

Higher Identity Types in Empirical Contexts

Andrei Rodin

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Historical and Philosophical Background

Kant, Cassirer and Friedman

Frege

Object Identification with Homotopy Type theory

Classical case

Relativistic case

Quantum case

Conclusion

Friedman 1998 on Kantian Revolution

“The relationship between the pure intellectual concepts of metaphysics and the world of phenomena has been reinterpreted [by Kant] in a profoundly radical fashion. Pure intellectual concepts no longer characterize an underlying reality situated at a deeper and more fundamental level than the phenomena themselves; on the contrary, such concepts can acquire a relation to an object in the first place only by being realized or schematized at the phenomenal level. And it is this this radical reinterpretation of the relationship between metaphysics and the phenomena - [...] that constitutes Kant’s truly decisive break with the Leibnizean-Wolffian tradition.”

Cassirer 1907 on Intuition

“The [Kantian] principle according to which our concepts should be sourced from intuitions means that they should be sourced from the mathematical physics and should prove effective in this field. Logical and mathematical concepts must no longer produce instruments for building a metaphysical “world of thought” : their proper function and their proper application lies only within the empirical science itself.”

20th century revival of metaphysics (i)

The idea according to which logic and mathematics provides a spectrum of conceptual possibilities, from which empirical observations pick up only few (mind the underdetermination of theories by empirical data), once again became common (Wigner). Even if the new metaphysics attempts to avoid dogmatism by being “descriptive” (Strawson) it tends to follow the traditional Aristotelian pattern of logical analysis of the everyday language rather than the Kantian model of philosophical critique of the contemporary mathematically-laden empirical science.

V.I. Arnold

“Mathematics is a part of physics. It is a part of physics where experiments are cheap. [...] In the middle of the 20th century there were attempts to separate mathematics from physics. The results turned to be catastrophic.”

Frege 1892

“The discovery that the rising Sun is not new every morning, but always the same, was one of the most fertile astronomical discoveries. Even today the identification of a small planet or a comet is not always a matter of course. Now if we were to regard identity as a relation between that which the names a and b designate, it would seem that $a = b$ could not differ from $a = a$ (provided $a = b$ is true).”

Venus Example

$a = \text{Morning Star}; b = \text{Evening Star}$
 $\text{Morning Star} = \text{Evening Star} = \text{Venus}$

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Cf. T. Budavari & A.S. Szalay, *Probabilistic Cross-Identification of Astronomical Sources*, *The Astrophysical Journal* 679 (2008) 301

Frege's solution

the sense (aka meaning) / reference distinction

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- ▶ an obscure nature of *sense* aka *meaning*;
- ▶ no account of how empirical or other evidences justify judgement $\vdash a = b$;
- ▶ linguistic examples from the everyday talk and a historical narrative (like “Napoleon recognized the danger to his right flank”) are used for fixing the notion of identity and the meaning of objecthood in empirical sciences; Frege’s logic is the old-fashioned logic of the “metaphysical world of thought” !

20th century revival of metaphysics (ii)

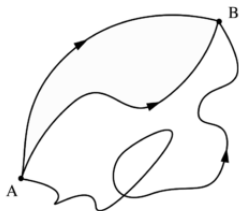
In the 20th century some people like Josef Bochenski embraced the Leibnizean-Wolffian-style dogmatic metaphysics (under the new cloths of Modern Logic and the new name of Analytic Philosophy) for religious, ideological and general philosophical reasons. However many others including Russell apparently did this for a merely technical reason : seeing that the Kantian philosophy in its original form was no longer adequate to new developments in pure mathematics and physics they gave up the whole program instead of trying to push it harder. I believe (following Cassirer) that this was a huge mistake.

Friedman 1998

“Euclidean geometry [...] is not to be compared with Hilbert’s axiomatization [of Euclidean geometry in his *Foundations of Geometry* of 1899], say, but rather with Frege’s *Begriffsschrift* [or another symbolic logical calculus]. It is not a substantive doctrine, but a form of rational representation : a form of rational argument and inference. [...] There remains a serious question about Euclid’s axioms, of course ; when pressed, Kant would most likely claim that they represent the most general conditions under which alone a concept of extended magnitude - and therefore a rigorous conception of an external world - is possible (see A163/B204). And, of course, we now know that Kant is fundamentally mistaken here.”

Claim : In the light of Cassirer's critique mathematics built with Univalent Foundations (UF) appears a more promising candidate for replacing the old good Euclidean geometry and traditional arithmetic than Frege's *Begriffsschrift*, Classical FOL (+ ZF) and their likes. In order to justify this claim I suggest how to build physical objects (objects of possible experience) with UF. More specifically I suggest how to think about identity types in empirical terms (i.e., "realistically").

Venus with Homotopy Type theory : the case of Classical particles



$$p_i, p_j : Id_T(A, B);$$

the “extensionality one dimension up” (Awodey), EX1 for short,
 “no higher identity types”, precisely

$$\vdash Id_{Id_T(A,B)}(p_i, p_j)$$

$$\vdash p_i = p_j : Id_T(A, B)$$

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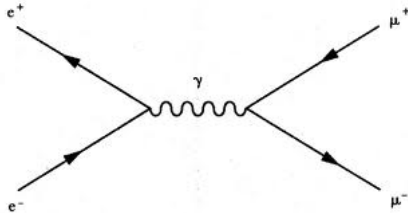
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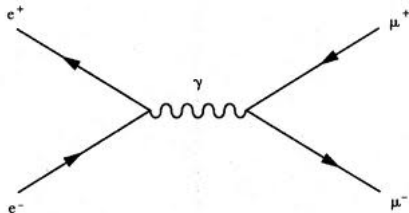
- ▶ no loops (= no time reversal), and
- ▶ no path intersections (= Classical particles are mutually impenetrable). The groupoid of paths reduces to a bare **set** up to homotopy equivalence (= worldlines are mutually disconnected). Dynamics is “frozen” : up to homotopy equivalence representations of moving and non-moving particles are the same.

Collision or Crash ?



In the Classical (Newtonian) space-time a collision of point-like particles is *sensu stricto* impossible because of the impenetrability condition. Crashing particles cease to exist and some new particles are born. A crash brakes the continuity of worldlines.

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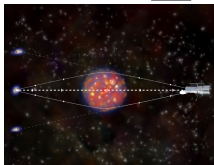


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A further research topic : to study the Scholastic (“metaphysical”) physics of 15-16th centuries and the Early Modern physics from a homotopical viewpoint.

Gravitational Lensing

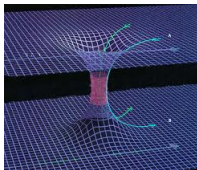
In GR-spacetime loops and intersections of worldlines are allowed ; moreover EX1 does not hold, and so higher identity types do



matter. One judges that the two “false images” are identical (i.e., are images of the same source) by comparing the corresponding two worldlines by their homotopies of type $Id_{Id_T(G,E)}(w_1, w_2)$ where G is the (“true”) galaxy, E the Earth (= the observer’s “image” of G), w_1, w_2 the two worldlines. In fact these images are no more “false” than are MS and ES !

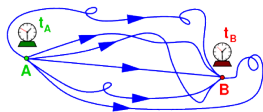
Wormhole Lensing

What happens if there is a spacetime hole (a “wormhole”) at the



place of G ? . Guess : the identity of G is of different character. Compare the case of Feynman’s path integral.

Homotopy theory of path integrals (after Suzuki 2011)



Consider a system of n free spinless indistinguishable particles in space \mathbb{R}^d and its configuration space X : of $x = f(x_1, ..x_n) \in X$ with $x_i \in \mathbb{R}^d$.

Theorem (Laidlaw&DeWitt 1971)

Let the configuration space X of a physical system be the topological space. Then the probability amplitude K for a given transition is, up to a phase factor, a linear combination

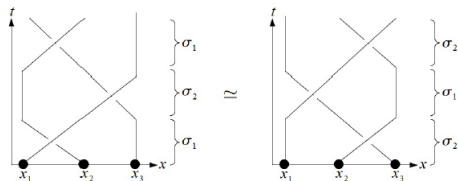
$$\sum_{\alpha \in \pi_1(X)} \chi(\alpha) K^\alpha$$

of partial probability amplitudes K^α obtained by integrating over paths in the same homotopy class in X , where the coefficients $\chi(\alpha)$ form a one-dimensional unitary representation of the fundamental group $\pi_1(X)$.

fundamental group by permutations

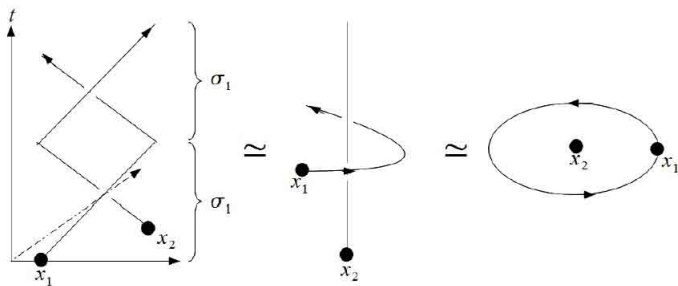
$$\sigma_i = S_{i,i+1}$$

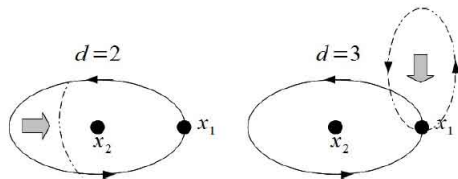
1. $\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}$
2. if $|i - j| > 1$ then $\sigma_i \sigma_j = \sigma_j \sigma_i$
3. $\sigma_i^2 = e$



(1); (2) is obvious

(3)





(3)

For $d \geq 3$ $\pi_1(X) = S_n$; since S_n has two 1D unitary representations we have two cases :

$\chi^B = 1$ for all $\alpha \in S_n$ (bosons);

$$\chi^F = \begin{cases} +1, & \text{when } \alpha \text{ is even} \\ -1, & \text{when } \alpha \text{ is odd} \end{cases}$$

(fermions)

For $d = 2$ $\pi_1(X) = B_n$ (anyons)

Conclusion

IF Homotopy theory manages to account for the physical spacetime at the fundamental level (and therefor also provides a new clear sense of objecthood and objectivity) then Kant and the whole idea of Critical Philosophy are vindicated. Stakes are high and it is worth trying!

THE END