact. (Given epistemological challenges of Quantum Mechanics the difference between Einstein's two relativistic theories with respect to objectivity might seem to be negligible at the time.) It is moreover remarkable that an official postulate of General Relativity reads as follows: laws of physics are *covariant* with respect to (rather than invariant under) transformations between local coordinate systems. In the Riemanean setting the difference between invariance and covariance is not easy to explain without going to mathematical details. Very roughly "covariance" means coherence of different variations. (But the covariance of physical laws in Einstein's understanding doesn't mean that the laws change!) However this difference becomes much clearer in a more general setting, which I am now going to present. Its mathematical background is provided by *Category theory* emerged in 1945 (see Marquis 2006). This theory is so far only tentatively applied in physics and other sciences. But one doesn't need to wait for further applications of Category theory in science to see that it provides a new generalised model of rationality.

Category theory has no immediate connection with the philosophical notion of category in the sense of Aristotle or Kant. In fact it is a general theory of transformations called in this theory *morphisms*. Here are basic definitions and axioms. A category comprises a class of *objects* and classes of morphisms specified for every (ordered) pair of objects. Morphisms are composable in the natural way: morphism f followed by morphism g is a new morphism fg. One should keep in mind that "followed by" implies the following condition: g starts exactly where f ends. So, generally speaking, not all morphisms of a given category are composable. The composition of morphisms is associative and each object A is supposed to have a special morphism into itself called *identity of A* with the following property: when it is composed (in the right sense) with morphisms coming into A and going out of A this gives the same incoming and outgoing morphisms (so for these morphisms the identity of A behaves as unit). This basic construction involves only very general assumption about transformations and serves for a plethora of very different special cases.

Talking earlier about transformations (of viewpoints or coordinate systems) I made an additional assumption, which remained hidden. Namely, I took it for granted that all transformations in question were *reversible*. This assumption is strongly supported by the usual spatial intuition: if I look at a stature and then make a move and take a different viewpoint I can always return to the first viewpoint. The same assumption of reversibility is involved into the notion of coding: a coded message can be decoded (at least in principle); otherwise the message is not coded but destroyed. The notion of category introduced above allows for the following precise definition of reversibility. A given morphism f is called reversible or *isomorphism* iff there exists morphism g (called the *reverse* of f) such that *both* compositions fg and gf are identities (possibly of different objects). Beware that none of the two conditions suffices if taken alone. Generally, morphisms in a category are not supposed to be reversible.

To see that the assumption of reversibility played indeed an essential role in the previous discussion let's come back to the example of a stature observed from different viewpoints. However naive this picture might be it serves as a good model of how objective and subjective features are distinguished both in Cartesian and Riemanean frameworks. In the Cartesian case one additionally assumes that the viewer has a magic power of vision, which allows her to see the whole thing at once from any perspective, while the more general Riemanean framework allows the viewer to be myopic. This picture immediately implies reversibility of transformations between different viewpoints because spatial motions of the observer (like all spatial motions) are reversible. When the reversibility of transformations is lifted one's spatial intuition gets completely confused. Imagine yourself travelling in an environment, which doesn't allow you (or someone else) to go back where you have already been before. This kind of environment cannot be anything like a space, no matter flat or curved. General Relativity allows for points of no return only exceptionally in limited areas known as black holes while I am now talking about a situation when, on the contrary, irreversibility is a rule and reversibility is an exception. Remarkably the notion of invariance under transformation, which is fundamental for Cartesian rationality and which in certain form also survives in Riemanean rationality, doesn't make sense when transformations are irreversible. Here is a proof. Any invariant structure like a pair of distant points on the Euclidean plane can be identified with a class of all its images obtained with different coordinate systems (different viewpoints). These images are all isomorphic in the sense that they are transformable into each other by reversible transformations. Isomorphism of objects (i.e. the existence of reversible morphism *A*-->*B* between given objects) is an equivalence relation, hence the classes. But the existence of general morphism *A*-->*B* is not equivalence relation on objects because it is not symmetric. So the usual way of thinking about invariance doesn't apply in this latter case. But the notion of covariance survives in the non-reversible case too as we shall shortly see. One might consider covariance as a suitable upgrade of invariance but in my view this is rather misleading since covariance has nothing to do with the idea of being constant.

Although irreversible transformations are ubiquitous in the everyday life - think of broken glasses and grown children - they become very problematic when one tries to take them seriously and avoids explaining them away in terms of some underlying reversible process like motion of atoms (Note 6). Let's consider an example of irreversible transformation, which not only shows the problem but also suggests a solution. This example is linguistic.

Think about translations between natural languages. Unlike coding linguistic translations is generally non-reversible. To see this consider the case of word-to-word translation between English and Russian. Russian word "porosha", which means a particular kind of snow (resembling a tiny hail), for the best of my knowledge don't have an exact equivalent in English, so its best English word-to-word translation is "snow". Translating "snow" back into Russian one gets not "porosha" but "sneg", which is the precise equivalent of

English "snow". So the translation of "porosha" into "snow" is irreversible (while the translation of "snow" into "sneg" is reversible). In the case of more involved text-to-text (rather then word-to-word) translations things get far more complicated, so without giving further examples I suggest that in this more general case translations aren't generally reversible either. So I assume that the non-reversibility of linguistic translations is a general phenomenon.

This phenomenon is problematic for the following reason. It is natural to think about linguistic translation (and paraphrase which is a translation from a given language into itself) as transformation, which preserve *meaning*. But as I have already proved invariants of transformations (like meaning) are allowed only in the reversible case. Now we can see how this general theorem applies to our example: when "porosha" is translated into "snow" its meaning is *partly* lost. This is the usual description of this situation. However it is clear that the naive mereology behind this "partly" is irrelevant. Whatever meaning might be it is not a kind of thing one can cut into parts. The understanding of irreversible translations as imprecise translations stems from the notion of meaning as invariant under translation. And this notion makes sense only when translations in question are reversible. Since this latter condition is not realistic one have to change the notion of meaning rather than say that translations are bad. From a categorical point of view the idea that irreversible morphisms are imprecise isomorphisms looks simply absurd.

The remedy is rather obvious. It is suggested by the very idea of *categorification* (construing concepts as categories): make the extension of a given concept (i.e. the class of all individuals falling under the given concept) into a category by providing these individuals with appropriate morphisms. Following this recommendation think about meaning not as a class of particular meanings but as a category of particular meanings, which transform into each other (without changing their identities) just like words (and texts) transform into each other by linguistic translations. So meaning becomes *dynamic*. I shall not specify how exactly to define morphisms of meanings: this depends on what kind of dynamics one wishes to take into consideration (it may and may not involve the diachronic aspect of language). The notion of morphism between languages is more straightforward: morphisms of languages are translations. Now consider *functors* from a given category of languages to an appropriate category of meanings. Functor is a morphism between different categories, which takes objects to objects and morphisms to morphisms in such a way that identities and composition of morphisms are respected. The notion of functor grasps that of covariance in its most general form. It allows for covariance where no invariance is available. I refer the reader to any standard introduction into Category theory for details (see Marquis 2006 for further references). I shall not further elaborate on this sketch of a categorical theory of meaning but only stress the idea: meaning should be thought of as covariant (i.e. linked functorially) with linguistic translation rather than invariant through translation.

Notice the double effect of categorification: it brings in irreversible transformations and makes things dynamic. The idea of reality as a stature, which remains immutable in the chaos of human opinions, is a Platonic reflex, which in my view must be definitely abandoned in science. Irreversible transformations of viewpoints turn the stature-like objective reality into an objective dynamic category, which has to be reasonably distinguished from a corresponding *subjective* category of viewpoints. In the current jargon of physicists there already exist appropriate terms describing this dynamic setting: they distinguish between *active* (objective) and *passive* transformations (transformations of viewpoints). The reality undergoing active transformations doesn't look at all as a stature. And its dynamics needs not to be reversible in order to be conceivable.

The idea to look at categories as relativistic frameworks is straightforward: think of a category of viewpoints where morphisms are transformations of viewpoints.

Categories of a particular type called *toposes* are particularly interesting when they are seen in this way. Why the question where the city of Kaliningrad is big doesn't have a definite answer but the question whether Kaliningrad is bigger than Reykjavik does? One may explain this providing a justified answer to the latter question and pointing to the fact that all possible answers to the former question are not justified. But this explanation doesn't shed any light on a more general problem: Which conceptual framework allow for well-posed questions and which don't? (I call a given question well-posed if it has a definite answer. This definite answer may be of the yes-no kind, but may be also different. In all cases considered earlier in this paper truth-values are connected to invariant features allowed by a given framework: when something is true or false this is the case independently of any particular viewpoint. This assumption is easy to challenge without being a sceptic: proposition *it rains* can be true from Kaliningrad's viewpoint and false from Reikjavik's viewpoint. Frege believed that this puzzle has an easy solution: proposition *it rains* is ill-formed, so it is not indeed a proposition but rather a piece of a proposition which needs to be completed; to make it into a full-fledged proposition one should specify *where* and *when* it rains; the truth-value of the resulting proposition obviously depends on such specification. This Frege's solution was already in odds with physics of his time, and it is moreover in odds with today's physics. The whole point of relativistic theories in physics is that an assumption of background spacetime frame which might allow for an uniform global indexing of events has no physical sense, and so the spacetime must be construed as a relativistic frame (Cartesian, Riemanean or other). This relativistic argument provides some support to the naive view according to which proposition (or *statement* if one prefers) *it rains* can be true today in Kaliningrad and false tomorrow in Reikjavik. In topos theory this naive idea of "local truth" is taken seriously and worked out technically. However the global aspect of topos logic shouldn't be overlooked either. In a topos (viewed as a relativistic frame) true-values are not simply independent from or invariant through viewpoints but they are covariant with viewpoints: the truth-value of proposition *it rains* changes *with* changing of a viewpoint and this works uniformly for the whole topos in question. The logical structure of a given topos hinges on this "with".

Toposes and other categories viewed as relativistic frameworks provide a new type of rationality, which I shall call *translational*. Let me stress the principal difference between the translational rationality and other rationalities considered in this paper. All these other rationalities hinged on the notion of *equivalence* of different viewpoints. This equivalence was construed either rather straightforwardly like in Classical and

Cartesian rationalities or in a more sophisticated way like in Riemanean rationality. Equivalence is not identity. Things may be drastically different but equivalent in some respect. For example, it may happen that texts looking very differently have the same meaning. Or it may happen that different systems of moral beliefs share some hidden common principles, which can be revealed through an appropriate reflection. One might think that without such underlying equivalences nothing like global rationality could be possible. The notion of translational rationality shows that this is not the case and suggests different solutions. Global rationalities can be built out of translations between viewpoints even when these translations are generally non-reversible and so don't allow for global equivalences. The strength of a given translational framework depends on assumed properties of translations. When one requires translations to be reversible this reduces the translational rationality to Cartesian or Riemanean (in a slightly generalised sense). However as the notion of topos clearly demonstrates there are strong global solutions, which don't require the reversibility of translations.

7) Conclusion: Rationality and Globalisation

The development of science during last few centuries can be viewed as a progressive exploration of our living environment allowing for better prediction and in some cases even for an effective control. The most obvious aspect of this exploration concerns spatial and temporal scales. Given science of his time Kant was perfectly right when he qualified cosmological questions as metaphysical, i.e., being out of the reach of the empirical physics. However since then the situation changed dramatically and nowadays cosmological theories became empirically testable like any other scientific. A similar point can be made about microscopic scales. Remind that the atomic hypothesis was seen by many physicists in late 19th century as highly speculative or even purely metaphysical in spite of its great explanatory power. Given the experimental work done in 20th century today this view is certainly untenable.

For an empirically-minded person like me it doesn't seem particularly surprising that the extension of the domain of science requires new physical principles as well as new mathematics. Euclidean geometry which works reasonably well in the scale interval between the human scale and the scale inter-planetary distances fails to do so both in the micro-world and at the cosmological scales. Einstein showed that at the cosmological scales Riemanean geometry works better; the former problem remains open.

In the human overall social and political development there takes place a similar process known today under the name of *globalisation*. By globalisation I mean the growth of integrity of the humankind and by integrity I mean the overall interaction between different parts of the humankind disregarding its character and nature. (So I don't think about the integrity of the humankind as necessarily stable and peaceful. This might be a desired end but not a matter of fact. In particular I count world wars as evidences of globalisation.) The globalisation so understood seems to be a long-term trend of development of the humankind, which presently passes through a critical phase. The crucial conceptual problem about globalisation seems to be this: just like physics of middle scales fails at the cosmological scales social, economic, cultural and political models developed for human communities called tribes, peoples, nations and the like fail at the scale of the humankind and even at lower social scales (as shows the sad end of multi-national empires in 20th century). Hence the urgent need of new global social, political and cultural models. I believe that mathematical approaches presented in this paper, which solve similar problems in physics, are very suggestive for developing such new global models. I tried to interpret mathematical notions in a form which facilitates such applications. The principle of relativity, which comes from physics, is, in my view, a very good general principle for organising the global human community and its environment.

Endnotes:

Note 1. The case of geometry involves the famous problem concerning the existence of non-commensurable magnitudes but I leave it now aside and only mention that an elaborated notion of ratio due to Eudoxus copes with this case too.

Note 2. The choice of units is not *always* arbitrary even in the traditional gauge systems. In particular in the case of time measurement there are *natural* astronomic gauges like day and year. In contemporary gauge systems all basic units are in some sense natural (while old arbitrary units like meters and kilograms remaining in the everyday use are specified in terms of these basic units).

Note 3. Some people suggested me to use terms "relational" and "relationalism" instead of "relativistic" and "relativism" but I cannot accept this suggestion because these terms refer to the logical notion of relation, which is too specific for my purpose. The standard logical notion of relation as a predicate of arity more than one doesn't account for many important relativistic frameworks. It covers Jon's relativism about being big and simplest forms of gauge relativity but it doesn't cover Einstein's relativism about spacetime or relativistic gauge theories developed in today's physics. "Relativism" sounds better than "relationalism", which is difficult to pronounce.

Note 4. I present here the method of coordinates in its modern form, which involves the notion of real number. Descartes himself didn't think about this method as a method of coding of geometrical figures into numbers; his idea was to apply algebra to geometry directly through a proper definition of sum and product of straight segments. For my purposes the difference between these two versions of the method doesn't matter.

Note 5. One of few philosophers who saw the philosophical significance of the notion of Riemanean manifolds was Husserl. He purported to develop a more general phenomenological notion of manifold (under the same name of *Mannifaltigkeit*) wholly independently from its mathematical source. In my view this was an unhappy decision, which made Husserl's ideas more difficult to understand. I managed to make a better sense of what Husserl mean by *Mannifaltigkeit* after I learnt that this Husserl's notion was motivated by Riemann's (see Miller 1982).

Note 6. Whether or not there exist fundamental irreversible physical processes remains an open question. The standard answer is in negative. It has been challenged many times on different grounds, most prominently by Prigogine (see his 1980) and his school. My philosophical worry about the claim of fundamental reversibility in physics is this: this claim is backed by strong epistemic assumptions and by existing mathematical apparatus of physics (which hardly allows for modelling non-reversible processes if any) and hence it is an a priori claim rather than an empirically testable hypothesis.

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