Why Category Theory Is “Unreasonably Effective”?

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November 9, 2011

Historical Background

Optical-Mechanical Analogy

Group theory and Category theory in physics

Why mathematics is "unreasonably effective" in natural sciences? We need to take into consideration a historical background in order to understand the question.

Galileo

The Book of Nature is written in the Language of Mathematics

Galileo

The Book of Nature is written in the Language of Mathematics
3 aspects of the claim: Ancient, Medieval and Modern

Ancient aspect: Plato

- Ideas (Ideal Forms) - dialectical Reason
- Mathematical Forms - hypothetical Understanding
- Sensible (Physical) Forms - Belief, Opinion, "Plausible Myth" ("Mathematical Physics" of *Timaeos*)

Ancient aspect: contra Aristotle

Mathematics is an abstract aspect of Physics; Basic Forms are Logical. Metaphysics is a foundation of Physics.

Medieval aspect

Studying Nature as reading and interpreting a Book. The meaning is hidden but my be revealed through an attentive reading if one knows the language.

Modern aspect

One understands a phenomenon when one can (re)produce it. Mathematically designed experiments force Nature to reveal its secrets. Using mathematics a scientists asks questions and get answers but not just reads the Book.

Kant

- Mathematics is a science of Time (Arithmetic) and Space (Geometry);
- Time and Space are a priori forms of possible experience, so everything mathematically possible is physically possible;
- Fundamental physics is a theory of physically possible but not only of physically actual (cf. Newtonian physics);
- Pure maths is not sufficient for doing natural science; but additional principles don't limit mathematical possibilia.

Methodological Turn

Full recognition of autonomy of science: the aim of philosophy (w.r.t. science) is *critique* of science but not providing first principles for science. The critique purports to explain how science is possible. It *is* possible because it is actual.
A new feature: critical philosophy must keep track of new scientific developments!

A problem

Invention of non-Euclidean geometries. Multiple geometrical spaces. Which one represents “the” physical space?

A problem (continued

More generally: Only particular mathematical constructions model physical phenomena. Mathematically possible is not necessarily physically possible. Relativity and QM make this problem particularly pressing for Kantianism.

Duhem

Saving Phenomena (1908)

> *Physical theory is not a [causal] explanation. It is a system of mathematical propositions, deduced from a small number of principles, which aim to represent as simply, as completely, and as exactly as possible a set of experimental laws Concerning the very nature of things, or the realities hidden under the phenomena a theorytells us absolutely nothing, and does not claim to teach us anything.*

Cassirer

A clue to the Nature of Man: a Symbol

Mathematically-laden natural science provides us with a *symbolic* representation of the underlying reality.

Russell

As I have attempted to prove in The Principles of Mathematics, when we analyse mathematics we bring it all back to logic. It all comes back to logic in the strictest and most formal sense. In the present lectures, I shall try to set forth in a sort of outline, rather briefly and rather unsatisfactorily, a kind of logical doctrine which seems to me to result from the philosophy of mathematics - not exactly logically, but as what emerges as one reflects: a certain kind of logical doctrine, and on the basis of this a certain kind of metaphysic.

a worry

DID QM BRING US BACK INTO MIDDLE AGES ??!!

this is where Wigner's puzzle becomes really pressing

Józef Maria Bocheński

history of Modern philosophy and science: 3 centuries of intellectual decline
(an argument in Cold War ideological battles)

Schrödinger, 1926

Maybe our classical mechanics is the full analog of geometrical optics, and, as such, wrong, not in agreement with reality. It fails as soon as the radii of curvature and the dimensions of the trajectory are not large anymore compared to a certain wavelength, to which one can attribute a certain reality in q-space. In that case, one has to search for an "undulatory mechanics"and the obvious way to this end is the wavetheoretical extension of Hamiltons picture.

Weyl on Schrödinger and Hamilton

Schrödinger's researches took as their point of departure the Hamiltonian theory of mechanics, which was originally obtained by Hamilton himself from an analogy with geometrical optics. He argued that since we replace geometrical optics, with the aid of which interference and diffraction cannot be treated, by wave optics, it is reasonable to attempt the analogous transition in mechanics. The results amply justified the attempt.

I this “analogy” a miracle?

I shall try to explain it in neo-Kantian terms by revising Kant’s original approach.

Weyl's neo-Kantian (?) view

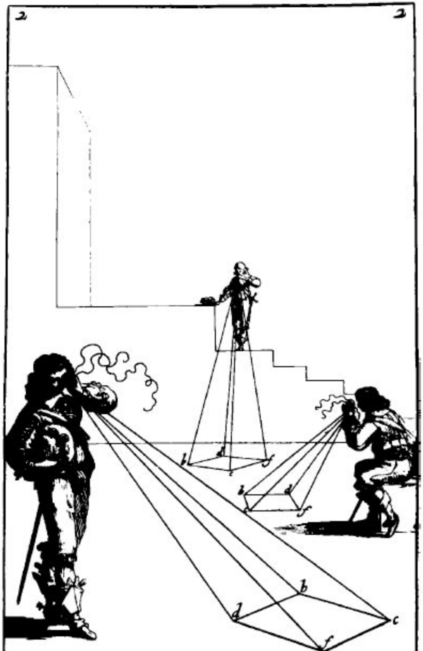
Natural science is of a constructive character. The concepts with which it deals are not qualities or attributes which can be obtained from the objective world by direct cognition. They can only be determined by an indirect methodology, by observing their reaction with other bodies, and their implicit definition is consequently conditioned by definite laws of nature governing reactions.

Euclid's *Optics*

Let it be assumed:

1. That rectilinear rays proceeding from the eye diverge indefinitely;
2. That the figure contained by a set of visual rays is a cone of which the vertex is at the eye and the base at the surface of the objects seen;
3. That those things are seen upon which visuals rays fall and those things are not seen upon which visual rays do not fall;
4. That things seen under a larger angle appear larger, those under a smaller angle appear smaller, and those under equal angles appear equal;
5. That things seen by higher visual rays appear higher, and things seen by lower visual rays appear lower;
6. That, similarly, things seen by rays further to the right appear further to the right, and things seen by rays further to the left appear further to the left;
7. That thing seen under more angles are seen more clearly.

Theory of Perspective



Theory of Perspective

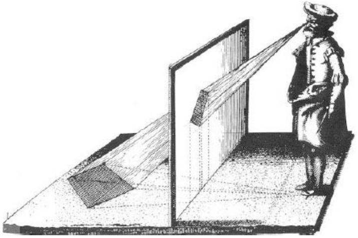


FIGURE VII.7. Marolois's illustration of the perspective model. Engraving by Hendrik Hondius, Marolois 1614, figure 7.

Convergence Theorem

Parallel lengths, seen from a distance, appear not to be equally distant from each other.

<u>Proof:</u>

Produce perpendiculars, then apply Post. 4. (Works only if the eye is placed between the parallels!)

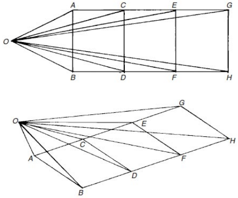


FIGURE 2. Euclid's convergence theorem. In the upper diagram the eye point and the parallel lines are situated in the same plane, and in the lower the eye point lies above the plane of the parallel lines.

Appearances and Objective Representations

are different things! Euclidean geometry works a relativistic scheme (like Minkowsky geometry in SR). Kant was too impressed by Newton's success and didn't pay enough attention (if any) to Leibniz' space relativism (perhaps because he rejected Leibniz' philosophy on more general grounds). An Euclidean straight line *is* a (segment of) light ray. This basic geometrical notion with all its properties and relations, in the last analysis, is empirical. However as long as it is not questioned in a given experiment it can be described as "a priori" w.r.t. the given experiment. Euclidean geometry is an empirically-grounded theory of vision applicable within certain limits. (A bold claim: Arithmetic has a similar character.)

A curvilinear perspective

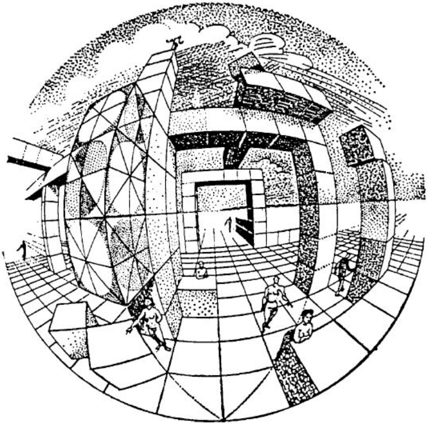


FIGURE III.26. Curvilinear perspective created by Albert Flocon and André Barre. Figure 65 in Flocon & Barre[S] 1987.

Relativity

SR and GR are alternative schemes of the same sort, which work better for large distances. They translate into the Euclidean (or pseudo-Euclidean) schemes through the assumption of smoothness (any smooth manifold is locally flat)

Optical-Mechanical Analogy

Since light rays at a closer look behave like waves straight lines (free trajectories) in mechanics at a closer look equally behave like waves. This is because mechanics uses the same geometrical

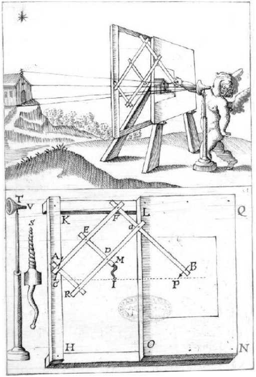


FIGURE VIII.12. Troili's idea of using a pantograph to draw in perspective. Contrary to Scheiner (figure VIII.10), Troili let the tracing pin also serve as the pen that draws. The *putto* moves the pen so that the eye point, a point in the object, and the other pin are collinear. For a large picture this process could require a longer arm than the *putto* actually has. Scheiner 1653, figure 2.

notion of straight line. No mystery!

Open Problem

Why this does not work throughout? A number of mathematically predicted physical entities are observed in experiments (particle physics). Why not *all* of them? What is going wrong? Probably because too many phenomena are *saved* by artificial mathematical means. Physics needs new *Mathematical Principles*.

Group theory

Groups of transformations are mainly used in 20th century physics for describing structures and magnitudes invariant under these transformations. This allows for *objective* representations compatible with but independent of certain classes of specific "appearances". In such context groups are "concrete" groups acting upon certain *spaces* (vector spaces, spaces of functions or geometrical spaces) and serve as an algebraic means of studying symmetries and detecting invariants in these spaces.

Transformation of coordinates in 1531 A.D.

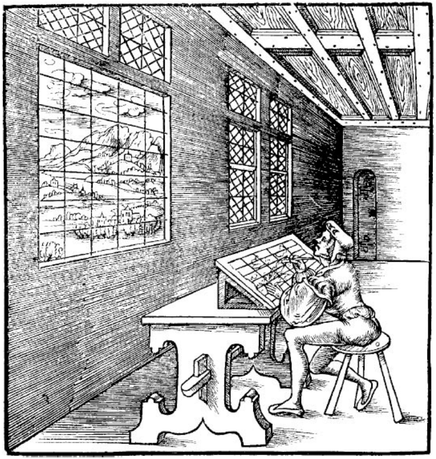


FIGURE V.61. Count Johann's interpretation of the Alberti idea of equipping the picture frame with a grid. As Lawrence Wright has remarked, the draughtsman in the picture is not reproducing what he sees through the window, but what we see (Wright[S] 1983, 314). Johann 1531, fol. Hii[v].

Category theory

A category can be seen as a double generalization of group: we get more objects (groupoid) and we allow for non-reversible transformations. A further generalization brings higher categories. This concept is already reach enough for capturing a lot of geometry without using external means (that cannot be done with a single group). This suggests a view on categories as general "relativistic schemes" in the above sense.

Mappings

A *mapping* is the most (?) general cognitive procedure playing a role in acquiring, updating and transmission of empirical data. Vision is a specific sort of mapping. It seems reasonable to assume (as a working hypothesis) that axioms of Category theory describe a protocol according to which humans (or at least scientists) collectively proceed empirical data (through space and time) and collectively manage their representations of these data. Any such protocol has not only descriptive but also normative (prescriptive) significance (like Euclidean Optics).

Spacetime

If this hypothesis turns to be tenable Category theory may serve also for building “the” physical space-time allowing for objective knowledge about the Nature. The idea that every physical object exists in space and time is not tenable if by space on understands Euclidean space and by time one understands Newtonian time. However the very idea of spacetime as an universal framework of objectivity in natural sciences is indispensable. The fact that QM has no proper notion of spacetime explains why QM only saves certain phenomena but does not give us their genuine explanation. (The incompatibility with GR is a part of the problem).

Conclusion

EPPUR SI MUOVE!