

Historical Reflexion, Projection of the Future and Philosophy of Mathematics

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Content :

Philosophy sub species aeternitatis and temporally-laden philosophy in the 19-21st c.

Are Mathematical Truths Eternal?

Re-Inventing Objectivity and Objecthood

Conclusion

Aeternitas adversus Janus



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- ▶ Cold War : Analytic Philosophy adversus Historical and Dialectical Materialism

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- ▶ Cold War : Analytic Philosophy adversus Historical and Dialectical Materialism
- ▶ Today : Analytic Philosophy adversus Postmodernism

The temporally-laden reasoning includes both reflecting upon the historical past and projecting / anticipating / predicting the future. The conflict between these two goals (i.e. between conservative and progressive ways of thinking) must not be confused with the conflict between the temporally laden and the atemporal modes of reasoning. The latter appears to me more profound.

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- ▶ Mathematics (along with mathematically-laden) sciences is a subject of fast progressive development. We know all what knew mathematicians of older generations and a LOT MORE.
- ▶ The progress of mathematics does NOT approximate an ideal state of “full” mathematical knowledge; there is no compelling reason for stipulating such an ideal state.

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- ▶ The renewal of foundations allows for solving problems, which cannot be solved with older foundations. Exs. : quadrature of circle (Cartesian geometry), solution of equations of degree ≥ 4 (Galois theory).
- ▶ The renewal of foundations allows for learning the earlier acquired knowledge in a new compactified form.

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- ▶ Mathematics interacts with Philosophy through its Foundations. (Idem for sciences.)
- ▶ The renewal of foundations involves a philosophical (speculative) work - or at least it always worked in this way in the past. I can see no reason why it should not work again in this way in the future. Speculative projects play a major role in the history of maths and sciences. Cf. Cantor's Set theory. Such projects may fail (ex. Cartesian Physics) but may also greatly succeed (Cartesian Geometry).

- ▶ The mainstream Analytic philosophy of maths is hardly appropriate for renewing foundations of today's maths. It eternalizes foundations of mathematics developed in early 1900ies (including the Set theory). It has little contact (if any) with the ongoing mainstream mathematical research. Moreover it has nothing to suggest for the future development of the discipline.

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- ▶ The metaphoric language of Postmodernism is not appropriate for discussing maths and sciences.
- ▶ We need a replacement for both! (Shall we look at Dialectical Materialism without ideological dogmas?)

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- ▶ Contra Kant : Objectivity reduces to logical validity . Objects are just things ; nothing but logical individuals.

Wigner's Puzzle (1960)

Why Mathematics is so unreasonably effective in Natural Sciences? Wigner's reply :

The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve. We should be grateful for it and hope that it will remain valid in future research and that it will extend, for better or for worse, to our pleasure..

My view : it cannot work for long without special efforts. Think of Newton, Riemann...

Dass unsere Begriffe sich auf Anschauungen zu beziehen haben, bedeutet daher, dass sie sich auf die mathematische Physik zu beziehen und in ihrer Gestaltung fruchtbar zu erweisen haben. Die logischen und mathematischen Begriffe sollen nicht länger die Werkzeuge bilden, mit denen wir eine metaphysische "Gedankenwelt" - aufbauen : sie haben ihre Funktion und ihre berechtigte Anwendung lediglich innerhalb der Erfahrungswissenschaft selbst.

E. Cassirer 1907, *Kant und die moderne Mathematik*

Mathematics is a part of physics. It is a part of physics where experiments are cheap. [...] In the middle of the 20th century there were attempts to separate mathematics from physics. The results turned to be catastrophic.

V.I. Arnold circa 2000

Wigner's Puzzle Solved

(trivially)

..

These views are STRONGLY incompatible with the Analytic Philosophy of Maths! To MAKE these views reasonable in the context of today's maths and mathematically-laden science (physics) is a challenging project. In particular it requires reconstituting the physical objectivity and the physical objecthood by using best available mathematical means. Solving the task in the context of QM and GR would contribute to Quantum Gravity problem.

My bet is on Category Theory and more specifically Voevodsky's Univalent Foundations..

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- ▶ My suggested project : reconstituting foundations of maths in a way that makes them (back) into foundations of physics.
- ▶ Try Category theory and Voevodsky's approach.

THE END