Abstract:

The geometrical theory of the first four books of Euclid's "Elements" prima facie does not fit the Classical Model of Science in the sense of de Johg and Betti. The principal reason is the following. Among first principles of this theory the are not only Axioms but also Postulates 1-3, which, being read literally, are not propositions but descriptions of certain primitive non-logical operations. The following theory built upon these principles contains not only Theorems, which are propositions, but also Problems, which are (not propositions but) descriptions of complex non-logical operations obtained through composition of iterated primitive operations. These non-logical operations produce new geometrical objects from a given set of primitive objects. (In Euclid's geometry such a set of primitive objects consists of two different points.) Euclid's geometry does not concern any fixed set of objects or any fixed domain of being(s); producing its proper objects (object-formation) is an essential part of this theory but not a preliminary. The structure of this theory does not allow for considering object-formation and proving theorems about these objects as two independent activities.

In the second part of my talk I use some hints from the first part for arguing that the Classical Model of Science cannot serve as an adequate epistemic model of theories of today's mathematics either. Among several modern examples I consider more specifically the case of wholly formalized mathematical theories like ZF, stress the importance of non-logical syntactic rules (including formation rules) in such theories and argue that formalized theories should be thought of as mathematical objects of certain kind rather than theories proper. Generalizing upon these examples I further argue that the issue of object-formation remains as much important in today's mathematics as in Euclid's geometry. The Classical Model of Science and the usual (modern) notion of axiomatic theory don't take the issue of object-formation into account and for this reason cannot serve as reliable epistemic models for doing mathematics and mathematically-laden science. An adequate model of science and axiomatic method useful for mathematical practice need to include certain principles that specify rules of object-formation in a way similar to which Postulates of Euclid's "Elements" specify rules of elementary geometrical constructions.