Abstract:

Awodey and some others read Voevodsky's Univalence Axiom in the sense that it "forces equivalences to be identities" and on this basis argue that the Univalent Foundations support a structuralist view on mathematics developed (among others) by Hellman, Reznik, and Shapiro. I argue that given the homotopical interpretation of the Univalence Axiom, which has been suggested by Voevodsky, the structuralist reading of this axiom is rather misleading. First of all it must be taken into account that equivalences in this setting are specific maps rather than bare logical relations; although using such a map one can straightforwardly obtain an equivalence relation this relation does not always carry all the relevant information and for this reason cannot, generally, replace the map. This shows that in the Univalent setting (and in fact in any higher categorical setting) the structuralist reasoning "up to equivalence" is not, generally, allowed.

A more satisfactory reading of the Univalence Axiom (which I borrow from Shulman) is that it "forces identities to be equivalences" (where "equivalences" are maps rather than relations). More specifically this axiom forces the hierarchy of identity types, which arise in the intensional version of Martin Löf's type theory, to be "essentially homotopic" (the axiom prevents a collapse of this hierarchy in homotopical models), and so reveals a topological aspect of the type-theoretic identity.

Using Voevodsky's insights I show that this non-standard notion of identity makes good sense not only in the pure mathematics but also in the context of an empirical study. For this reason I go back to Frege's famous "Venus" example (which involves the Morning Star and the Evening Star as two different descriptions of the same planet Venus) and provide its alternative analysis in type-theoretic terms. I argue that Frege's distinction between sense and reference is coherent with Frege's own views on identity but not compatible with the new view on identity suggested by works of Martin-Löf and Voevodsky.

As a remedy I suggest a neo-Kantian approach according to which reasoning in the pure mathematics and in mathematized natural sciences involves "construction of concepts with intuitions". Unlike Kant I don't make an essential difference between the pure and the empirical intuition and take all intuitions to be ultimately empirical. On this epistemological basis I argue that Martin-Löf-Voevodsky's identity may better serve needs of empirical sciences than Frege's "god-given" identity.