

# Mathematics, Physics, and Metaphysics from the Neo-Kantian and the Analytic Perspectives

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Critical versus Dogmatic Philosophy

From Neo-Kantianism To Analytic Philosophy

Logic and the Cold War

21st century: a Future for Critical Philosophy?

# Critical versus Dogmatic Philosophy

Full recognition of autonomy of science: the aim of philosophy (w.r.t. science) is *critique* of science but not providing first principles for science. The critique purports to explain how science is possible. It *is* possible because it is actual.  
A new feature: critical philosophy must keep track of new scientific developments!

## Friedman 1998 on Kant's Critical Philosophy

“The relationship between the pure intellectual concepts of metaphysics and the world of phenomena has thus been reinterpreted in a profoundly radical fashion. Pure intellectual concepts no longer characterize an underlying reality situated at a deeper and more fundamental level than the phenomena themselves; on the contrary, such concepts can acquire a relation to an object in the first place only by being realized or schematized at the phenomenal level. And it is this this radical reinterpretation of the relationship between metaphysics and the phenomena - [...] that constitutes Kant's truly decisive break with the Leibnizean-Wolffian tradition.”

## Friedman 1998 on Kant's Critical Philosophy

“Kant locates his break with the dogmatic metaphysics of the Leibnizean - Wolffian philosophy at just this point. Dogmatism is “the presumption that it is possible to make progress with only a pure knowledge from concepts (philosophical knowledge), according to principles”. The error of dogmatism is its attempt to proceed with general logic alone, independently of transcendental logic.”

# Kant 1781 on philosophical and mathematical cognition 1

Die philosophische Erkenntnis ist die Vernunftkenntnis aus Begriffen, die mathematische aus der Konstruktion der Begriffe. Einen Begriff aber konstruieren, heisst: die ihm korrespondierende Anschauung a priori darstellen. Zur Konstruktion eines Begriffs wird also eine nicht empirische Anschauung erfordert, die folglich, als Anschauung, ein einzelnes Objekt ist, aber nichtsdestoweniger, als die Konstruktion eines Begriffs (einer allgemeinen Vorstellung), Allgemeingültigkeit für alle möglichen Anschauungen, die unter denselben Begriff gehören, in der Vorstellung ausdrücken muss.

## Kant 1781 on philosophical and mathematical cognition 2

So konstruiere ich einen Triangel, indem ich den diesem Begriffe entsprechenden Gegenstand, entweder durch blosser Einbildung, in der reinen, oder nach derselben auch auf dem Papier, in der empirischen Anschauung, beidemal aber völlig a priori, ohne das Muster dazu aus irgendeiner Erfahrung geborgt zu haben, darstelle. Die einzelne hingezzeichnete Figur ist empirisch, und dient gleichwohl den Begriff, unbeschadet seiner Allgemeinheit, auszudrücken, weil bei dieser empirischen Anschauung immer nur auf die Handlung der Konstruktion des Begriffs, welchem viele Bestimmungen, z.E. der Grösse, der Seiten und der Winkel, ganz gleichgültig sind, gesehen, und also von diesen Verschiedenheiten, die den Begriff des Triangels nicht verändern, abstrahiert wird.

## Kant 1781 on philosophical and mathematical cognition 3

Die philosophische Erkenntnis betrachtet also das Besondere nur im Allgemeinen, die mathematische das Allgemeine im Besonderen, ja gar im Einzelnen, gleichwohl doch a priori und vermittelt der Vernunft, so dass, wie dieses Einzelne unter gewissen allgemeinen Bedingungen der Konstruktion bestimmt ist, ebenso der Gegenstand des Begriffs, dem dieses Einzelne nur als sein Schema korrespondiert, allgemein bestimmt gedacht werden muss. (A713-4/B741-2).



# Kant 1781 on mathematical and speculative reasoning 1

Man gebe einem Philosophen den Begriff eines Triangels, und lasse ihn nach seiner Art ausfindig machen, wie sich wohl die Summe seiner Winkel zum rechten verhalten möge. Er hat nun nichts als den Begriff von einer Figur, die in drei geraden Linien eingeschlossen ist, und an ihr den Begriff von ebensoviel Winkeln. Nun mag er diesem Begriffe nachdenken, so lange er will, er wird nichts Neues herausbringen. Er kann den Begriff der geraden Linie, oder eines Winkels, oder der Zahl drei zergliedern und deutlich machen, aber nicht auf andere Eigenschaften kommen, die in diesen Begriffen gar nicht liegen. Allein der Geometer nehme diese Frage vor. Er fängt sofort davon an, einen Triangel zu konstruieren.

## Kant 1781 on mathematical and speculative reasoning 2

Weil er weiss, dass zwei rechte Winkel zusammen gerade so viel austragen, als alle berührenden Winkel, die aus einem Punkte auf einer geraden Linie gezogen werden können, zusammen, so verlängert er eine Seite seines Triangels, und bekommt zwei berührende Winkel, die zweien rechten zusammen gleich sind. Nun teilt er den äusseren von diesen Winkeln, indem er eine Linie mit der gegenüberstehenden Seite des Triangels parallel zieht, und sieht, dass hier ein äusserer berührender Winkel entspringe, der einem inneren gleich ist, usw. Er gelangt auf solche Weise durch eine Kette von Schlüssen, immer von der Anschauung geleitet, zur völlig einleuchtenden und zugleich allgemeinen Auflösung der Frage. (A 716 / B 744)

## Friedman 1998 on Kant on empirical intuition

[I]t is not pure intuition, but only empirical intuition that is capable of providing a model for the truths of mathematics. It follows that pure intuition, at least by itself, can in no way demonstrate or exhibit the real possibility of mathematics. For Kant, such real possibility means that mathematics is applicable to empirical objects (objects of experience), and this cannot be shown except on the basis of transcendental philosophy itself.

# Summary on Mathematics and Physics

- ▶ Mathematics is a science of Time (Arithmetic) and Space (Geometry);
- ▶ Time and Space are a priori forms of possible experience, so everything mathematically possible is physically possible (i.e., belongs to the possible experience);
- ▶ Fundamental physics is a theory of physically possible but not only of physically actual (cf. Newtonian physics);
- ▶ Pure maths is not sufficient for doing natural science; but additional principles don't limit mathematical possibilities.

# Summary on Logic and Metaphysics

- ▶ Well-founded Metaphysics is based on Physics but not the other way round;
- ▶ It involves not only concepts belonging to objects of possible experience but also ideas of reason that guide the construction of concepts (like “world”);
- ▶ The general (formal) Logic does not distinguish between the possible experience and the bold speculation, the Transcendental Logic does.

## Cassirer 1907 on the Future of Critical Philosophy

Das Schicksal und die Zukunft der kritischen Philosophie wird durch ihr Verhältnis zur exakten Wissenschaft bedingt. Wenn es gelänge, das Band zwischen ihr und der Mathematik und mathematischen Physik zu zerschneiden, so wäre sie damit ihres Wertes und Inhalts beraubt. Wie hier die geschichtlichen Wurzeln ihrer Entstehung liegen, so kann auch ihre Fortdauer nur durch diesen lebendigen Zusammenhang gesichert werden. Der Bestand ihrer Sätze bildet somit keinen fertigen und ein für alle Mal gesicherten Besitz, sondern er muss sich gegenüber den Wandlungen der wissenschaftlichen Überzeugungen und Begriffe stets von Neuem rechtfertigen. Hier gibt es keine selbstsicheren Dogmen, die auf ihre "unmittelbare Evidenz" hin angenommen und für alle Zeiten festgestellt werden könnten: dauernd ist allein die Aufgabe der ständig erneuten Prüfung der wissenschaftlichen Grundbegriffe, die für die Kritik zugleich zu strenger subjektiver Selbstprüfung wird.

# New Mathematics and Science

Invention of non-Euclidean geometries. Multiple geometrical spaces. Which one represents “the” physical space? Set theory, Quantum Mechanics...

## A problem

More generally: Only particular mathematical constructions model physical phenomena. Mathematically possible is not necessarily physically possible. “Unreasonable Effectiveness”



## Duhem 1908 on Saving Phenomena

Physical theory is not a [causal] explanation. It is a system of mathematical propositions, deduced from a small number of principles, which aim to represent as simply, as completely, and as exactly as possible a set of experimental laws [..] Concerning the very nature of things, or the realities hidden under the phenomena [..] a theory [..] tells us absolutely nothing, and does not claim to teach us anything.

## Russell 1903 pro Leibniz contra Kant 1

It seemed plain that mathematics consists of deductions, and yet the orthodox accounts of deduction were largely or wholly inapplicable to existing mathematics. Not only the Aristotelian syllogistic theory, but also the modern doctrines of Symbolic Logic, were either theoretically inadequate to mathematical reasoning, or at any rate required such artificial forms of statement that they could not be practically applied. In this fact lay the strength of the Kantian view, which asserted that mathematical reasoning is not strictly formal, but always uses intuitions, i.e. the a priori knowledge of space and time. Thanks to the progress of Symbolic Logic, especially as treated by Professor Peano, this part of the Kantian philosophy is now capable of a final and irrevocable refutation. By the help of ten principles of deduction and ten other premisses of a general logical nature (e.g. "implication is a relation"), all mathematics can be strictly and formally deduced.

## Russell 1903 pro Leibniz contra Kant 2

The general doctrine that all mathematics is deduction by logical principles from logical principles was strongly advocated by Leibniz... But owing partly to a faulty logic, partly to belief in the logical necessity of Euclidean Geometry, he was led into hopeless errors in the endeavour to carry out in detail a view which, in its general outline, is now known to be correct. The actual propositions of Euclid, for example, do not follow from the principles of logic alone ; and the perception of this fact led Kant to his innovations in the theory of knowledge.

## Russell 1903 pro Leibniz contra Kant 3

But since the growth of non-Euclidean Geometry, it has appeared that pure mathematics has no concern with the question whether the axioms and propositions of Euclid hold of actual space or not ..... What pure mathematics asserts is merely that the Euclidean propositions follow from the Euclidean axioms, i.e., it asserts an implication. .... We assert always in mathematics that if a certain assertion  $p$  is true of any entity  $x$  or of any set of entities  $x, y, z, \dots$ , then some other assertion  $q$  is true of those entities ; but we do not assert either  $p$  or  $q$  separately of our entities.

## Russell 1918 on Metaphysics

As I have attempted to prove in *The Principles of Mathematics*, when we analyse mathematics we bring it all back to logic. It all comes back to logic in the strictest and most formal sense. In the present lectures, I shall try to set forth in a sort of outline, rather briefly and rather unsatisfactorily, a kind of logical doctrine which seems to me to result from the philosophy of mathematics - not exactly logically, but as what emerges as one reflects: a certain kind of logical doctrine, and on the basis of this a certain kind of metaphysics.

## Russell 1945 on Analytic Philosophy

Modern analytical empiricism [...] differs from that of Locke, Berkeley, and Hume by its incorporation of mathematics and its development of a powerful logical technique. It is thus able, in regard to certain problems, to achieve definite answers, which have the quality of science rather than of philosophy. It has the advantage, as compared with the philosophies of the system-builders, of being able to tackle its problems one at a time, instead of having to invent at one stroke a block theory of the whole universe. Its methods, in this respect, resemble those of science. I have no doubt that, in so far as philosophical knowledge is possible, it is by such methods that it must be sought; I have also no doubt that, by these methods, many ancient problems are completely soluble.

A MERELY TECHNICAL ADVANTAGE? OTHERWISE THE  
OLD-FASHIONED DOGMATIC APPROACH

## Cassirer 1907 on Russell 1

Damit aber ist ein Problem gestellt, das völlig ausserhalb des Gesichtskreises der “Logistik” liegt und das somit von ihrer Kritik auch nicht berührt wird. Alle empirischen Urteile liegen jenseit ihres Bereiches: sie macht an der Grenze der Erfahrung Halt. Was sie entwickelt, ist ein System hypothetischer Voraussetzungen, von denen wir aber niemals wissen können, ob sie sich jemals in irgend einer Erfahrung verwirklicht finden, ob sie daher jemals irgend eine mittelbare oder unmittelbare konkrete Anwendung verstatten werden.



## Cassirer 1907 on Russell 2

So fällt nach Russell schon der allgemeine Begriff der Grösse aus dem Umkreis der reinen Mathematik und Logik heraus: er enthält ein empirisches Element, das uns nur durch die sinnliche Wahrnehmung gegeben werden kann. Nach der Grundansicht der Logistik ist die Aufgabe des Denkens beendet, wenn es gelungen ist, unter all seinen Gebilden und Erzeugungen eine strenge deduktive Verknüpfung herzustellen. Die Sorge um die Gesetzlichkeit der Welt der Objekte dagegen bleibt gänzlich der direkten Beobachtung überlassen, die allein uns innerhalb ihrer eigenen, sehr eng gesteckten Grenzen zu lehren vermag, ob auch hier bestimmte Regelmässigkeiten sich finden, oder aber ein reines Chaos herrscht.

## Cassirer 1907 on Russell 3

Logik und Mathematik haben es nur mit der Ordnung der Begriffe zu tun; die Ordnung oder Verwirrung unter den Gegenständen ficht sie nicht an und braucht sie nicht zu beirren.

So bleibt, wieweit man auf diesem Standpunkt die Analyse der Begriffe auch treiben mag, das empirische Sein ein ewig unbegriffenes Problem.

UNREASONABLE EFFECTIVENESS!

# Summary on Mathematics and Physics

- ▶ Mathematics is based upon or even reduces to (formal mathematized) Logic (a logico-mathematical core of sciences)
- ▶ Logic comes with (dogmatic) Metaphysics
- ▶ This Logic and Metaphysics is a foundation of Physics.

## a worry

Are we back to dogmatic metaphysic? Russell 1900 (*A Critical Exposition of the Philosophy of Leibniz*): Leibniz was right in principle but used a wrong (subject-predicate) logic. With the new logic (Frege, Peano et al.) Leibniz' project is doable!  
Is it justified to abandon Kantian philosophy of mathematics only because it “does not work” with the new mathematics and physics?  
No.

## Ex.: Frege 1892 on Sense and Reference 1

$a = a$  and  $a = b$  are obviously statements of different epistemic value;  $a = a$  holds *a priori* and, according to Kant, is to be labeled analytic, while statements of the form  $a = b$  often contain very valuable extensions of our knowledge and cannot always be established *a priori*. The discovery that the rising sun is not new every morning, but always the same, was one of the most fertile astronomical discoveries. Even today the identification of a small planet or a comet is not always a matter of course. Now if we were to regard identity as a relation between that which the names  $a$  and  $b$  designate, it would seem that  $a = b$  could not differ from  $a = a$  (provided  $a = b$  is true).

## Ex.: Frege 1892 on Sense and Reference 2

When we found  $a = a$  and  $a = b$  to have different epistemic values, the explanation is that for the purpose of knowledge, the sense of the sentence, viz., the thought expressed by it, is no less relevant than its reference, i.e. its truth value. If now  $a = b$ , then indeed the reference of  $b$  is the same as that of  $a$ , and hence the truth-value of  $a = b$  is the same as that of  $a = a$ . In spite of this the sense of  $b$  may differ from that of  $a$  and thereby the thought expressed in  $a = b$  differs from that of  $a = a$ . In that case the two sentences do not have the same epistemic value. If we understand by judgment the advance from the thought to its truth value, as in the above paper, we can also say that the judgments are different.

FREGE'S SOLUTION IS WHOLLY BASED ON HIS ANALYSIS OF PATTERNS OF THE EVERYDAY TALK AND HISTORICAL NARRATIVE! WHY THE COMMON LANGUAGE MUST WORK IN SCIENCE?



## Bochenski on Dialectical Materialism

Perhaps this supervision is to be blamed also for the mediocrity of the philosophers in this school; it is in any case responsible for the extreme dogmatism, chauvinism, and aggressiveness of dialectical materialists. Even more significant, however, than these peculiarities, which could be accidental, is the reactionary character of dialectical materialism, for this philosophy leads straight back to the mid-nineteenth century and seeks to restore the intellectual situation of that time without the slightest alteration.

(*Contemporary European Philosophy*, 1956, Russian translation 1959)

# Bochenski on Neo-Thomism and Revival of Metaphysics

Thomism experienced a rebirth about 1880 [...] and organized a large school which was soon become extremely powerful. Its Fribourg organ, *La Revue Thomiste*, was started in 1893 and *La Revue Néoscolastique de Philosophie* in Louvain in 1894. It represents the direct realism and the traditional metaphysics. Thomism did not stand alone, however, for in England G.E. Moore produced his famous essay, *The Refutation of Idealism*, in 1903 and along with Bertrand Russell he put forward an almost Platonic philosophy. (*Contemporary European Philosophy*, 1956, Russian translation 1959)

## Janovskaya on Hilbert and Ackermann

Д. Гильберт разрабатывал аппарат математической логики в надежде с его помощью оправдать свою формалистическую и идеалистическую точку зрения на математику, как на совокупность лишенных содержания формул, которые пишутся по определенным правилам. Однако действительное развитие логики, и притом с помощью построенного самим же Гильбертом аппарата, обнаружило неосуществимость его надежд. Развитие науки и в этой области неизменно подтверждает правильность философских установок марксизма-ленинизма, Но буржуазные ученые не хотят признавать этого, Они упорно борются против всякого проявления материализма. И притом все более и более агрессивно.

## Janovskaya on Hilbert and Ackermann (continued)

В руках исследователя, вооруженного передовой марксистско-ленинской философией, и математическая логика становится не только орудием открытия новых систем, но и средством разоблачения реакционной идеологии. В применении к математической логике нам особенно следует помнить указание, сделанное товарищем А.А. Ждановым <..> Современная буржуазная наука, - говорит А.А. - снабжает поповщину, фидеизм новой аргументацией, которую необходимо беспощадно разоблачать. Кому же как не нам, стране победившего марксизма и ее философам, возглавить борьбу против растленной и гнусной буржуазной идеологии, кому, как не нам, наносить ей сокрушающие удары. (1947)

Did Analytic Philosophy go out of the Cold War?

# Problems

The standard Set-Theoretic foundations of mathematics based upon the Formal Axiomatic Method are wholly detached from the foundations of physics. David Hilbert who has shaped the Axiomatic Method in its present form saw the problem and expected his Formal Axiomatic Method be applicable to Physics and other Natural Sciences too (the Sixth Problem). But did not succeed. Today's mainstream philosophy of mathematics is detached from the philosophy of physics, biology and other natural sciences.

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# Problems

NATURALISM VS. FUNDAMENTALISM WITHIN THE  
ANALYTIC METAPHYSICS (QUINE).

A problem for Naturalism: the logico-mathematical apparatus of  
Analytic Philosophy has nothing to do with the mathematical  
apparatus of today's science!



# Needs

- ▶ Fundamental physic: Quantum Gravity, Quantum Cosmology;
- ▶ Theoretical Biology: Mathematization;
- ▶ The board line between science and speculation.

# Conclusion

EPPUR SI MUOVE!