Andrei Rodin

Formal Logic versus Categorical logic

1) Formal logic

Logic is usually described as a discipline concerned with codification, systematisation of and theorising on general forms of reasoning. This is, of course, a very imprecise description. Nevertheless I'm going to argue that the above description is too restrictive and suggest a more general and in my view more appropriate notion of logic. For reasons which will become clear I shall call the traditional notion of logic implied by the above description "formal logic" and the enlarged notion of logic proposed here "categorical logic".
A more precise term for "general forms of reasoning" (taken in the relevant sense) is "logical form". The concept of logical form stems from the fundamental observation that reasonings like many other linguistic, social and natural phenomena come in patterns, that is, are in certain sense repeatable. Given a sufficiently large set of individual organisms one can make up appropriate equivalence relations between these organisms, and so classify them into species, types, etc. Obviously not every equivalence relation brings a reasonable classification and a reasonable notion of biological form but some of them do so. Similarly, the notion of logical form is brought by an appropriately chosen equivalence relation between reasonings.
Logic studies logical forms rather than particular reasonings just like biology studies species rather than individual organisms. No biology manual mentions my girlfriend's beloved goldfish Sisyphus, and no logic book normally mentions debates about any other specific scientific issue but logical. However in both cases this lack of interest to individuals should be understood with a good pitch of salt. Clearly biology says something about individual organisms (including Sisyphus), moreover it has an ambition to say something true about all of them! Since medieval times logical textbooks keep telling us that since Socrates is human and humans are mortal, therefore Socrates is mortal. Although the purpose of this story is *not* to tell the reader about Socrates but to demonstrate a logical form called *perfect syllogism* the very notion of syllogism couldn't be justified without appropriate examples. So the claim of disinterestedness of sciences (including logic) with respect to particular cases should be rather understood as a way to make a useful distinction between a theory and its applications in various particular contexts. That a given theory must correctly apply to its subject matter goes without saying in case of empirical sciences (at least in the case of straightforward applications like the application of general facts about goldfishes to Sisyphus). I suggest that some notion of applicability is indispensable in the case of logic too. In any event the applicability of logic in different semantic contexts is assumed in the traditional (Aristotelian) notion of logic: it goes here without saying that logical inferences and other logical forms can be exemplified and illustrated by different linguistic examples.
One might try to disapprove the suggested analogy between logic and biology pointing to the fact that while biology is an empirical science logic is not. So, the argument goes, logic and biology have very different aims: while biology and other empirical sciences look for an adequate account of Nature logic doesn't purport to describe how people actually reason but purports to teach people how to reason correctly. In my view these differences are less dramatic than it is often claimed. Certainly logic doesn't reduce to a

sociological, psychological or cognitive science study of how people reason in different situations. However any logical rule suggested as a norm is tested against certain practice (epistemic, social, cognitive, etc.), which it purports to normalise. Unless the degree of adequacy between the suggested norm and the practice is sufficiently high to begin with the normalisation is a non-starter. Think (for another analogy) about linguistic norms of contemporary English fixed in dictionaries and grammar books, on the one hand, and of how different people actually speak English, on the other hand. Obviously there are all kinds of deviations from the norm in actual speech practices; these deviations justify the distinction between the norm and a mere description. However the range of these deviations is limited in principle: if a speaker deviates too far from norms of English he no longer speaks bad English but he doesn't speak English at all (and so no longer deviates from norms of English)! Obviously there is a sense in which dictionaries and grammar books describe how people speak English but not only prescribe how they should do it. Similarly, the idea of logical norm of reasoning may make sense only if, first, it sufficiently fits the existing practice of reasoning, and second, this fit is not perfect, so one can detect logical fallacies, etc. So there is a sense in which logic describes how people reason too. Coming back to biology remark that the notion of norm (as opposed to pathology) is not completely alien for this science either; this notion turns to be basic in medicine whose links with biology are evident. Just like logic biology and any other empirical science involves a complicated dialectic between description, prescription and intervention. I assume that these remarks (based on my broadly empirical views on logic) are sufficient for justification of the analogy between logic and biology just made.

I make this analogy here in order to show that the notion of form is not specific for logic: logic uses its specific notion of logical form, biology uses its own specific notion of biological form but the general notion of form is shared by the two disciplines in common. Why then the popular description of logic as "formal science"? Let's look more precisely on how the notion of form works in biology and logic correspondingly. Talking about goldfishes one (but my girlfriend) can forget that she is talking about Sisyphus or any other particular goldfish and refer to common features of goldfishes as if they all would belong to the same collective individual. However one can hardly say anything of real scientific value about a generalised abstract organism forgetting that there are different kinds of organisms. No tentative general theory of organisms can be taken seriously by biologists unless it is tested against the whole variety of known biological forms. It remains actually unclear whether the pre-theoretical or a purely speculative notion of organism can be developed into a full-fledged biological concept at all.

Logic, on the contrary, works with forms of much higher levels of generality. For "general forms of reasoning" is supposed to apply to any reasoning (or at least to any correct reasoning) and one may reason (and in particular reason correctly) about anything. Isn't this too much? What seems worrying is that unlike the case of biology or any other empirical science there is apparently no systematic control in logic which might justify or rule out a given general concept through checking it against some independent background.

2) Logic, ontology and rationality

Before discussing what could realise such control function in logic let me point to some well-known constraints of what counts as "reasoning" here. The most fundamental constraint is this: reasonings falling under the scope of logic involve truth-values. This rules out non-assertive discourses and arguably also assertive fictious discourses (but not intentional lies). However what remains is still a lot. In particular we are still left with any assertive talk about anything that there is. For one can assume that about any entity something can be truly or falsely asserted. This assumption provides a link between logic and ontology. In Aristotle's account logic and ontology are so much mixed up that it is not always easy to distinguish between the two. His approach can be roughly described as follows: general rules of reasoning about what there is must reflect general features of what there is. This "reflection" in Aristotle's logic reduces nearly to identity, so he considers basic logical rules like the ban of contradiction as basic laws of being.

This shows that ontology provides indeed some control on logic (taking Aristotle's approach one might rule out certain systems of logic as ontologically absurd) but this control mechanism is hardly effective for two different reasons. Firstly, ontology is, generally speaking, not quite independent of logic as Aristotle's example clearly shows. This is even clearer in Russell's philosophy of "logical atomism": in order to promote a system of logic (other than Aristotle's) Russell built up an ontology perfectly reflecting basic features of the given logic like in Aristotle's case. Secondly, and more importantly, one might wish to make logic neutral with respect to ontology. For as far as I can see, the major argument in favour of traditional Aristotelian idea of context-independent formal logic, which is crucial for today's defenders of this idea who usually rely on other system of logic than Aristotle's, is pragmatic and epistemological rather than ontological. This argument is roughly the following.

Unless a system of context-independent rules of reasoning is assumed and respected by all members of a given community this community cannot support a rational discussion and so cannot develop sciences and philosophy in anything like the usual sense. It might be further argued that without sharing common logical principles a given community cannot support democracy, independent juridical system and any other social institution based on rational dialog. I shall not try now to define the relevant notion of rationality in all its generality here - actually I don't think that this can be done in a non-circular way - but I want to stress how much is put here at stake. In addition I want to point to the unifying function of logic which becomes clear in this context. As far as any esoteric system of knowledge is concerned it can be well organised on its own internal principles and so it doesn't need anything like formal logic with an unlimited domain of application. Logic is needed when knowledge becomes public, that is, when different systems of knowledge are brought into a contact and supposed to be merged in a certain way. In fact the unification provided by a shared logic is very weak - and exactly for this reason it is fundamental. Because of this weakness I prefer to use the term "integration" instead of "unification". This integration doesn't require compatibility of integrated parts in the usual epistemic sense. In particular it allows for mutually incompatible beliefs. As far as proponents of these alternative beliefs share a basic logic they can rationally discuss and in case revise their beliefs. Whether or not this dialog results into a compromise view on which all involved parties might agree all these parties turn to be linked through their mutual dialog. And arguably this is exactly the way in which science and philosophy may mostly successfully develop.

The choice of rational discussion as a proper mean for dealing with truth in sciences and elsewhere is a very basic choice, so it is very difficult to test it against any neutral ground. One can point to the history of philosophy and sciences and claim that these things wouldn't be possible without rational discussion. However in the present context it is more appropriate to focus on a particular feature of the model of rationality just mentioned, namely on the idea of formal logic as a backbone of rationality. Historical evidences in favour of this latter idea are less convincing than evidences in favour of rationality tout court. Logic was much promoted and advanced in Europe in late Middle Ages but these advances in logic were not accompanied neither by important developments in natural sciences nor by a rise of democratic institutions. Moreover the important developments in sciences, which took place later in 17th century, openly opposed the medieval scholastic tradition in general and medieval logic in particular. The impact of logic on science and society in 20th century is equally very controversial. In fact in the 20th century the situation about logic changed in a dramatic way, so everything I told so far about logic meaning the traditional notion of logic dating back to Aristotle needs now to be revised.

In 20th century Russell and later his followers working under the brand of "analytic philosophy" made an attempt to revive the traditional way of doing ontology and metaphysics through replacement of Aristotelian logic by another system of formal logic generally known today as "classical first-order logic" (FOL). Some people involved in this movement (like Bochenski) explicitly relied on scholastic tradition and tried to justify it; critics of this movement often point to the link between analytic and scholastic philosophy stressing the bad fame of the later. I deter myself from going into the history of analytic philosophy here but I want to stress one of its internal problem which seems me very important for the future of logic and of philosophy of logic. Once the possibility of alternative systems of logic has been discovered, and once people realised that given a system of logic it is possible to develop an ontology and metaphysics supporting this system, anything like traditional Aristotelian or scholastic understanding of logic as *the* logic became untenable (at least if *the* logic is identified with a particular system of formal logic). So analytic people faced the problem which Aristotle and schoolmen never had: how to chose among the multiplicity of available logical systems (which in 20th century were massively produced through newly developed symbolic means) a "basic" one which might play the role of backbone of rationality? Although FOL in many senses was a reasonable choice the very possibility of making choice at this point was at odds with the view on logic as a neutral background on which all open-minded people would agree in order to discuss their different beliefs and different philosophical and other positions (the view, which in my understanding gave to analytic philosophy a good deal of its credibility). A sheer logical pluralism doesn't resolve the problem; in fact it often reduces to indifferent or skeptical attitude. Since logic ceases to pretend to be universal in the old good sense it ceases to function as logic in the old sense, in particular it cannot any longer pretend to be a backbone of rationality. Then one may reasonably wonder whether various symbolic calculus selling itself as systems of logic really deserve this name. If the older notion of logic is given up one needs to develop a new one rather than just accept that anything goes.

3) Categorical logic

Let me now propose a solution. I claim that a shared system of formal logic is not in fact necessary for supporting a rational discussion. Imagine a community of speakers where each speaker reasons according to his or her own private (formal) logic. By private formal logic I mean a system of formal rules applied by a single person with respect to his or her own reasoning, so these rules remain invariant through the discourse of any single participant's but they are different for different participants. It might seem that in this situation no rational discussion would be possible. However this is not the case, at least if one is not too dogmatic about the notion of rationality. What can allow for a rational discussion in the given situation is a properly functioning network of *translations* between speakers. To see how it may work think first about a group of speakers speaking different languages. Again it might be argued that in this situation no linguistic communication would be possible. But this is obviously false since the speakers may successfully translate between their languages. Now it may be argued that as far as the given languages allow for mutual translations they are basically equivalent (at least in the semantic domain under consideration), and so the situation will not change if the speakers choose one of their languages for their communication or in order to avoid the choice would speak some artificial language equivalent to their own languages. This argument is also false for the following reason. It is reasonable to call given languages equivalent when their mutual translations are reversible; a stronger notion of equivalence may require this reversible translation to be unique. In this latter case differences between languages reduce to notational differences and so can be treated as contingent. One may call a chair "chair" in English or "chaise" in French or "stul" in Russian or invent a new artificial word with the same meaning. Translation from French to English arguably reduces in this case to substitution of "chair" for "chaise"; this operation is obviously reversible (one may equally substitute "chaise" for "chair") and (provided some additional conditions which I leave aside) unique. The problem is that translations between natural languages are generally not reversible nor unique, they certainly don't reduce to substitutions word-by-word. Any reader knowledgeable in two different languages will find such examples easily.

Now one may argue that the problem I'm rising here has to do with semantic vagueness and redundancy in natural languages (but certainly not with logic) and suggest to improve on it in the following way. Distinguish in each given language classes of semantically equivalent expressions (equivalent up to a paraphrase) and then look for an unique one-to-one correspondence between such equivalence classes belonging to different languages (rather than between single words) and so form new equivalence classes of equivalence classes. Ultimately these later classes are replaced by expressions in an artificial language. As far as one is free to manipulate with languages (which is easy at private level but very difficult at social scale) this might be at certain degree feasible (at the price of leaving some linguistic stuff out as inappropriate for the task). My question is whether such an operation is a prerequisite for a successful communication, in particular for a rational dialog. My answer is that it is not. We can do better with non-reversible translations. Generally I think that linguistic transformations including paraphrases and translations are fundamental for functioning of languages, and that the popular understanding of non-reversibility and multiplicity of translations between languages as semantic ambiguities is totally wrong. I cannot provide a theory of meaning justifying this claim but let me give a hint. Given group *G* of natural languages think about artificial language *T* such that any sentence of any language *L* from *G* translates into *A* in a unique (but not reversible) way. Then think about another artificial language *I* such that it itself translates uniquely (but not reversibly) into any *L*. Both artificial languages *T* and *I* can be quite useful (for different purposes) and they need not be the same (albeit they may be). None of them replace any of *L*'s or purports to catch anything like "core structure" shared by all *L*'s. This and similar but more involved constructions don't require anything like core structure shared by *L*'s.

How to extend this approach to logic? How to translate between different logics? First of all remark that whatever can be reasonably meant by translation between systems of logic if there exist a unique reversible translation between two such systems they are the same (just like in the above linguistic example the differences in this case can be only notational). For a simple non-trivial example consider FOM and intuitionistic first-order logic. An immediate observation is that the former is a particular case of the latter but I think that this fact is not quite relevant for the task. What seems me more relevant is the usual distinction between logic and meta-logic, which I suggest to rethink in terms of translation or interpretation of one system of logic in another. Taking FOM as basic framework one may introduce intuitionistic calculus. One may equally turn things the other way round taking intuitionistic logic as basic and developing classical logic on this basis. So my suggestion amounts to relativisation of the notion of being "basic". As far as translations between logics work properly we simply don't need anything like basic system of logic common for everybody (in particular in order to support a rational dialog). Remark that the major difference between the two suggested constructions concerns the precise meaning of "develop". But there is no need to keep it fixed. Remark also that the suggested translations between the two systems of logic are non-reversible since the relation between logic and meta-logic is not symmetric.

I shall not elaborate on this example because I think that in fact we can go further in rethinking the traditional notion of logic than translating between known systems of logic. Namely we can rid of the doubtful notion of private logic just introduced and assume that any individual as well as any collective reasoning is accountable in terms of translations rather than in terms of logical forms and formal rules. To continue the above linguistic analogy remark that the notion of translation between different languages naturally extends to paraphrases, which could be so viewed as translations from a language into itself. However the notion of translation is itself too general (like the notion of form), so it is important to specify what kind of translation - or rather what kind of system of translations - deserves to be called "logical" (just like in the traditional framework it is important to specify which forms are logical).

Let me now point on a piece of mathematics, which will help me to make the suggested approach more precise. This is category theory. The mathematical notion of category captures pre-theoretical notions of translation, transformation, mapping and the like. A category comprises a class of objects provided by transformations (morphisms) between the objects. Each morphism has its source object and its target object but it is not identified by these data because there are generally might many different morphisms having the same source and the same target. Each object is equipped with identity morphism to itself. Morphisms are composable, the composition is associative and works as it should with identities. However weak the notion of category might seem it turns to be very reach and useful from a mathematical viewpoint. What I want to stress here is the fact that the notion of category generalises upon that of form. As I have already explained any specific notion of form (logical, biological or other) comes with an equivalence relation which holds between items told to have the "same form". Now I claim that the notion of form also brings a corresponding notion of transformation between these items, namely an *isomorphism* by which I understand here any *reversible* transformation. This is particularly evident in the case of geometrical forms like that of circle. Circles can be moved around and scaled up and down (so their size may change) without ceasing to be circles: importantly the transformations just mentioned are reversible. One can also think of non-reversible transformations of circles like contraction of a circle into a point but in this case the form is no longer preserved. In the case of logical form the corresponding reversible transformation is substitution (like in the case of any algebraic form). Writing the scheme (form) of perfect syllogism as

A is *B,* all *B'*s are *C*, therefore *A* is *C*

one is supposed to substitute at places of *A*, *B*, *C* English words provided the obtained sentence is meaningful. Substitutions of this sort are always reversible (and I think that their role in logic and mathematics is not yet sufficiently understood.) However imperfect this only slightly modernised version of Aristotle's formalism might be from the viewpoint of today's standards of being formal it well shows my point. Remark that not every equivalence relation is naturally equipped with a notion of isomorphism, for example there is no obvious isomorphism between items of the same colour. I suggest to use this fact for explaining why colours aren't forms and to assume that an associated notion of isomorphism is indeed a specific feature of the notion of form. By the way this explains why highly symmetric forms like that of circle are distinguished: they bring more isomorphisms with them. (The obvious fact that isomorphisms associated with any form always form a group by composition - in the algebraic sense of the term - usually remains hidden in formal logical analysis.) Now the notion of category is obtained as a generalisation of that of form through allowing for non-reversible morphisms along with reversible ones (with isomorphisms). This generalisation changes the situation quite dramatically, in particular given a category one cannot, generally speaking, consider any equivalence relation between its objects. Objects of a category don't share anything like common form unless all morphisms of the given category are reversible (such categories are called groupoids; a groupoid with just one object is a group). Since categories are generally not forms logical categories are generally not logical forms and categorical logic is not formal logic. By logical category I understand a category having reasonings as objects and logical morphisms between reasonings as morphisms. Just like in the case of traditional formal logic I shall not try to specify what are reasonings from the outset but instead try to find an appropriate notion of logical morphism hoping that this help me to understand what are reasonings afterwards (in the traditional case this is done mutatis mutandis with the notion of logical form). What kind of morphism one might reasonably called logical? What is again a hint is the notion of logic as a science about truth. So the answer that readily comes to mind is that a logical morphism should preserve truth-values or at least translate true sentences into true sentences. However in order to apply this proposal we would need to come back to the idea of translation between different systems of logic assuming that reasonings which are objects of our category involve truth-values from the outset. But there is another way to develop categorical logic, which seems me more appropriate. Instead of thinking what kind of structure logical morphisms should "preserve" and with respect to what kind of data logical morphisms should be indifferent one may look for specification of logical morphisms in terms of properties of their composition. A proper specification of this latter kind (which is found in any standard textbook) brings the notion of topos, which is a category having a distinguished "truth-value object". In addition we get an "internal logic" of given topos (one for all its objects) which can be written down in a form of formal logical calculus. I think this is a result of great philosophical importance, which has no obvious counterpart in traditional formal logic where truth values and logic are viewed either as god-given (fixed) or as put by (human) fiat.

One might argue that in the case of categorical logic we finally face the same problem as in the case of formal logic, namely the problem of choice of a particular system of such logic. But this is not correct: in the case of categorical logic the problem of choice doesn't exist in the same form. In the case of formal logic it is indeed essential that all its users use one and the same system of such logic. There might be different groups of users using different systems of formal logic but there'll be no obvious logical links between such groups. Some links of this sort can be construed but they are not provided by systems of formal logic themselves. Categorical logic, on the contrary, is based on the idea of translation. The issue is how to manage translations in the best way, in particular how to treat truth-values through translations. These questions are not supposed to have unique answers since these answers are, generally, context-dependent.

Categorical logic is a flexible framework that allows for multiple specific solutions, so the problem of choice of an unique universal solution is no longer pertinent.

4) Conclusion

The notion of form first stressed by Plato is deeply embedded into what we call science in spite of all changes, which this basic epistemic concept underwent during its long history. Plato's notion of form was mostly motivated by mathematical examples (however his general notion of form or idea doesn't wholly reduce to mathematical form). Aristotle opposed to Plato's view according to which any science is ultimately a science of forms, in particular Aristotle stressed that the variety of life and of everything else he considered as domain of physics (general natural science) doesn't reduce to mathematical or other forms but requires additional fundamental principles. Aristotle's idea of formal logic (tightly connected, as I have already mentioned, to his metaphysics and ontology) is in fact grounded on a rather mild version of Plato's philosophy of form: unlike Plato Aristotle didn't consider formal principle as fundamental or at least he didn't consider them as sufficient. Think about Aristotle's ban of switching between different genera. An important motivation of Aristotle's invention of formal logic was, in my view, his conviction that to the contrary to Plato's opinion physics (including biology) cannot and shouldn't be done mathematically or at least only mathematically. So Aristotle put forward logic as an alternative universal conceptual tool, which was in his view appropriate for physics.

This never worked indeed, and all the medieval advances in logic had little if anything to do with natural sciences. The scientific revolution of 17th century showed that Plato was more right about physics than Aristotle thought he was. When the new science became well established and was accompanied by great advances in mathematics the common interest to Aristotle's logic naturally declined. One needed to have the genius of Leibniz for looking both ahead of and back from his time and trying in 18th century to make a new sense of outdated Aristotelian and scholastic doctrines.

The revival of formal logic in late19-20th century is a very interesting phenomena, which I think is still waiting for a serious historical analysis. The new formal logic developed through a renewal of the old contact between logic and mathematics: in 20th century logicians learned from their mathematical colleagues about more effective tools than Aristotle's hopelessly outdated "figures" (of syllogism) and "means" (the middle term of syllogism). This development was partly promoted by mathematicians themselves who like Hilbert hoped to find in logic a firm background wanted in their own science since old basic mathematical principles like axioms of Euclidean geometry dramatically lowered their epistemic status from that of first principles to that of merely hypothetical assumptions, and since mathematics became flowed by risky concepts earlier commonly viewed as contradictory like that of infinite set. Without going to the discussion on whether or not the idea of logical foundations of mathematics is justified in principle I claim that the project of building such foundations pursued in 20th century completely failed for the simple reason that very similar development occurred in logic itself: available systems of logic rapidly multiplied, so there is now no more certainty in logic than in mathematics. Some people working in category theory propose it as a foundation but these proposals are not relevant to my present point.

Multiple links between category theory and logic have already brought about categorical logic (including topos logic) as a respectable field of mathematical research. My principle task in this paper is to suggest a view showing actual and potential epistemic impact of these developments. My principal thesis is that the notion of category takes us beyond form-based science and mathematics in general and beyond formal logic in particular. Whether this theory can be rightly seen as a foundation of a kind or not it certainly functions as an integrating device linking together different fragments of our knowledge. That's why I think that to develop these new possibilities as far as possible, in particular in logic, is a worthy and challenging task of the moment.