

Abstract:

In his famous paper “On Sense and Reference” Frege asks: In which sense (if any) the Morning Star (MS) and the Evening Star (ES) are the same thing? As Frege observes the assertion of identity $MS=ES$ unlike the assertion of identity $MS=MS$ (or $MS=MorningStar$) has empirical content and in this sense is not trivial (in Frege’s time similar questions concerning some comets and asteroids still remained open). It is not clear how the same (?) notion of identity may apply in these two very different cases. Frege tries to solve the problem by distinguishing between the sense and the reference of linguistic expressions. However this distinction does not solve the whole problem because it does not explain how empirical data may possibly justify the claim that two linguistic expressions like “MS” and “ES” are co-referential. Since Frege’s theory of identity does not take the issue of empirical justification into account it remains largely irrelevant to the practice of empirical sciences. Given that the naive pre-theoretical understanding of identity proves insufficient in some areas of the modern science (think about the particle physics) the need of a new approach to identity seems me obvious.

Using the homotopy- type-theoretic theory of identity developed by Martin-Löf, Voevodsky and others I suggest an alternative approach to Frege’s problem. While earlier attempts to modify the classical identity were mostly motivated by the idea of its “weakening” (as in the case of Krause’s theory of quasi-sets, for example), the homotopy type theory suggests a view on identity as a construction from available empirical data. Suppose after Frege that identities of MS and ES are fixed beforehand. This means that one is in a position to identify two independent observations of MS as observations of one and the same object; similarly for ES. In terms of Martin-Löf’s theory such a presupposed identity is described as definitional. The identity $MS=ES$ we treat as propositional in Martin-Löf’s sense. In order to establish $MS=ES$ we construct invertible maps from observations of MS to observations of ES (by mapping the corresponding frames of reference), and then proceed by constructing further maps between those maps, etc., always checking the appropriate coherence conditions. Whether or not a certain map satisfying the corresponding coherence conditions is

constructible depends on the available empirical data: this is how $MS=ES$ is justified empirically. So we arrive to a higherorder homotopy groupoid, which represents an identity type and makes explicit its topological properties. One may expect that identity types of physical entities of different types (e.g., mesons and baryons) differ in their topological properties (this is so far a hypothesis to be checked).

Tentative applications of higher identity types in empirical contexts shed some light on the distinction between the definitional and the propositional identity. Admittedly we cannot proceed a scientific reasoning without taking some notion of identity for granted. In Martin-Löf's theory this role is played by the definitional identity. However as the above example suggests there is no reason to consider the definitional identity as something fundamental. We treat the identity $MS=MorningStar$ as definitional because in the given context we have decided (after Frege) not to analyze the way in which different phenomena observed at different times by different people are identified as observations of the same Morning Star. In a different context the non-trivial empirical character of this identity can be similarly taken into account. Thus a definitional identity has the character of explicit assumption (that can be questioned and analyzed if needed) rather than that of ultimate foundation.