Mathematics, Physics, and Metaphysics After Kant

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6 апреля 2012 г.

Friedman on Kant

Kant, 20th century Science and Cold War

Optical-Mechanical Analogy

Group theory and Category theory in Physics

(1)Friedman on Kant

Wigner: [T]he enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it.
Why indeed mathematics is so “unreasonably effective”?
Kant has an answer...

Kant

- Mathematics is a science of Time (Arithmetic) and Space (Geometry);
- Time and Space are a priori forms of possible experience, so everything mathematically possible is physically possible;
- Fundamental physics is a theory of physically possible but not only of physically actual (cf. Newtonian physics); however physical possibilities are grounded upon (and systematically tested in) actual observations and experiments.
- Pure maths is not sufficient for doing natural science; but additional principles don't limit mathematical possibilia.

Critical versus Dogmatic Philosophy

Full recognition of autonomy of science: the aim of philosophy (w.r.t. science) is *critique* of science but not providing first principles for science. The critique purports to explain how science is possible. It *is* possible because it is actual.
A new feature: critical philosophy must keep track of new scientific developments!

Friedman on Kant

The relationship between the pure intellectual concepts of metaphysics and the world of phenomena has thus been reinterpreted in a profoundly radical fashion. Pure intellectual concepts no longer characterize an underlying reality situated at a deeper and more fundamental level than the phenomena themselves; on the contrary, such concepts can acquire a relation to an object in the first place only by being realized or schematized at the phenomenal level. And it is this this radical reinterpretation of the relationship between metaphysics and the phenomena - [..] that constitutes Kant's truly decisive break with the Leibnizean-Wolffian tradition. (*Kant and Exact Sciences*)

Friedman on Kant

Kant locates his break with the dogmatic metaphysics of the Leibnizean - Wolffian philosophy at just this point. Dogmatism is "the presumption that it is possible to make progress with only a pure knowledge from concepts (philosophical knowledge), according to principles". The error of dogmatism is its attempt to proceed with general logic alone, independently of transcendental logic. (*Kant and Exact Sciences*)

Kant on philosophical and mathematical cognition

Philosophical cognition is rational cognition from concepts, mathematical cognition is that from the construction of concepts. But to construct a concept means to exhibit a priori the intuition corresponding to it. For the construction of a concept, therefore, a non-empirical intuition is required, which consequently, as intuition, is an individual object, but that must nevertheless, as the construction of a concept (of a general representation), express in the representation universal validity for all possible intuitions that belong under the same concept, either through mere imagination, in pure intuition, or on paper, in empirical intuition..[..] Philosophical cognition thus considers the particular only in the universal, but mathematical cognition considers the universal in the particular, indeed even in the individual... (A713-4/B741-2).

Friedman on Kant on Euclid

Euclidean geometry [..] is not to be compared with Hilbert's axiomatization [of Euclidean geometry in his *Foundations of Geometry* of 1899], say, but rather with Frege's *Begriffsschrift* [or another symbolic logical calculus]. It is not a substantive doctrine, but a form of rational representation: a form of rational argument and inference. Accordingly, its propositions are established, not by quasi-perceptual acquaintance with some particular subject matter, but, as far as possible, by the most rigorous methods of proof - by the proof-procedures of Euclid, Book I, for example. There remains a serious question about Euclid's axioms, of course; when pressed, Kant would most likely claim that they represent the most general conditions under which alone a concept of extended magnitude - and therefore a rigorous conception of an external world - is possible (see A163/B204). And, of course, we now know that Kant is fundamentally mistaken here. (*ib.*)

Friedman on Kant

[I]t is not pure intuition, but only empirical intuition that is capable of providing a model for the truths of mathematics. It follows that pure intuition, at least by itself, can in no way demonstrate or exhibit the real possibility of mathematics. For Kant, such real possibility means that mathematics is applicable to empirical objects (objects of experience), and this cannot be shown except on the basis of transcendental philosophy itself.

Kant on Galilean Science

The understanding does not extract its laws (a priori) from, but prescribes them to, nature.
Ex.: the law of gravitation. The Galilean Science as contrasted to Ptolemean Science saving phenomena.

Friedman on Kant on Galilean Science

[T]he law of gravitation has a particular kind of mixed status: it is derived from a combination of a priori laws of the understanding (the analogies of experience), a priori laws of sensibility (Euclidean geometry), and a posteriori given data of experience. Because of its essential dependence on the latter, the law of gravitation is an empirical law and hence, strictly speaking, is neither a priori nor necessary. Nevertheless, because of its equally essential dependence on the above-mentioned a priori laws, the law of gravitation still enjoys a particular kind of "empirical" or "material" necessity in virtue of which it is more firmly established and secure relative to the a posteriori given data than any mere inductive generalization or hypothesis.

(2) Kant, 20th century Science and Cold War

Invention of non-Euclidean geometries. Multiple geometrical spaces. Which one represents “the” physical space? Set theory, Quantum Mechanics...

A problem (continued)

More generally: Only particular mathematical constructions model physical phenomena. Mathematically possible is not necessarily physically possible.

Duhem

Physical theory is not a [causal] explanation. It is a system of mathematical propositions, deduced from a small number of principles, which aim to represent as simply, as completely, and as exactly as possible a set of experimental laws [..] Concerning the very nature of things, or the realities hidden under the phenomena [..] a theory [..] tells us absolutely nothing, and does not claim to teach us anything. *Saving Phenomena* (1908)
Back to Ptolemy?

Cassirer

With this arises a problem, which lies completely outside the scope of logistics [..] Worrying about the rules that govern the world of objects is completely left to direct observation, which is the only one that can teach us [..] whether we can find here certain regularities or a pure chaos. Logic and mathematics deal only with the order of concepts; they don't contest the order or the disorder of objects and they don't need to confuse themselves with this issue. (1907)

Cassirer

A clue to the Nature of Man: a Symbol
Mathematically-laden natural science provides us with a *symbolic* representation of the underlying reality.

Russell

As I have attempted to prove in *The Principles of Mathematics*, when we analyse mathematics we bring it all back to logic. It all comes back to logic in the strictest and most formal sense. In the present lectures, I shall try to set forth in a sort of outline, rather briefly and rather unsatisfactorily, a kind of logical doctrine which seems to me to result from the philosophy of mathematics - not exactly logically, but as what emerges as one reflects: a certain kind of logical doctrine, and on the basis of this a certain kind of metaphysic. (*The Philosophy of Logical Atomism*, 1918)

my worry

Are we back to dogmatic metaphysic? Russell 1900 (*A Critical Exposition of the Philosophy of Leibniz*): Leibniz was right in principle but used a wrong (subject-predicate) logic. With the new logic (Frege, Peano et al.) Leibniz' project is doable!
Is it justified to abandon Kantian philosophy of mathematics only because it "does not work" with the new mathematics and physics? No.

Bochenski on History of Modern Philosophy

and science: 3 centuries of intellectual decline
(an argument in Cold War ideological battles)

Bochenski on Dialectical Materialism

Perhaps this supervision is to blamed also for the mediocrity of the philosophers in this school; it is in any case responsible for the extreme dogmatism, chauvinism, and aggressiveness of dialectical materialists. Even more significant, however, than these pecularities, which could be accidental, is the reactionary character of dialectical materialism, for this philosophy leads straight back to the mid-nineteenth centruy and seeks to restore the intellectual situation of that time without the slightest alteration.
(*Contemporary European Philosophy*, 1956, Russian translation 1959)

Bochenski on Neo-Thomism and Revival of Metaphysics

Thomism experienced a rebirth about 1880 [..] and organized a large school which was soon become extremely powerful. Its Fribourg organ, *La Revue Thomiste*, was started in 1893 and *La Revue Néoscolastique de Philosophie* in Louvain in 1894. Its represents direct realism and traditional metaphysics.
Thomism did not stand alone, however, for in England G.E. Moore produced his famous essay, The Refutation of Idealism, in 1903 and along with Bertrand Russell he put forward an almost Platonic philosophy. (*Contemporary European Philosophy*, 1956, Russian translation 1959)

Janovskaya on Hilbert and Ackermann

Д. Гильберт разрабатывал аппарат математической логики в надежде с его помощью оправдать свою формалистическую и идеалистическую точку зрения на математику, как на совокупность лишенных содержания формул, которые пишутся по определенным правилам. Однако действительное развитие логики, и притом с помощью построенного самим же Гильбертом аппарата, обнаружило неосуществимость его надежд. Развитие науки и в этой области неизменно подтверждает правильность философских установок марксизма-ленинизма, Но буржуазные ученые не хотят признавать этого, Они упорно борятся против всякого проявления материализма. И притом все более и более агрессивно.

Janovskaya on Hilbert and Ackermann (continued)

В руках исследователя, вооруженного передовой марксистко-ленинской философией, и математическая логика становится не только орудием открытия новых систем, но и средством разоблачения реакционной идеологии. В применении к математической логике нам особенно следует помнить указание, сделанное товарищем А.А. Ждановым <..> Современная буржуазная наука, - говорит А.А. - снабжает поповщину, фидеизм новой аргументацией, которую необходимо беспощадно разоблачать. Кому же как не нам, стране победившего марксизма и ее философам, возглавить борьбу против растленной и гнусной буржуазной идеологии, кому, как не нам, наносить ей сокрушающие удары. (1947)

GAPS

The standard Set-Theoretic foundations of mathematics based upon the Formal Axiomatic Method are wholly detached from the foundations of physics. David Hilbert who has shaped the Axiomatic Method in its present form saw the problem and expected his Formal Axiomatic Method be applicable to Physics and other Natural Sciences too (the Sixth Problem). But did not succeed. Today's mainstream philosophy of mathematics is detached from the philosophy of physics, biology and other natural sciences.

(3) Optical-Mechanical Analogy

An evidence in favor of the Kantian view?

Schrödinger on Optical-Mechanical Analogy, 1926

Maybe our classical mechanics is the full analog of geometrical optics, and, as such, wrong, not in agreement with reality. It fails as soon as the radii of curvature and the dimensions of the trajectory are not large anymore compared to a certain wavelength, to which one can attribute a certain reality in q-space. In that case, one has to search for an "undulatory mechanics" and the obvious way to this end is the wave-theoretical extension of Hamilton's picture.

Weyl on Schrödinger and Hamilton

Schrödinger's researches took as their point of departure the Hamiltonian theory of mechanics, which was originally obtained by Hamilton himself from an analogy with geometrical optics. He argued that since we replace geometrical optics, with the aid of which interference and diffraction cannot be treated, by wave optics, it is reasonable to attempt the analogous transition in mechanics. The results amply justified the attempt.

Is this "analogy" a miracle?

I shall try to explain it in neo-Kantian terms by revising Kant's original approach.

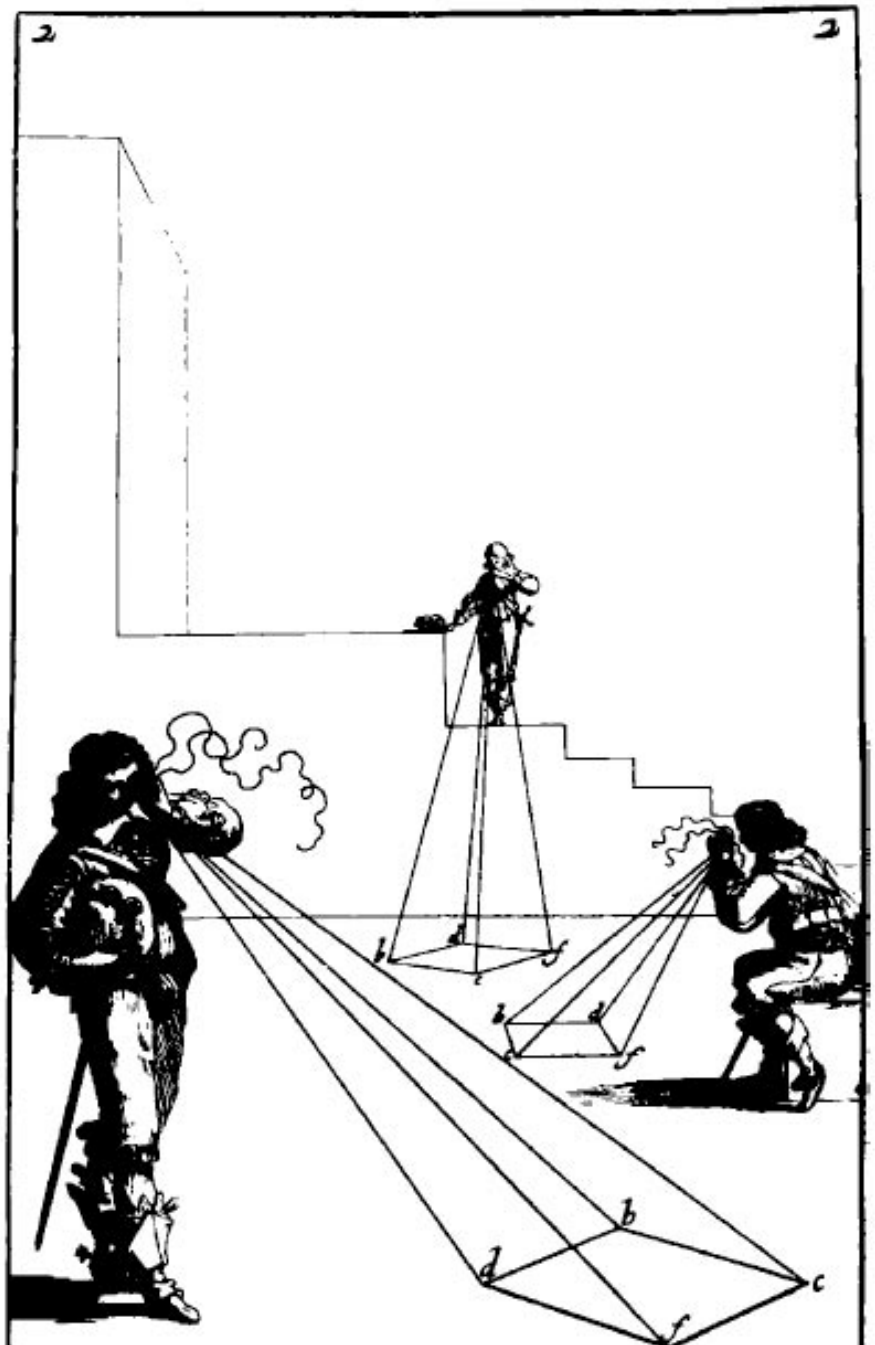
Weyl's neo-Kantian (?) view

Natural science is of a constructive character. The concepts with which it deals are not qualities or attributes which can be obtained from the objective world by direct cognition. They can only be determined by an indirect methodology, by observing their reaction with other bodies, and their implicit definition is consequently conditioned by definite laws of nature governing reactions.

Euclid's *Optics*

Let it be assumed:
1. That rectilinear rays proceeding from the eye diverge indefinitely;
2. That the figure contained by a set of visual rays is a cone of which the vertex is at the eye and the base at the surface of the objects seen;
3. That those things are seen upon which visuals rays fall and those things are not seen upon which visual rays do not fall;
4.That things seen under a larger angle appear larger, those under a smaller angle appear smaller, and those under equal angles appear equal;
5. That things seen by higher visual rays appear higher, and things seen by lower visual rays appear lower;
6. That, similarly, things seen by rays further to the right appear further to the right, and things seen by rays further to the left appear further to the left;
7. That thing seen under more angles are seen more clearly.

Theory of Perspective



Convergence Theorem

Parallel lengths, seen from a distance, appear not to be equally distant from each other.

Proof:

Produce perpendiculars, then apply Post. 4. (Works only if the eye is placed between the parallels!)

Appearances and Objective Representations

are different things! Euclidean geometry works a relativistic scheme (like Minkowsky geometry in SR). An Euclidean straight line *is* a (segment of) light ray. This basic geometrical notion with all its properties and relations, in the last analysis, is empirical. However as long as it is not questioned in a given experiment it can be described as "a priori" w.r.t. the given experiment. Euclidean geometry is an empirically-grounded theory of vision applicable within certain limits. (A bold claim: Arithmetic has a similar character.)

Relativity

SR and GR are alternative schemes of the same sort, which work better for large distances. They translate into the Euclidean (or pseudo-Euclidean) schemes through the assumption of smoothness (any smooth manifold is locally flat)

Optical-Mechanical Analogy

Since light rays at a closer look behave like waves straight lines (free trajectories) in mechanics at a closer look equally behave like waves. This is because mechanics uses the same geometrical notion of straight line. No mystery!

Open Problem

Why this does not work throughout? A number of mathematically predicted physical entities are observed in experiments (particle physics). Why not *all* of them? What is going wrong? Probably because too many phenomena are *saved* by artificial mathematical means. Physics needs new *Mathematical Principles*.

(4) Group theory and Category theory in Physics

Hypothesis: Category theory provides a joint foundation for mathematics and natural sciences.

Group theory

Groups of transformations are mainly used in 20th century physics for describing structures and magnitudes invariant under these transformations. This allows for *objective* representations compatible with but independent of certain classes of specific "appearances". In such context groups are "concrete" groups acting upon certain *spaces* (vector spaces, spaces of functions or geometrical spaces) and serve as an algebraic means of studying symmetries and detecting invariants in these spaces.

Transformation of coordinates in 1531 A.D.

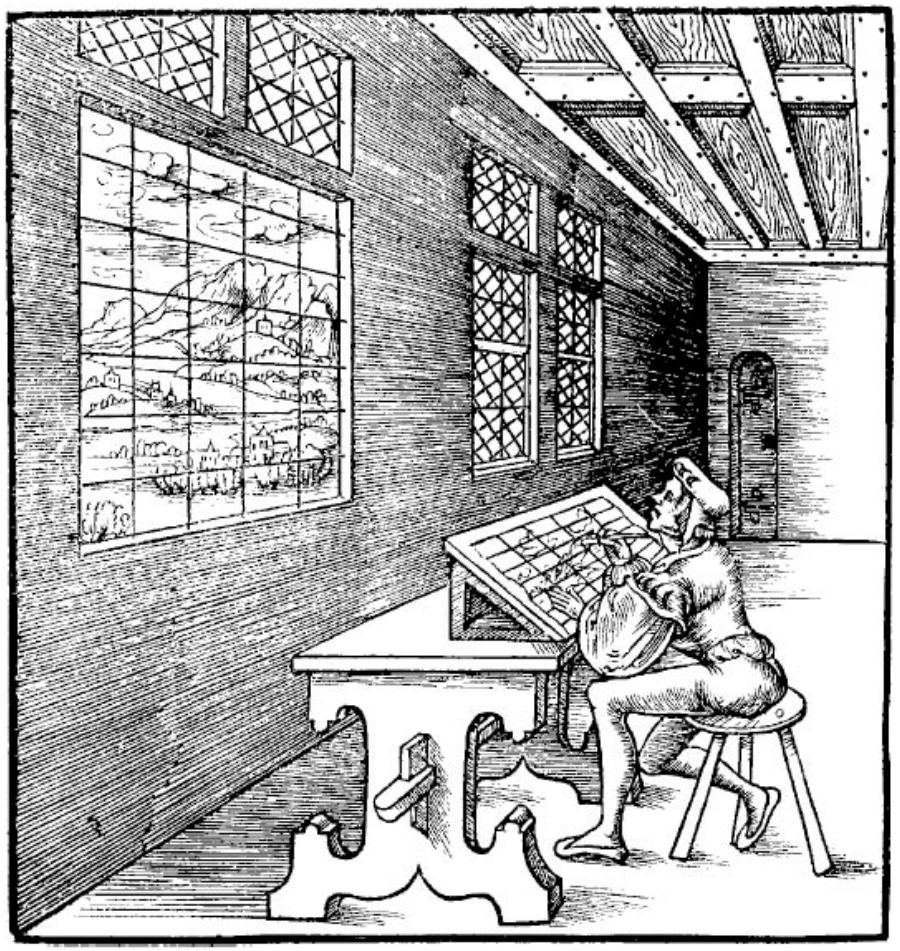


FIGURE V.61. Count Johann's interpretation of the Alberti idea of equipping the picture frame with a grid. As Lawrence Wright has remarked, the draughtsman in the picture is not reproducing what he sees through the window, but what we see (Wright[S] 1983, 314). Johann 1531, fol. Hii[v].

Category theory: an alternative foundation of mathematics?

A category can be seen as a double generalization of group: we get more objects (groupoid) and we allow for non-reversible transformations. A further generalization brings higher categories. This concept is already reach enough for capturing a lot of geometry without using external means (that cannot be done with a single group). This suggests a view on categories as general "relativistic schemes" in the above sense.

Mappings

A *mapping* is the most (?) general cognitive procedure playing a role in acquiring, updating and transmission of empirical data. Vision is a specific sort of mapping. It seems reasonable to assume (as a working hypothesis) that axioms of Category theory describe a protocol according to which humans (or at least scientists) collectively proceed empirical data (through space and time) and collectively manage their representations of these data. Any such protocol has not only descriptive but also normative (prescriptive) significance (like Euclidean Optics).

Spacetime

If this hypothesis turns to be tenable Category theory may serve also for building "the" physical space-time allowing for objective knowledge about the Nature. The idea that every physical object exists in space and time is not tenable if by space on understands Euclidean space and by time one understands Newtonian time. However the very idea of spacetime as an universal framework of objectivity in natural sciences is indispensable. The fact that QM has no proper notion of spacetime explains why QM only saves certain phenomena but does not give us their genuine explanation. (The incompatibility with GR is a part of the problem).

Conclusion

EPPUR SI MUOVE!