

Abstract:

I compare the published Bourbaki's volume on set theory (the version of 1968) with an unpublished Bourbaki's draft that treats the same topic in an informal manner, which better reflects the contemporary practice of using sets in mathematics in general and in geometry in particular. Both texts use set-theoretic concepts for developing a theory of mathematical (set-based) structure rather than a set theory for its own sake. The draft begins with a philosophical introduction explaining the central notion of structure, which is followed by a description of various procedures allowing for obtaining further set-theoretic constructions from a given finite family of base sets. Finally the draft presents a general form of set-theoretic construction called mathematical structure and introduces some operations with structures such as induction, transport and product.

In the published volume the notion of set-theoretic construction reduces to a mere metaphor and gets replaced by a notion of syntactic construction, which is treated in great detail. Crucially, these syntactic constructions represent not informal set-theoretic constructions themselves but certain propositions referring to sets and relations between sets as well as certain relations between such propositions (like the relation of logical inference). This is, of course, a very general feature of the formal axiomatic method, which dates back to Hilbert's work. Although it is not specific for Bourbaki's approach comparing the two versions of Bourbaki's set theory gives an opportunity to evaluate the effect of formalization.

I argue that the Hilbert-style formalization of the informal set theory is not an improvement and hence must be abandoned and replaced by a different theoretical setting based on non-propositional postulates rather than usual axioms (I have in mind here the difference between postulates and axioms in Euclid). In addition to stressing the fact that such a constructive setting better fits the current mathematical practice I explain why the Hilbert-style formalization makes mathematics inapplicable in empirical sciences and show how to fix the problem.