Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

What Comes After Structural Realism?

Homotopy Type Theory and Constructive Identity

Andrei Rodin

23 novembre 2012

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

Friedman 1998 on Kantian Revolution

"The relationship between the pure intellectual concepts of metaphysics and the world of phenomena has been reinterpreted [by Kant] in a profoundly radical fashion. Pure intellectual concepts no longer characterize an underlying reality situated at a deeper and more fundamental level than the phenomena themselves ; on the contrary, such concepts can acquire a relation to an object [and reality ? - AR] in the first place only by being realized or schematized at the phenomenal level. And it is this this radical reinterpretation of the relationship between metaphysics and the phenomena - [..] that constitutes Kant's truly decisive break with the Leibnizean-Wolffian tradition."

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

Cassirer 1907 on Intuition

"The [Kantian] principle according to which our concepts should
be sourced from intuitions means that they should be sourced from
the mathematical physics and should prove effective in this field.
Logical and mathematical concepts must no longer produce
instruments for building a metaphysical "world of thought" : their
proper function and their proper application lies only within the
empirical science itself."
NO POSSIBLE WORLDS !

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

20th century revival of metaphysics (i)

The idea according to which logic and mathematics provides a spectrum of conceptual possibilities, from which empirical observations pick up only few (mind the underdetermination of theories by empirical data), once again became common (Wigner). Even if the new metaphysics attempts to avoid dogmatism by being "descriptive" (Strawson) it tends to follow the traditional Aristotelian pattern of logical analysis of the everyday language rather than the Kantian model of philosophical critique of the contemporary mathematically-laden empirical science.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

Critical Philosophy discerns the notion of OBJECTIVITY (and closely related notion of OBJECTHOOD) from that of REALITY. The view according to which scientific objectivity implies reality is known as SCIENTIFIC REALISM. A Critical (Transcendental) philosopher can but must not be a scientific realist. It can be also either an idealist or systematically neutral w.r.t. the realism debate (dismissing this debate as metaphysical).

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

Thus from the Critical perspective the question of REALITY splits
into two separate issues :

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

Thus from the Critical perspective the question of REALITY splits into two separate issues :

▶ General question about the relationships between the (scientific) objectivity and reality ;

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

Thus from the Critical perspective the question of REALITY splits into two separate issues :

- ▶ General question about the relationships between the (scientific) objectivity and reality ;
- ▶ (Given that scientific realism is granted :) a plethora of specific questions about what (in a given discipline) is objective (and hence real) and what qualifies as an object (and hence as an entity).

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

Observations :

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

Observations :

▶ The issue of objectivity and objecthood can be also discussed INDEPENDENTLY from the realism debate.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

Observations :

▶ The issue of objectivity and objecthood can be also discussed INDEPENDENTLY from the realism debate.

▶ The current debate on Structural Realism focuses on this later issue within Scientific Realism.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

The crucial question behind the current realism debate : What objecthood and objectivity are or can be in today's science ?

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

Pure mathematics provides candidate forms of objecthood and objectivity for empirical sciences. Historically, new forms of objecthood/objectivity usually first emerge in mathematics. Examples :

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

Pure mathematics provides candidate forms of objecthood and objectivity for empirical sciences. Historically, new forms of objecthood/objectivity usually first emerge in mathematics. Examples :

▶ Barrow's Geometry of Curves (1665) and Newton's Physics

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

Pure mathematics provides candidate forms of objecthood and objectivity for empirical sciences. Historically, new forms of objecthood/objectivity usually first emerge in mathematics. Examples :

▶ Barrow's Geometry of Curves (1665) and Newton's Physics

▶ Riemann's Geometry and Einstein's GR

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

How To Be a Realist After Kant ?
Two Issues About Scientific Realism
Mathematical and Physical Objecthood and Objectivity

Pure mathematics provides candidate forms of objecthood and
objectivity for empirical sciences. Historically, new forms of
objecthood/objectivity usually first emerge in mathematics.
Examples :

- ▶ Barrow's Geometry of Curves (1665) and Newton's Physics
- ▶ Riemann's Geometry and Einstein's GR
- ▶ Mathematical structures (Klein, Hilbert, Bourbaki) and
  Structural Realism

Claims :

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

Claims :

▶ Mathematical Structuralism is outdated ; even if it still serves
as a conceptual framework for a bulk of existing mathematics
it does not any longer serve as a suggestive idea for further
research.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

Claims :

► Mathematical Structuralism is outdated ; even if it still serves as a conceptual framework for a bulk of existing mathematics it does not any longer serve as a suggestive idea for further research.

► Category Theory provides a new form of objecthood and objectivity, which historically emerges from Structuralism but does not qualify as a form of Structuralism.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

Claims :

- ▶ Mathematical Structuralism is outdated ; even if it still serves as a conceptual framework for a bulk of existing mathematics it does not any longer serve as a suggestive idea for further research.

- ▶ Category Theory provides a new form of objecthood and objectivity, which historically emerges from Structuralism but does not qualify as a form of Structuralism.

- ▶ This new mathematical form of objecthood and objectivity is a strong candidate for a new form of objecthood and objectivity in physics.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

Cultural Observation

Structuralism began to constitute itself as a philosophical doctrine
in the English-speaking world in early 1970-ies (Sneed, Suppe) just
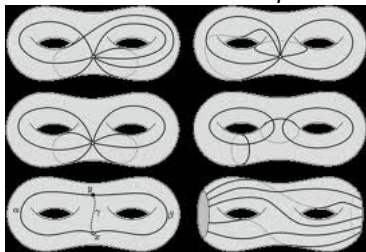after it lost its suggestive power and went out of fashion in French
intellectual life.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

Are isomorphic structures identical ? Does identity replaces
isomorphism ? Or it is rather isomorphism that replaces identity ?
How it works exactly ? The "wobble about identity" (Lawvere) is a
clear symptom that in the structural mathematics the objecthood
is not properly fixed. Structures have objective (=invariant)
properties (Nozik) but they are not well-constituted *qua* objects.
Instead one equates (after Bolzano, Frege, Russell and Parsons and
ignoring Kantian Critical view) objects with logical individuals.
Thus objectivity and objecthood are set apart (French : structures
are <u>not</u> objects).

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

A way out suggested by Category theory :
Consider an isomorphism as a *transformation* rather than relation.
(Then one sees that an isomorphism is an invertible *morphism*, i.e.,
an invertible general transformation.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

Example : categorification of fundamental group.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

Fundamental group

Fundamental group G_T^0 of a topological space T :

- a base point P ;
- loops through P (loops are circular paths $l : I \rightarrow T$) ;
- composition of the loops (up to homotopy only ! - see below) ;
- identification of homotopic loops ;
- independence of the choice of the base point.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

Fundamental (1-) groupoid

G_T^1 :

- all points of T (no arbitrary choice) ;
- paths between the points (embeddings $s : I \to T$) ;
- composition of the *consecutive* paths (up to homotopy only ! - see below) ;
- identification of homotopic paths ;

Since not all paths are consecutive G_T^1 contains more information about T than G_T^0 !

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

Path Homotopy and Higher Homotopies

$s : I \rightarrow T$, $p : I \rightarrow T$ where $I = [0, 1]$: paths in T

$h : I \times I \rightarrow T$: homotopy of paths s, t if $h(0 \times I) = s$, $h(1 \times I) = t$

$h^n : I \times I^{n-1} \rightarrow T$: n-homotopy of $n - 1$-homotopies h_0^{n-1}, h_1^{n-1} if $h^n(0 \times I^{n-1}) = h_0^{n-1}$, $h^n(1 \times I^{n-1}) = h_1^{n-1}$;

<u>Remark :</u> : Paths are zero-homotopies

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

Higher Groupoids and Omega-Groupoids

- all points of T (no arbitrary choice) ;

- paths between the points ;

- homotopies of paths

- homotopies of homotopies (2-homotopies)

- higher homotopies up to n-homotopies

- higher homotopies ad infinitum

G_T^n contains more information about T than G_T^{n-1} !

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

Grothendieck Conjecture :

G_T^ω contains all relevant information about T; an omega-groupoid is a complete algebraic presentation of a topological space.

Morals :

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

Morals :

▶ Klein's Program does not work out straightforwardly : the group concept is not powerful enough to capture topology ;

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

Morals :

- ▶ Klein's Program does not work out straightforwardly : the group concept is not powerful enough to capture topology ;

- ▶ Category theory has been described in 1945 (i.e., at the very moments of its creation !) by Eilenberg and MacLane as a continuation of Erlangen Program ; however, categories unlike groups, generally, have no invariants. In the category-theoretic context the notion of invariance is replaced by that of functoriality (i.e. co- and contra variance). The (standard) structuralist thinking is no longer relevant.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Structuralism is Outdated
Structuralism : Wobble about Identity

Morals :

▶ Klein's Program does not work out straightforwardly : the group concept is not powerful enough to capture topology ;

▶ Category theory has been described in 1945 (i.e., at the very moments of its creation !) by Eilenberg and MacLane as a continuation of Erlangen Program ; however, categories unlike groups, generally, have no invariants. In the category-theoretic context the notion of invariance is replaced by that of functoriality (i.e. co- and contra variance). The (standard) structuralist thinking is no longer relevant.

▶ Topology motivates Higher Category theory ("Grothendieck's Program")

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Voevodsky (circa 2008)

Higher (homotopical) groupoids model higher identity types in
Martin-Löf's Constructive Type theory !

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Definitional and Propositional Identity

- ▶ $x : A$ - term x of type A; double interpretation of types : "sets" and propositions

- ▶ Definitional identity of terms (of the same type) and of types $x = y : A$; $A = B :$ *type* (substitutivity)

- ▶ Propositional identity of terms x, y of (definitionally) the same type A :
  $Id_A(x, y) :$ *type*;
  Remark : propositional identity is a (dependent) type on its own.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Higher Identity Types

- $x', y' : Id_A(x, y)$
- $Id_{Id_A}(x', y') : type$
- and so on

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Extensional and Intensional theories

- $x = y : A$ implies $z : Id_A(x, y)$;
- Extensionality : $z : Id_A(x, y)$ implies $x = y : A$;
- Intensionality : otherwise (beware that in the given framework this is a purely negative concept !);
- Remark : In extensional theories the hierarchy of higher identity collapses (is trivial); in intensional theories it, generally, does not.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Homotopical interpretation of Constructive Type theory

- types are topological spaces
- $Id_A(x, y)$ is a space of paths between points x, y in space A
- $Id_{Id_A}(x', y')$ is a space of homotopies of paths x', y'
- and so on

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

In HTT **objects** are connected components of topological spaces ; each object is provided with a groupoid of identities that can be highly non-trivial and that reflects its topological structure. NO WOBBLE ABOUT IDENTITY

Constructive Type theory provides a formal logical framework for such objects ; Homotopy theory provides an intuitive background ; (Higher) Category theory provides a bridge between these two aspects of HTT.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Classical case
Relativistic case
Quantum case

Venus with Homotopy Type theory : the case of Classical particles
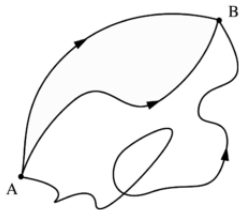

$$p_i, p_j : Id_T(A, B) \, ;$$

the "extensionality one dimension up" (Awodey), EX1 for short,
"no higher identity types", precisely
$$\vdash Id_{Id_T(A,B)}(p_i, p_j)$$

$$\overline{\vdash p_i = p_j : Id_T(A, B)}$$

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Classical case
Relativistic case
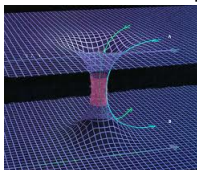Quantum case

Gravitational Lensing

In GR-spacetime loops and intersections of worldlines are allowed ; moreover EX1 does <u>not</u> hold, and so higher identity types do
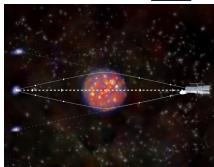
matter. One judges that the two "false images" are identical (i.e., are images of the same source) by comparing the corresponding two worldlines by their homotopies of type $Id_{Id_T(G,E)}(w_1, w_2)$ where G is the ("true") galaxy, E the Earth ($=$ the observer's "image" of G), w_1, w_2 the two worldlines. In fact these images are no more "false" than are MS and ES !

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Classical case
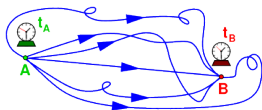**Relativistic case**
Quantum case

Wormhole Lensing

What happens if there is a spacetime hole (a "wormhole") at the

place of G ? . Guess : the identity of G is of
different character. Compare the case of Feynman's path integral.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Classical case
Relativistic case
Quantum case

Homotopy theory of path integrals (after Suzuki 2011)

Consider a system of n free spinless indistinguishable particles in space $\mathbb{R}^d$ and its configuration space X : of $x = f(x_1, ..x_n) \in X$ with $x_i \in \mathbb{R}^d$.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Classical case
Relativistic case
Quantum case

Theorem (Laidlaw&DeWitt 1971)

Let the configuration space X of a physical system be the topological space. Then the probability amplitude K for a given transition is, up to a phase factor, a linear combination
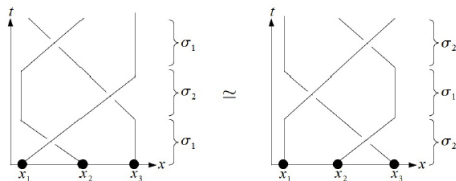
$$\sum_{\alpha \in \pi_1(X)} \chi(\alpha) K^\alpha$$

of partial probability amplitudes K^α obtained by integrating over paths in the same homotopy class in X, where the coefficients $\chi(\alpha)$ form a one-dimensional unitary representation of the fundamental group $\pi_1(X)$.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Classical case
Relativistic case
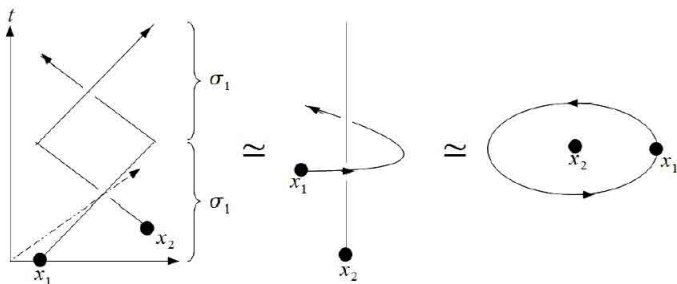**Quantum case**

fundamental group by permutations

$\sigma_i = s_{i,i+1}$
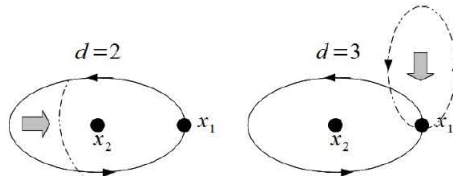
1. $\sigma_i \sigma_{i+1} \sigma_i = \sigma_{i+1} \sigma_i \sigma_{i+1}$
2. if $|i - j| > 1$ then $\sigma_i \sigma_j = \sigma_j \sigma_i$
3. $\sigma_i^2 = e$

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Classical case
Relativistic case
Quantum case

(1); (2) is obvious

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Classical case
Relativistic case
Quantum case

(3)

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Classical case
Relativistic case
Quantum case

(3)

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Classical case
Relativistic case
Quantum case

For $d \geq 3\pi_1(X) = S_n$; since S_n has two 1D unitary representations we have two cases :
$\chi^B = 1$ for all $\alpha \in S_n$ (bosons) ;

$$\chi^F = \begin{cases} +1, & \text{when } \alpha \text{ is even} \\ -1, & \text{when } \alpha \text{ is odd} \end{cases}$$

(fermions)
For $d = 2\pi_1(X) = B_n$ (anyons)

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Classical case
Relativistic case
Quantum case

Moral :

Quantum particle happen to be indistinguishable in different ways. Each particular way in which particles (forming a given quantum system) are indistinguishable is determined by the (topological properties of the) ω-groupoid of their identities. The real object is the system, not the particles !

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

Conclusion

IF Homotopy theory manages to account for the underlined physical spacetime at the fundamental level then it can provides a new clear sense of objecthood, objectivity and reality. It is worth trying.

Outline
General Issues : Objectivity, Objecthood and Reality
Structuralism and Structural Realism
Homotopy Type Theory (HTT)
HTT and Physics
Conclusion

THE END