Euclid and Radical Translation

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Content :

EUCLID *ELEMENTS* SERVED AS THE BIBLE OF MATHEMATICS FOR CENTURIES UNTIL LOBACHEVSKY AND BOLYAI DISCOVERED NON-EUCLIDEAN GEOMETRIES... ???

In fact Euclid's Letter was almost never respected. Unlike philosophers, theologians, literature critics and other *hommes de lettre* mathematicians usually did not care about older texts. They didn't try to preserve older writings in their original form. They typically tried to revise and complement older texts, sometime by re-writing these older texts wholly anew.

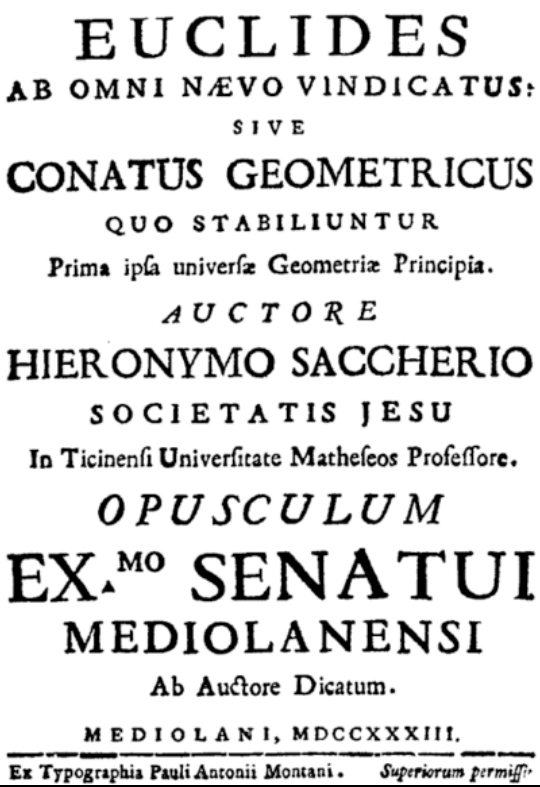
In particular, this happened when Euclid's *Elements* were translated from one natural language into another (in particular, when the *Elements* were translated into Arab and into Latin). However repeated translations of the *Elements* into new "mathematical languages" (in particular, translations of geometrical books of the *Elements* into the language of *algebra* during 16th and 17th centuries) from a mathematical point of view were even more important. In Section 3 I shall consider these different kinds of translation from a general viewpoint. Now I shall provide some examples.

Euclides Restitutus Denuo Limatus ab Omni Naevo Vindicatus

Giovanni Alfonso Borelli 1658 : *Euclides Restitutus Denuo Limatus* (Euclid Revived and Newly Polished)

Euclides Restitutus Denuo Limatus ab Omni Naevo Vindicatus

Girolamo Saccherri 1733, *Euclides ab Omni Naevo Vindicatus* (Euclid Cleared of Every Flaw)



EUCLIDES
AB OMNI NÆVO VINDICATUS:
SIVE
CONATUS GEOMETRICUS
QUO STABILIUNTUR
Prima ipſa univerſæ Geometriæ Principia.
AUCTORE
HIERONYMO SACCHERIO
SOCIETATIS JESU
In Ticinenſi Univerſitate Matheſeos Profeſſore.
OPUSCULUM
EX.MO SENATUI
MEDIOLANENSI
Ab Auctore Dicatum.
MEDIOLANI, MDCCXXXIII.
Ex Typographia Pauli Antonii Montani. *Superiorum permiſſu*

Tacquet and Dechales : Identity Problem

Comparing once popular *Elements of Geometry* published by A. Tacquet in 1654 and the edition of Euclid's *Elements* (the first eight books thereof) published by M. Dechales 6 years later in 1660 it is difficult to say why the later work has Euclid's name in its title while the former doesn't. The difference between the two titles seems to be unrelated to the content of the two books although it might point to different intentions of their authors. When Tacquet's book was republished in 1725 (long after the authors death) it actually got Euclid's name on its cover !

This example shows that the question of whether or not to put Euclid's name on a geometry textbook, in 17-18th centuries was seen as a secondary issue. A more important issue was the choice between teaching geometry after older versions of Euclid's *Elements* and producing new revised versions of this book.

Isaac Barrow

Curiously, Isaac Barrow tried to do both things at once. He believed, not without some reasons, that Theon of Alexandria who edited Euclid's *Elements* in the end of 4th century A.D. had made a serious damage to the original text, which influenced all the later editions of this book. Barrow also believed that by rewriting the *Elements* anew without looking at any available historical source but with a clear mathematical mind he could reproduce the lost Euclid's original. It is hardly surprising that Barrow's newly produced version of the *Elements* (1733), however valuable it could be, didn't reproduce Euclid's original text as we know it today....

Isaac Barrow

Perhaps Barrow in fact aimed at revival of Euclid's spirit rather than reproduction of Euclid's letter ?

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- Euclid's *urtext* has been (reasonably) fixed, translated into several new languages and extensively commented.
- *Elements* are alive as a mathematical genre but its connection to Euclid (and to the earlier tradition of translating Euclid) became very superficial.

The Urtext

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- I.L. HEIBERG and H. MENGE (an assistant) : Euclid's *Complete Works with a new Latin translation* : 1883-1916 ("Heiberg was Professor of Classical Philology at the University of Copenhagen from 1896 until 1924.)

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- I.L. HEIBERG and H. MENGE (an assistant) : Euclid's *Complete Works with a new Latin translation* : 1883-1916 ("Heiberg was Professor of Classical Philology at the University of Copenhagen from 1896 until 1924.)
- Modern commented translations based on Heiberg's Urtext :
  English translation : 1908, by Th. Heath
  Russian translation :
  D.D. Morduhai-Boltovskoi, 1950
  French translation :
  Vitrac, continued

New *Elements*

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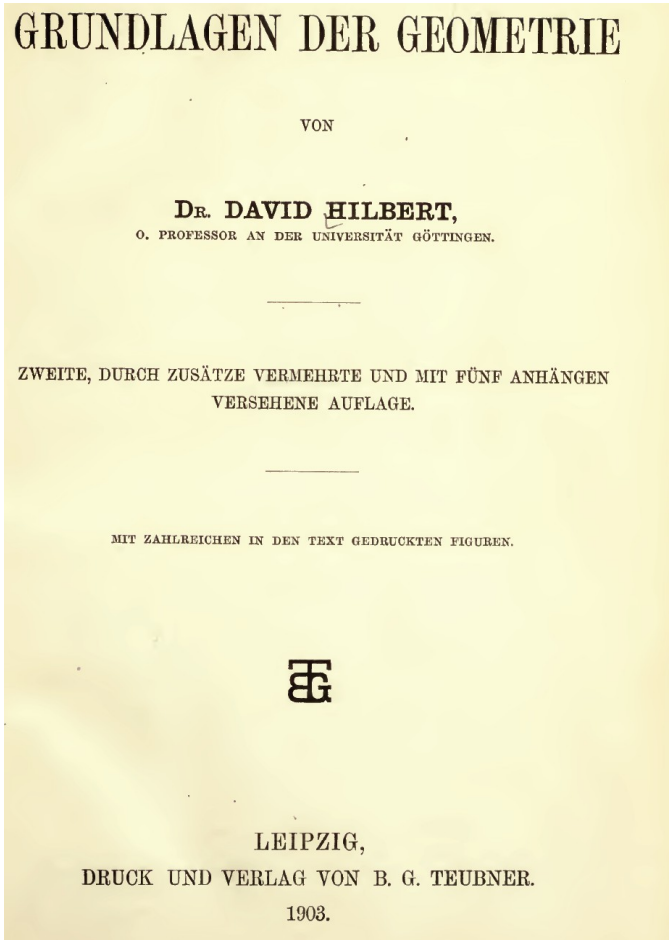
- N. Bourbaki, *Éléments de mathématique* (sic !), Paris 1939 - *circa* 2000

D. Hilbert : *Grundlagen der Geometrie*



GRUNDLAGEN DER GEOMETRIE

VON

DR. DAVID HILBERT,
O. PROFESSOR AN DER UNIVERSITÄT GÖTTINGEN.

ZWEITE, DURCH ZUSÄTZE VERMEHRTE UND MIT FÜNF ANHÄNGEN VERSEHENE AUFLAGE.

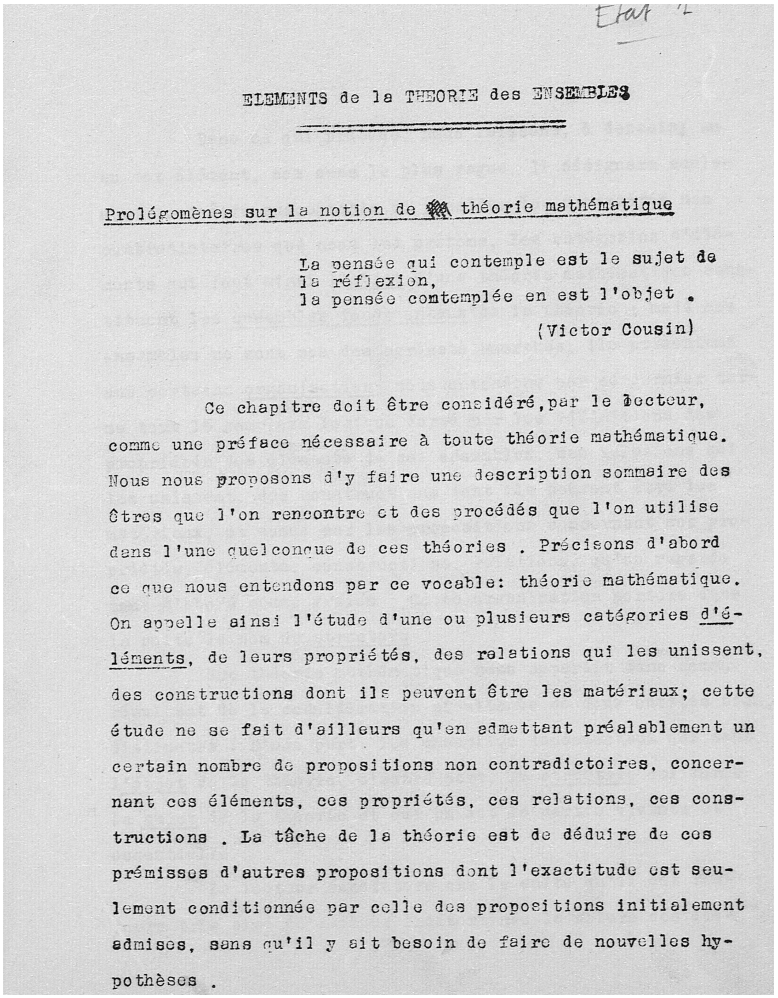
MIT ZAHLREICHEN IN DEN TEXT GEDRUCKTEN FIGUREN.

LEIPZIG,
DRUCK UND VERLAG VON B. G. TEUBNER.
1903.

Veblen and Whitehead

The starting point of any strictly logical treatment of geometry (and indeed of any branch of mathematics) must then be a set of undefined elements and relations, and a set of unproved propositions (=axioms) involving them, and from these all other propositions (theorems) are to be derived by the methods of formal logic. Moreover, since we assumed the point of view of fromal (i.e. symbolic) logic, the undefined elements are to be regarded as mere symbols devoid of content..

N. Bourbaki, *Éléments de mathématique* (sic !)



Etat 1

ELEMENTS de la THEORIE des ENSEMBLES

Prolégomènes sur la notion de théorie mathématique

La pensée qui contemple est le sujet de la réflexion,
la pensée contemplée en est l'objet .

(Victor Cousin)

Ce chapitre doit être considéré, par le lecteur, comme une préface nécessaire à toute théorie mathématique. Nous nous proposons d'y faire une description sommaire des êtres que l'on rencontre et des procédés que l'on utilise dans l'une quelconque de ces théories . Précisons d'abord ce que nous entendons par ce vocable: théorie mathématique. On appelle ainsi l'étude d'une ou plusieurs catégories d'éléments, de leurs propriétés, des relations qui les unissent, des constructions dont ils peuvent être les matériaux; cette étude ne se fait d'ailleurs qu'en admettant préalablement un certain nombre de propositions non contradictoires, concernant ces éléments, ces propriétés, ces relations, ces constructions . La tâche de la théorie est de déduire de ces prémisses d'autres propositions dont l'exactitude est seulement conditionnée par celle des propositions initialement admises, sans qu'il y ait besoin de faire de nouvelles hypothèses .

N. Bourbaki, *Éléments de mathématique* (sic !)

Un mot "élément" ....désignera seulement tout être susceptible de posséder les propriéités non contadictoires que nous lui prétons. Les catégories d'éléments qui font ainsi l'objet d'une théories mathématiques constituent les ensembles fondamentaux de la théorie ; ... ces ensembles .... présentent une certaine organisation : ... tout le complexe logique formé par les définions des propriétés des éléments des ces ensembles, des relations qui les unissent, des construction dont ils peuvent être les matériaux.... Cette organisation portera dans la suite le nom de structure.

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- Modern Euclid studies lack a genuine mathematical expertise. They provide no space for updating current *mathematical* perspectives on Euclid.
- (More importantly !) New *Elements* are written *as if* they were eternal and going to survive forever (even if nobody believes this really). The new *Elements* lack a historical expertise and a historical reflection. As a result mathematics and its philosophy lack strategic thinking. Without reflecting on the past on cannot reasonably anticipate the future.

Renewing Fondations

As a matter of fact foundations of mathematics undergo today a continuing revision, which is more intensive than ever. Bourbaki's *Éléments de mathématique* written in the second half of the 20th century are commonly seen by research mathematicians as hopelessly outdated ! The issue of conceptual and historical *continuity* of passing from older to newer *Elements* become vital.

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Renewing Fondations

Having said that I would like also to stress that the *radical character* of recent foundational renewals in mathematics is wholly justified. The possibility of a radical renewal of foundations must be also granted for the future. Such a renewal should no longer be seen as a catastrophe destroying the erroneous past and bringing mathematics into the Millennial Kingdom of unproblematic progressive development. The renewal of foundations should be rather recognized as a permanent process making part of the mathematical progress.

Problem

How to combine the continuity of foundational changes with the radical character of these changes ?

Solution

RADICAL TRANSLATION

Radical Translation

After Quine I call translation *radical* when it takes nothing for granted but rebuilds the translated content wholly anew.

Example : Three versions of the (statement of the) Pythagorean theorem

Version 1 : Euclid

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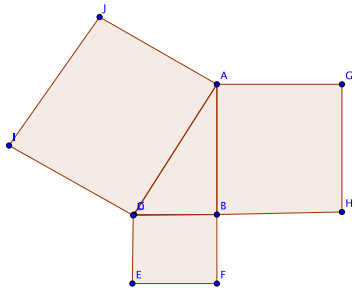
In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.

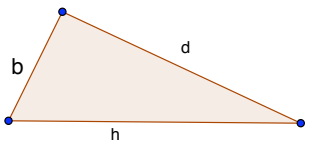
(*Elements*, Proposition 1.47)



Three versions of the (statement of the) Pythagorean theorem : Version 2 : Arnauld (1667)

The square of hypothenuse is equal to (the sum of) squares of the two (other) sides (of the given rectangular triangle) : $bb + dd = hh$.

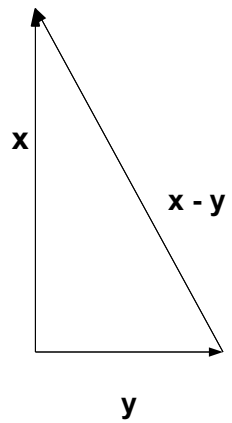
(*New Elements of Geometry, Proposition 14.26.4*)



Three versions of the (statement of the) Pythagorean theorem : Version 3 : Doneddu (1965)

Two non-zero vectors x and y are orthogonal if and only if $(y - x)^2 = y^2 + x^2$

(Donnedu, *Euclidean plane geometry*)



Versions 1-3 of the Pythagorean theorem differ in their **foundations**, i.e., differ *radically*. Still they translate the same theorem !

How radical translation helps to solve mathematical problems

Translation V1$\rightarrow$V2 , which translates traditional geometrical constructions into the language of algebra, allowed people in 19 th century to settle great open geometrical problems of Antiquity, including the problem of quadrature of circle. Using algebraic methods one shows that this and other similar problems are unsolvable by the required means (i.e. by compass and ruler). Such results could not be in principle obtained in the same foundational setting, in which these problems were first posed !

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- It brings a setting necessary for enquiring a new mathematical knowledge ;
- It allows an earlier enquired knowledge to survive in this new setting.

Both these conditions are crucial for mathematical progress !

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Answer : Versions 1-3 of the Pythagorean theorem share a common *history*, which is a history of *translation* of older contents into new conceptual frameworks. When such a new framework qualifies as *foundational* the corresponding translation qualifies as *radical*. Different version of the Pythagorean theorem do NOT share in common anything like an eternal *essence* or an invariant *structure*. They share nothing but mutual translations ! This assumption is sufficient for explaining how our mathematical knowledge grows and persists through time. It implies a historically-laden view on mathematics, which seems to me more appropriate and more responsible than the dominant essentialist and structuralist views.

Thank You !