A.Rodin

On the grammatology of mathematics and logic

It would be unfair to say that modern thinkers pay little attention to the works of Jaques Derrida. However historians and philosophers of mathematics have ignored this author till now - in spite of the fact that Derrida himself repeatedly refered to mathematics in his early works. To understand the importance of the phenomenon of mathematics for Derrida it is sufficient to say that in mathematical texts we deal with non-phonetic (geometrical drafts, algebraic formulae) as well as phonetic writing (discourses in natural language written by alphabetic writing). That is why all principal themes of "Grammatology" [1] are present in mathematics: phonetization of writing, submission of graphics by logos of mathematical definitions and proofs, making non-phonetic graphics play a supplementary "illustrative" role. The formula "geometry is the art of good reasoning with bad drafts" popular among mathematicians is the exact expression of mathematical logocentrism. It is also important that the drama of logocentrism should be not only fundamental but external for mathematics - this drama can be clearly seen in mathematical arguments on "problems" and "theorems" in the 1-st and 2-nd centuries, between "algebraists" and "geometrists" in the 18-th and 19-th centuries, between mathematical logicism and intuitionism in the 20-th century. As for the recent discussions we can mention the discussion concerning B.Mandelbrot's "fractal geometry". All this makes it clear that Derrida's "Grammatology" allows us to take a new look at the history and philosophy of mathematics. Since an old question of the relationship between mathematics and logics occures in this context, we mentioned it in the name of the article.

Mathematics

The most important event in development of mathematics is the appearance of mathematical proofs. It is widely considered that at the moment mathematics is gaining a theoretical status [2]. What were the first proofs of Thales, which are mentioned by Proclus (Comm. in Eucl.), it remains unknown. The only thing one can be confident about is as follows: Thales added speech to graphics (drafts), "spoke about" graphics. The transition to the point of view, according to which such "speaking about" is an integral part if not the main content of mathematics distinguishing itself as a free art (knowledge epistemh) from applied calculating skills (tecnai) , we shall call *logization* of mathematics (deriving the term not from "logic", but directly from "logos"). While the logization of geometry we ascribe to Thales, the logization of the second basic ancient mathematical discipline - arithmetic, should be obviously ascribed to Pythagoras. Logization of arithmetic is "speaking about" graphic count. In fact outside logos arithmetic and geometry can't be distinguished at all: any arithmetical script can be considered as geometrical draft, for example the cipher "0" as an oval. Geometrical drafts and arithmetical scripts are distinguished only by logos and below we'll point out the way it does it.

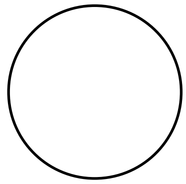
What is the logization of mathematics, what is its structure? The answer that seems to be natural is as follows: before Thales they had been accepting mathematical truths dogmatically; Thales, being imbued with critical spirit, called these truths in question and tried to investigate the matter independently, to find the reasons for these truths or to refute them, to convince of these truths himself and his critically thinking audience or to discover their falsity. However after Derrida we have to say : the notion of truth itself presupposes the logization; even more so the mentioned above platonic distinction between the "knowledge" and the "truth opinion", between truths obtained by critical work of the reason and uncritically accepted, but true dogmata. Actually the truth outside speech is unthinkable. If a truth is written down it becomes a kind of "second rate truth", a "truth opinion" or "dead dogma" as opposed to a dialectic of the "living plenitude of the logos", and its truth anyway is preserved only by reference to speech, i.e. by the phonetic nature of alphabetic writing. Graphics itself cannot be true or false, for example a primitive ornament cannot be true or false. That is why the approach to logization mentioned above is a particular self-interpretation of the logos; to see what stands under logization here and not to miss this under-standing itself we have to examine this problem from a different point of view. (It cannot be of course an "objective" approach, because the objectivity obviously depends on the notion of truth.)

Thus the logization of mathematics which is usually called "the appearance of mathematical proofs" is not a transition from dogmatic acceptation of truths to critical understanding of them, but this is logization that puts into mathematics the truth itself. Briefly speaking propositions claiming to be true appear together with proofs (1). Speaking only about "the appearance of proofs" they obviously ignore the other side of the coin (2).

The fact, that mathematics can somehow exist outside truth without being under a delusion, but rather outside the field of truth in general allows us to consider the truth in mathematics as one of participants of the game, but not as a referee. As a matter of fact such an approach justifies a mathematical status of B.Mandelbrot's "fractal geometry" [3] and of his followers' works. The main part of Mandelbrot's work is a development of the new type of mathematical graphic with computer. Mandelbrot's critics do not agree to call a development of the new graphic a mathematical result. Surely all this staff can be reduced to the terminological strife on the meaning of "mathematics"; if however not to limit yourself by the frameworks of logocentrism - and mathematicians are inclined to do it less than others recognizing "their matter" in Egyptian hieroglyphs as well as in primitive ornaments [5,6] - Mandelbrot's results are to be considered as mathematical ones in the very important sense of the word.

We say: seeking for truth is a free business of a free man. So the logization appears to be the "liberation" of mathematics, its transformation from skill to "free art" mentioned by Proclus (ib.) and ascribed by him to the younger Thales' contemporary Pythagoras. However this "liberation" also has its reverse side: the notion of skill and that of technical means in opposition to the truth as an absolute goal presuppose the logization. In other words the interpretation of non-logizied mathematics as applied, serving to an external goal and for this reason "slavish" is a self-interpretation of the logos. The field of truth given by logization is at the same time the field of (any possible) teleology where speech as a goal opposes graphics as a means serving this goal. What we call freedom here is a domination of the speech over graphics. This cultural situation Derrida calles "logocentrism".

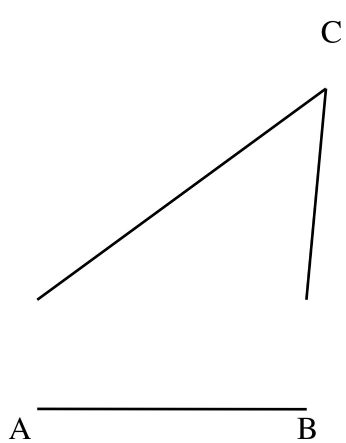
So the history of *theoretical* mathematics starts with "speaking about" graphics. However as early as in the 5-th or 4-th centuries B.C. the first mathematical texts appear, i.e. speeches going with mathematical graphics are written down in usual phonetic alphabetic writing. That is the fact the cause of which we are not going to discuss here that phonetic writing i.e. recording of speech appears to be linear: speech in writing is transformed to a linear sequence of alphabet's letters and punctuation marks; all two-dimensional differences are limited by the line's pitch and fixed. Phonetization of writing at the same time appears to be its linearization. So we can consider a (theoretical) mathematical text as a linearized mathematical graphics. The definition "the circle is a flat figure bounded by the line such as all the straight lines drawn from an internal point of this figure to this line are equal each others" (Cf. Eucl.Elem. def.1.15) "transforms the circle's draft into the line":



The circle is a flat figure ...

Perhaps there are no other texts of the Western culture except mathematical ones were linearization of writing mentioned by Derrida can be seen so clearly.

We have just pointed to the two extremes: geometrical drafts (non-phonetic non-linear graphics) and written reasonings in natural language about these drafts (phonetic linear graphics). However in mathematical texts there are also intermediate levels of writing. First of all let us take into consideration notations in letters of geometrical figures . When in a mathematical text the triangle's apexes are lettered as A, B and C, we can speak about a non-phonetic using of phonetic alphabet. Because it does not matter what sounds correspond to these alphabet's characters, but one has only to be able to distinguish these characters between each others and to differ these characters from other ones (if there is any). That is why for such purpose one can use the different letters corresponding to one and the same sound, for example lower case and capital letters. Nevertheless phonetic nature of the alphabet characters enables to linearize and sound the script:



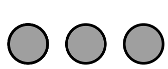
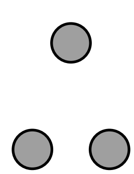
as "triangle ABC" .

(If letters with the same phonetic value are used, linearization and phonetization demands some additional efforts. For example one needs to say "A-capital" and "a-small" instead of making the sound [ei], which corresponds to the characters of Latin alphabet "A" and "a".)

"ABC" in linear record appears to be a *proper name* of geometrical figure. This mathematical example confirms the fact that proper names have a scriptural but not oral nature. When one says "triangle" it is always "triangle in general". To specify this noun (onoma) it is turned into speech (logos), in the given example into the definition of triangle. Only in this case it is fair to say that the noun has sense. Proper names can not be reduced to speech. That's why they are arbitrary and have no sense (3). So proper names mediate between graphic and speech: they can be used in speech and even have some glimpse of sense (cf. nicknames), but as such they are senseless and have a graphical nature.

The mediative role of the notation in letters in geometry can also be seen from the other point of view. As we said above the name "ABC" regarding the noun "triangle" realizes an individuating function: "triangle ABC" is not a "triangle in general", but is a concrete triangle. However regarding the triangle's draft with apexes notated as "A","B" and "C" this function of the name "ABC" is reversed. We say "the triangle ABC is given" and then prove some proposition about this triangle. Obviously this proposition must relate not to the used draft but to the "triangle in general". Although we notate as "ABC" the triangle on our draft we understand that we can use another draft (for example drown near the old one) continuing to use the same name for anew drown triangle. Thus regarding speech, the notation in letters realizes an individuating function; and regarding draft the same notation realizes generalizating (inductive) function (4).

Let us now consider arithmetic. What differs arithmetic from geometry is not the nature of its graphic but the way of logization of graphic. As we said above there is no graphema which could not be taken as a geometrical object. For example every letter can be taken as a geometrical object: what makes a letter to be itself is not its graphical peculiarities but a reference to the corresponding sound. The same is true for any recording of numbers and arithmetical operations' signs. What is a cipher then? Cipher is a proper name of a graphema of corresponding type. For example "3" in so-called "Arabic" or "III" in "Roman" system of numeration is the name of a graphema such as

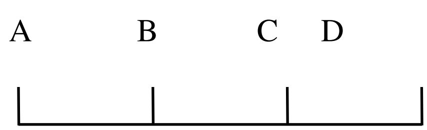
 or  and so on.

Note, that as well as a notation in letters for triangle, this name is not collective one of the graphemata of corresponding kind, but that of a concrete though arbitrary graphema of the kind. Ciphers mediate between graphic and speech as well as notations in letters of geometrical figures. On the one hand the cipher "III" is only linearized graphema of the initial level. On the other hand this graphema corresponds to the numeral "three". The visual connection between a cipher (a graphema of the second level) and a graphema of the initial level is not obligatory here. On the contrary it is rudimental - as well as the name of a man is not necessarily connected with his real features. We use the word *nickname* for any name of a person which describes some aspect of him or her, but yet is not his/her own (proper). So a nickname remains partly a common noun, even if for a certain group of people it is associated with a certain individual. Therefore we can call the cipher "III" (but not "3") a nickname of a graphema of initial level. In case there is no visual connection between graphemata of the first and of the second level, i.e. between "numerical images" and ciphers, ciphers also are not a completely phonetic graphemata. Actually the main claim laid to a cipher is a possibility to define a corresponding number, i.e. to distinguish ciphers and corresponding numbers between each other. For that purpose the sound value of a cipher does no matter. (We saw the same in notation of geometrical figures in letters.) But on the other hand any cipher may be sounded. So a cipher as a proper name of the number mediates between spoken (common) name of the number and the graphema of the initial level. Notice that it is Greek mathematics that makes our analogy between notation of geometrical figures in letters and use of ciphers more accurate because the same letters with a special mark are used as ciphers there.

The possibility to sound ciphers makes it possible to record big numbers with the limited set of ciphers and to organize calculations. The linear script "1995" for instance does not refer directly to a graphema of the initial level like the cipher "5", but presupposes an explanation of the numeration. While separate cipher is sounded as a number's name, a numerical script with several ciphers is sounded as a number's name with its explanation. (It concerns not only positional but any other numeration as well.) On the other hand numeration is an independent graphic: one can know nothing about initial level's graphic and about phonetic value of numerical scripts and operate with these scripts by the rules of calculations. It is the speech which mediates and distinguishes the two levels of graphic. Outside speech a cipher or any other numerical script does not differ from any other image. More precisely speaking they differ from any other images just the same way that any image differs from another one.

What we have said above concerned not the theoretical arithmetic, but a calculating practice which is found in ancient epochs all over the world. That is why logization must be considered here not in the way we did it in the case of Greek theoretical geometry: there is no aim to "transform" graphic into logos here; there is no reason to consider numeration as a technical mean and an oral calculational discourse as a goal, an explanation of numeration and calculations can be considered otherwise as an auxiliary mean of the count. (It limits our analogy between notation of geometrical figures in letters and ciphers .) What is the *theoretical* arithmetics that had been developing in ancient Greece at least from Pythagoras' times? As far as Pythagoras' arithmetic is concerned it is hard to say something above the statement that it included some kind of "speaking about" calculations. Meanwhile just as in Thales' case the very fact that such "speaking about" was the main part of Pythagoras' activities (for Pythagoras as arithmetician is known not because of invention of some new numeration or calculating method - among Greek mathematician only Archimedes dealt with this) is of great importance.

Let us now consider the original texts, first of all the arithmetical books of Euclid's "Elements" (books 7-9). Euclid does not use the numerical notation in letters that was in use at his times for calculating purposes. Instead he uses a linearized geometrical graphic representing numbers as segments made up with unit segments and notation in letters as in geometry:



Nevertheless while Euclid speaks here not about segments but about numbers, we are to consider a segment-number as representing the class of graphemata corresponding with the number. In this case we have a linearization that is not directly connected with phonetization. We call such a linearization a *primary* linearization to differ it from the *secondary* linearization the alphabetic recording of speeches about mathematical graphic. We can call Euclid's segment-number again a *nickname* of the numerical graphema of the initial level because here as in the case of the roman cipher "III" a signe of the number (graphema of the second level) the same time is a suitable graphema of the first level. However it is important to distinguish two levels of graphemata here for otherwise it would appear to be that in arithmetical books of "Elements" Euclid proofs theorems not about numbers but about segments. As we just said for the secondary linearization and phonetization Euclid uses notation of segments in letters like in geometry. So in arithmetic Euclid does not break with numerical graphic of the first level. So although Euclid probably knew Aristotel's definition of geometrical point as a "unit with position" (for example An.Post. 87a35), we can say that in the "Elements" the unit does not lose finally its position in arithmetic as well. Otherwise in later Diofant's and Nicomachus' arithmetic they usually speak about a numerical graphic of the second level, namely numbers notated in letters, and numerical graphic of the first level appears there only incidentally. Thus arithmetic differs from geometry much more definitely there.

Let us consider algebra now. We can agree with these scholars who ascribe "the idea of algebra" to al-Khorezmi [6], but from grammatological point of view algebra appears together with special algebraic symbolism. First of all we mean notation in letters of variables. A notation in letter of variable is a proper name of a script of some number - just in the same sense that "ABC" may be called a triangle's name. As in the case of notation in letters of geometrical figures letters are used quasi-phonetically here: on the one hand we have only to differ one letter from another one and letter's phonetic value does not play any role here, on the other hand letters-variables with their phonetic values participate in the speech. While we have defined the arithmetical graphic as the second level graphic, notation in letters of algebraic variables appears to be a third level graphic. As in the case of arithmetic we called a cipher proper name of the first level graphema, and called a numerical script common name which presuppose an explanation of its sense, now we call a notation in letter of variable proper name of the cipher or the combination of ciphers, while an algebraic formula is sounded as a full proposition.

The number of graphic's level can be increased: in modern "abstract" algebra not only numbers are notated in letters, but complex construction using themselves notation in letters as well. We must not forget here that every time we mean an intermediate . level of graphic because the graphic's levels are limited not only "below" by geometrical graphic but also "above" by phonetic graphic of alphabetic writing with which mathematical reasonings are written.

It is hard to say when appears a theoretical algebra. In al-Khorezmi's algebra there are proofs, but they concern even not arithmetical but geometrical graphic. On the other hand in Europe algebra for the long time is developing not as theoretical but practical discipline (from the ancient logocentric point of view), the kind of calculating art. In 17-th century algebra come in theoretical sphere again; however since algebra is closely connected with arithmetic and geometry this time, it is hardly possible to consider it as an independent theoretical science. Perhaps samples of the "pure" theoretical algebra we can find only in our century. "Pure" algebra is a speaking about algebraic graphic, i.e. graphic of the third and higher levels, without any appeal to the first and second level graphic, i.e. to geometry and arithmetic. We do not at all mean that such purity is a great achivement and that appeals to the lower levels graphic is a sign of backwardness. Quite the contrary mainly the interaction of graphic of different levels makes the content of mathematics. Mathematics as a theoretical discipline with written tradition should to have at least two levels of graphic - "spoken about" and "speaking about" (alphabetic phonetic writing). Obviously including the number of intermediate levels enriches the situation.

This approach allows us to justify the modern school formula of mathematics as "algebra and geometry". Actually geometry and algebra are the opposite sides of mathematics in the sense of the levels of graphic that is "spoken about". The level of graphic that is "spoken about" in arithmetic lays between geometrical and algebraic ones, so arithmetic appears to be included in this formula of mathematics. "The pure algebra" and "the pure geometry" are the extreme cases; the modern mathematics in the whole is developing between the two using all the levels of graphic and creating the new ones.

Because of the pseudo-phonetic nature of algebraic graphic algebraic formulae have partly linear and partly non-linear form. Especially expressive from this point of view are upper and lower indexes, tildes and asterisks, which overstep the limits of non-linear variation put down by logos. The same can be said about the signs like the signs of sum and of integral. The final delinearization of algebraic formulae we can see in the modern mathematical categories theory where algebraic scripts become completely two-dimensional. A diagramma of category can not be sounded as an usual algebraic formula. Perhaps this situation may cause some unexpected theoretical difficulties. To remain the theoretical discipline in the classical sense of the word the category theory have to use an additional level of graphic which would connect the diagrammata with the phonetic alphabetic writing (something like notation in letters in geometry). However this mean would hardly be effective, because the process of dephonetization and delinearization of graphic in the modern algebra involves not only its intermediate but also the upper level - the level of phonetic alphabetic writing. Ousting of phonetic alphabetic writing undermines the theoretical status of modern mathematics in the most essential way: mathematical texts becomes "unreadable", they do not have any more their final foundation in the dialectic of living speech able to clear everything. So mathematics appears to be senseless not only for ignoramus but in some way also for the specialists:

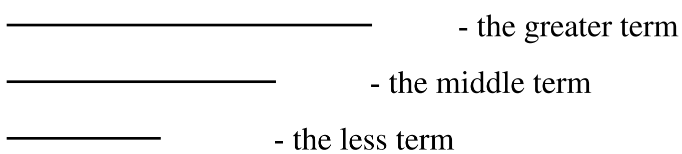
mathematics becomes "the play of symbols" which is not any more founded by the logos comprehending and significating any possible mathematical sign. (In this connection the modern mathematics rather looks like the "cossists'" algebra of 16-th century than the classical Greek geometry.) Logocentrism correctly sees its decay here and does its best to comprehend and to significate existing mathematical graphic. Derrida's "Grammatology" allows to look at the situation from another point of view and to get the more tolerant position in this question.

Logic

Mathematics and logic always had rather strange relationships. Their mutual pretensions to be the criterion of strictness have been making a competition between them. One of the two disciplines tries to absorb another: logisists (Russel) consider mathematics as a part of logic while "mathematists" (Brower) consider logic as a part of mathematics. The competition between mathematics and logics became clear not only in the 20-th century: Proclus' "The Commentary to Euclid" and Galen's logical treaties give us the sufficient information to speak about the contest of logicians and mathematicians for the right to be called "strict" in the firsts centuries A.D. The beginning of this controversy is the debate between Plato who ontologizes the mathematical figures and Aristotel who ontologizes the logical figures. The debate resumes in the modern time: we see here the return of the platonic mathematism after the époque of dominance of aristotelian logicism which we call the epoque of scholasticism. The Descartes' and Galiley's argument against the scholasticism is the argument for mathematism against the logicism, and Galiley independently considers it as an argument for platonism against the aristotelism. Namely the return of platonic mathematism made possible the modern science. This short survey is enough to show that the question of the relationships between logic and mathematic is very important. "Grammatology" allows to give a new look at this problem as well. First of all we try to present "the grammatology of logic" the same way that we presented the grammatology of mathematics above.

While mathematics is usually considered as nothing but discipline - the texts and the oral tradition practised by the community (although the limits of texts, tradition and community are under discussion) - the word "logic" has much wider rate of meanings. For example one can say as an universal objection: "there is no logic here". We explain such a using below and now let us to use the word "logic" like the word "mathematics" - in the sense of the discipline. The logic as a discipline has two parts - they speak about formal and non-formal logic. Both are presented in works of "the father of logic" Aristotel: the formal logic is a content of *Prior Analitics* and non-formal is a content of *Posterior Analitics* and some other texts, first of all the *Metaphysics*. We consider the two parts of logic one after another.

From the grammatological point of view the formal logic is nothing but "geometry otherwise". Since the (theoretical) geometry as we said above is speaking about and (in the written case) linearization of graphic, the formal logic is a dephonetization of speech and a delinearization of the linear phonetic alphabetic writing which is used to write speech down. That is why we after Aristotel use the term "logical *figure*" (schema) and that is why all the other basic ancient logical terms the same time were important mathematical terms [8]. There are evidences that aristotelian figures of syllogisms was actually drown in ancient schools as non-linear drafts [8]. "The perfect syllogism" for example was drown as follows (the vertical dimension is important here to show "the natural order of terms"):



Once again: geometry "transforms" the graphic into the speech and further into the linear graphic of phonetic alphabetic writing, and the formal logic otherwise transforms the oral or written in the usual way speech into the non-phonetic graphic which does not differ from geometrical one but is poorer. The pseudo-phonetical notation in letters connecting the speech (and linear writing) with non-linear graphic is used in formal logic as well as in geometry. It is interesting to compare the ways of using of such notation in geometry and in logic. When we speak not simply about "a triangle" but about "triangle ABC" "triangle" is individuated, appears to be not "the triangle in general" but some concrete triangle with the proper name "ABC". (As well as saying "Andrei" instead of "a man" we point at some concrete man even if our audience previously had no idea about Andrei.) Everything stands otherwise in logic. When Aristotel says "let us A to be inherited in B" "A" and "B" may signify "man", "Socrates", "paleness", "education" and so on. So since in geometry notations in letter realize the individuating function regarding to the speech, in logic they realize the generalizating (inductive) function regarding to the speech (4). As we said above in geometry notations in letters realizes the inductive function regarding to the geometrical draft, and that allows us to speak about the given draft with its proper name and so to consider the concrete drown triangle as an example of "the triangle in general". Otherwise in logic: every term is denoted by line; denoting now a line by the letter "A" we demonstrate that this line signify not "the term in general" but the concrete term A. So in logic notations in letters realizes individuating function regarding to drafts. What was said above about the role of notations in letters in geometry and in logic can be represented with the figure as follows:

geometry	logic	geometrical example	logical example
term	draft	"line"	———
letter	letter	"A"	"A"
draft	term	———	"Socrates"

(upwards - induction, downwards - individuation)

For Aristotle himself the problem of mathematics has rather simple solution: while in mathematics there is a speech speaking about graphics, this speech can be "outlined", i.e. described with speech figures like any other speech. However such approach misses the inverse ratio between mathematic and logic that we just have revealed. To reveal it it is not sufficient to put mathematical graphic under consideration (Aristotle does it developing the theory of mathematical object - his theory of abstraction): it is necessary also to consider the graphic of logic itself. Aristotle's logical terminology shows that one could notice the connection between mathematical and logical graphic mentioned above with a very superficial look . So it is nothing surprising that such a deep thinker as Aristotle missed it.

Misunderstanding of the different correlation of graphic and speech in mathematics and in logic leads to the points of view mentioned above according to which mathematics is a part of logic or otherwise. If to expel from mathematics the initial level of the "spoken about" graphic, it actually appear to be a part of logic (Russell). If to expel from logic the initial "outlined" level of speech it appear to be a part of mathematics (and very poor one! - Brower). The question about a mathematical logic - is it a part of mathematics or a part of logic? - can be solved the same way. Bool (?) who developed "the algebra of propositions" perhaps the first time realized the connection between logical and mathematical graphic and tried to use in logic the new type of mathematical graphic, namely algebraic one. We know that such a "superficial" from the logocentric point of view step as a change of logical graphic greatly influenced the development of logic. When mathematical logic begins to develop its operations without any reference to speech and to use speech only for speaking about these operations, it becomes a part of mathematics. So if you are engaged in the field of mathematical logic and you do not know where you are engaged - in mathematics or in logic, ask yourself about your task - is it to speak about your graphic or to outline your speech. In the first case you are mathematician, in the second you are logician. However this question and the both responses stay in the existing frameworks. Perhaps it is better to leave them and to try to comprehend the mathematical logic in some different way. Notice that grammatological structure of formal logic revealed in the case of Aristotle's syllogistic is the same in modern "non-classical" logics. Actional (?) or deontic logic for example develops as follows: there are the notions of action and of duty in common discourse; logician formalizes such "intuitive" notions, i.e. develops some operations with non-phonetical and pseudo-phonetical graphical objects which he makes to correspond with these notions; then logician compares again the developed operationalism with facts of speech [9]. As we show below non-classical nature of modern logic that differs it from the aristotelian paradigm rather concerns not formal but non-formal side of logic.

Formal logic seems to be an anti-logocentric enterprise. For formal logic "transforms a living speech into dead schemes" - everybody can hear this logocentric protest in his or her soul. However such a concealed dissatisfaction is not the only form of this protest. In response to formal logic's initiative the logocentrism gains in range that is quite unthinkable in the frameworks of the simple phonetization of writing. It even suggests the idea that formal logic is nothing but a logocentric provocation. So logical figures immediately are again spoken about. It begins with their introduction because it is necessary to explain their definition and using. We can say that here logos restores its rights that it have in usual phonetic writing. However then it goes further. The Aristotle's "Prior Analytics" are followed by "Posterior Analytics" where non operational but deep, ontological meaning of logical figures is explained. The figure of "the perfect syllogism" appears to be the figure of *essence*. "Dead logical schemes" are revived here by the dialectical logos. Logical graphic perishes in the narrow space between the epistemic (being outlined) and the philosophical (speaking about logical schemes) logos.

Thus there are two steps of logization here: in the first (epistemic) step the geometrical and arithmetical graphic is spoken about, then the epistemic logos is outlined by logical figures, at last in the second (philosophical) step of logization the logical figures are spoken about. It is just the logization of logical graphic used in the formal logic that is the task of non-formal logic. It is the secondary logization that presents logic as something primordial, natural and universal. That is why the meaning of the word "logic" is much wider than "logic as a discipline".

Plato also tried to adjoin to mathematical level of logization which is not able to suppress graphic completely and continues to deal with "images", dialectical one which would deal only with "forms" (Rep.510b-511d, 533b-e). To be consistent in his logocentrism Plato some moment refuses from any writing connected with the project and develops an oral learning which is known by the evidences of his pupils and contemporaries (and first of all of Aristotel) as "unwritten" (agrafa dogmata) [10]. Evidently the Plato's project was to reduce all the variety of geometrical graphic to one or a few of figures (for example the "perfect" figure of the circle) in the mathematical level of logization (5) and then to logizate dialectically the given "basic" graphic. So Plato tried "to continue" the mathematical logization to make it perfect and so to justify the name "epistemai" of mathematical disciplines while according Plato in their present state they actually remain to be a kind of skills (tecnai) (ib.).

Aristotle on the contrary to Plato leaves the idea of the "true" i.e. completely logizated mathematics and find another way to the final logization of graphic. He firstly develops an independent discipline (later named formal logic) where all the epistemic speeches are reduced to a few number of simple graphemata (logical figures), and secondly speaks about the given simple graphic so subordinating all the possible epistemai (6) to the only logos and completing the secondary logization of their graphic (non-formal logic or metaphysics). The "ontological" figure for Aristotel is not the geometrical figure of the circle but the logical figure of the perfect syllogism. Thus Aristotel try to complete the total logization not directly as Plato does it but trough the intermediate delogization (formalization). However we can comprehend the situation another way: probably the secondary logization was not an initial Aristotel's task but his reaction to his own antilogocentric step.

We see that Plato's mathematism and Aristotel's logicism are both equally logocentric projects. However the platonic mathematism in its modern form appeared to be more available to antilogocentric alternatives than the aristotelian logicism in its medieval form: although a platonic mathematism of modern scientists remain to be logically founded (Galiley for example calls mathematics *the language of Nature*) it causes a powerful development of mathematical graphic and scientific and technical blow up (as Derrida notice it is writing that gives a paradigm for techniques in the logocentric scale [1]). But the aristotelian logicism allows for a non-logocentric development as well. Such development one can see in the modern "non-classical" logics which are non-classical first of all because they do not subordinate the task of the formalisation of speech to the task of secondary logization (7). Thus we have a choice again: to make all possible to logizate the immense mass of logical graphic produced by modern logicians or to refuse from the logocentric thinking.

Notes:

(1). Surely we also can put and solve a question about a mathematical papyrus as follows: is the solution of a problem there true or false? But why have we to suggest that the papyrus' author (if we can correctly speak about author there) puts the question the same way? And why have we always to hear behind a mathematical texts a dialog (questions and responses) at all?

(2). We ignore here the fact that the couple "proposition-proof" does not completely describes the speech structure of theoretical mathematics. Its another important element is for example a definition.

(3). As we know our proper names somewhere meant something. That is why we can say that proper names *lose* their meanings. While nouns enrich their meanings in speech, proper names lose their meanings in writing. Cf.[7]

(4). Such an individuation in ancient geometry and logic has a name of "ekqesis" (layout see Proclus ib.). The inverse procedure namely giving a general proposition on the basis of individual cases Aristotle calls "epagwgh" (induction). Notice that the individuation is not a singling out of a particular case as well as a generalization mentioned above in the text is not a looking for a common rule for a number of particular cases.

(5). In our [11] we showed that the theory of books 1-4 of Euclid's "Elements" can be interpreted this way.

(6). Notice that with the Aristotel's approach the status of epistema gain not only the mathematical but also the "physical" (in the aristotelian sense of the word) disciplines such as biology and meteorology.

(7). We must not however think that the trend of modern logic to deal with formalization but not the secondary logization is absolutely new. Obviously it can be derived at least from stoics.