

2017 EUROPEAN SUMMER MEETING
OF THE ASSOCIATION FOR SYMBOLIC LOGIC
LOGIC COLLOQUIUM '17

Stockholm, Sweden

August 14–20, 2017

Logic Colloquium '17, the annual European Summer Meeting of the Association of Symbolic Logic, was hosted by the University of Stockholm. It formed part of a three week Logic in Stockholm 2017 event, which also featured the Third Nordic Logic Summer School (NLS 2017) and the The 26th Annual Conference of the European Association for Computer Science Logic (CSL 2017). The meeting took place from August 14th to August 20th, 2017, at the main campus of the university. Logic Colloquium 2017 was organized and hosted jointly by the Departments of Mathematics and Philosophy at Stockholm University and also supported by the KTH Royal Institute of Technology.

Major funding for the conference was provided by the Association for Symbolic Logic (ASL), the US National Science Foundation, Stockholm University, Prover Technology, Stockholm City Hall, and the G.S. Magnuson Foundation.

The success of the meeting was due largely to the excellent work of the Local Organizing Committee under the leadership of the Co-Chairs, Valentin Goranko and Erik Palmgren, from the University of Stockholm. The other members were Stefan Buijsman, Mads Dam, Jacopo Emmenegger, Dilian Gurov, Sven-Ove Hansson, Eric Johannesson, Vera Koponen, Johan Lindberg, Roussanka Loukanova, Peter LeFanu Lumsdaine, Anders Lundstedt, Karl Nygren, Peter Pagin, and Dag Westerståhl.

The Program Committee consisted of Rod Downey (University of Wellington), Mirna Džamonja (University of East Anglia, Chair), Ali Enayat (University of Gothenburg), Fernando Ferreira (University of Lisbon), Valentin Goranko (Stockholm University), Martin Hils (University of Münster), Sara Negri (University of Helsinki), Assaf Rinot (Bar-Ilan University), and Igor Walukiewicz (University of Bordeaux).

The conferences centered on the classical subjects of mathematical and philosophical logic, as well as on many connections between these subjects and computer science. A distinctive feature of the conference was a special LC2017-CSL2017 highlight session organized on the morning of August 20th, at which each of the two conferences invited two highlight speakers to present highlights of their subject intended for the broader community represented by the two conferences. The speakers at this session were as follows:

Verónica Becher (University of Buenos Aires), *Normal numbers, logic and automata*.

Phokion Kolaitis (University of California Santa Cruz and IBM Research-Almaden), *Schema mappings: structural properties and limits*.

Pierre Simon (University of California at Berkeley), *Recent directions in model theory*.

Wolfgang Thomas (RWTH Aachen), *Determinacy of infinite games: perspectives of the algorithmic approach*.

The program featured two 3-hour tutorials and eleven plenary lectures. There was a special session on Category Theory and Type theory in honor of Per Martin-Löf on his 75th birthday, and special sessions on Computability, History of Logic, Model Theory,

Philosophical Logic, Proof Theory, and Set Theory. There were 246 participants, and ASL travel grants were awarded to 29 students and recent Ph.D.'s.

The following tutorial courses were given:

Patricia Bouyer-Decitre (LSV, ENS Cachan), *On the verification of timed systems—and beyond*.

Mai Gehrke (University of Paris Diderot (Paris 7)), *On stone duality in logic and computer science*.

The following invited plenary lectures were presented:

David Asperó (University of East Anglia), *Generic absoluteness for Chang models*.

Alessandro Berarducci (University of Pisa), *Surreal differential calculus*.

Elisabeth Bouscaren (University of Paris Sud (Paris XI)), *A stroll through some important notions of model theory and their applications in geometry*.

Christina Brech (University of São Paulo), *Families on large index sets and applications to Banach spaces*.

Sakaé Fuchino (Kobe University), *Set-theoretic reflection of mathematical properties*.

Denis Hirschfeldt (University of Chicago), *Computability theory and asymptotic density*.

Wilfrid Hodges (British Academy), *Avicenna sets up a modal logic with a Kripke semantics*.

Emil Jeřábek (Czech Academy of Sciences), *Counting in weak theories*.

Per Martin-Löf (Stockholm University), *Assertion and request*.

Dag Prawitz (Stockholm University), *Gentzen's justification of inferences and the ecumenical systems*.

Sonja Smets (University of Amsterdam), *The Logical basis of a formal epistemology for social networks*.

The conference featured a rich social program, including a reception in the Stockholm City Hall, excursions, and a conference dinner. More information about the meeting can be found at the conference website: <https://www.math-stockholm.se/en/konferenser-och-akti/logic-in-stockholm-2/logic-colloquium-201/logic-colloquium-2017-august-14-20-1.717657>.

Abstracts of invited and contributed talks given in person or by title by members of the Association follow.

For the Program Committee
MIRNA DŽAMONJA

Abstracts of Invited Talks

- ▶ DAVID ASPERÓ, *Generic absoluteness for Chang models*.
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The main focus of the talk will be on extensions of Woodin's classical result that, in the presence of a proper class of Woodin cardinals, \mathcal{C}_ω^V and $\mathcal{C}_\omega^{V^P}$ are elementarily equivalent for every set—forcing P (where \mathcal{C}_κ denotes the κ -Chang model).

1. In the first part of the talk I will present joint work with Asaf Karagila in which we derive generic absoluteness for \mathcal{C}_ω over the base theory $\text{ZF} + \text{DC}$.

2. Matteo Viale has defined a strengthening MM^{+++} of Martin's Maximum which, in the presence of a proper class of sufficiently strong large cardinals, completely decides the theory of \mathcal{C}_{ω_1} modulo forcing in the class Γ of set—forcing notions preserving stationary subsets of ω_1 ; this means that if MM^{+++} holds, $P \in \Gamma$, and P forces MM^{+++} , then $\mathcal{C}_{\omega_1}^V$ and $\mathcal{C}_{\omega_1}^{V^P}$ are elementarily equivalent. MM^{+++} is the first example of a “category forcing axiom.”

In the second part of the talk I will present some recent joint work with Viale in which we extend his machinery to deal with other classes Γ of forcing notions, thereby proving

the existence of several mutually incompatible category forcing axioms, each one of which is complete for the theory of \mathcal{C}_{ω_1} , in the appropriate sense, modulo forcing in Γ .

- ▶ ALESSANDRO BERARDUCCI, *Surreal differential calculus*.

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I will report on joint surreal work with Vincenzo Mantova. We recall that Conway's ordered field of surreal numbers contains both the real numbers and the ordinal numbers. The surreal sum and product of two ordinals coincide with the Hessenberg sum and product, and Cantor's normal form of ordinals has a natural extension to the surreals. In [1] we proved that there is a meaningful way to take both the derivative and the integral (antiderivative) of a surreal number, hence in particular on an ordinal number. The derivative of the ordinal number ω is 1, the derivative of a real number is zero, and the derivative of the sum and product of two surreal numbers obeys the expected rules. More difficult is to understand what is the derivative of an ordinal power of ω , for instance the first epsilon-number, but this can be done in a way that reflects the formal properties of the derivation on a Hardy field (germs of nonoscillating real functions). In [2] we showed that many surreal numbers can indeed be interpreted as germs of differentiable functions on the surreals themselves, so that the derivative acquires the usual analytic meaning as a limit. It is still open whether we can interpret all the surreals as differentiable functions, possibly changing the definition of the derivative.

[1] A. BERARDUCCI and V. MANTOVA, *Surreal numbers, derivations and transseries*. *Journal of the European Mathematical Society*, to appear, [arXiv:1503.00315](https://arxiv.org/abs/1503.00315), p. 47.

[2] ———, *Transseries as germs of surreal functions*, [arXiv:1703.01995](https://arxiv.org/abs/1703.01995), p. 44.

- ▶ ELISABETH BOUSCAREN, *A stroll through some important notions of model theory and their applications in geometry*.

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In this talk, we will try to explain the use of some important model-theoretic notions, focusing on the model-theory of finite rank groups and on the notion of orthogonality. Their use in applications to algebraic geometry will be gently illustrated by some examples. This talk is partly inspired by a series of recent joint articles with Franck Benoist (Paris-Sud) and Anand Pillay (Notre-Dame), giving new model theoretic proofs of the original results of Ehud Hrushovski on the Mordell–Lang Conjecture for function fields (1994).

- ▶ DENIS HIRSCHFELDT, *Computability theory and asymptotic density*.

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The notion of generic-case complexity was introduced by Kapovich, Myasnikov, Schupp, and Shpilrain to study problems with high worst-case complexity that are nevertheless easy to solve in most instances. They also introduced the notion of generic computability, which captures the idea of having a partial algorithm that halts for almost all inputs, and correctly computes a decision problem whenever it halts. Jockusch and Schupp began the general computability-theoretic investigation of generic computability and also defined the notion of coarse computability, which captures the idea of having a total algorithm that might make mistakes but correctly decides the given problem for almost all inputs (although this notion had been studied earlier in Terwijn's dissertation). Two related notions, which allow for both failures to answer and mistakes, have been studied by Astor, Hirschfeldt, and Jockusch (although one of them had been considered in the 1970's by Meyer and by Lynch). All of

these notions lead to notions of reducibility and associated degree structures. I will discuss recent and ongoing work in the study of these reducibilities.

► PER MARTIN-LÖF, *Assertion and request*.

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Think of the content of an assertion as something that is to be done: let us call it a task. Peirce's explanation of the speech act of assertion as the assuming of responsibility then takes the form: by making an assertion, you assume the responsibility, or duty, of performing the task which constitutes the content of the assertion, when requested to do so by the hearer. Thus a duty on the part of the speaker appears as a right on the part of the hearer to request the speaker to perform his duty: this is an instance of what is called the correlativity of rights and duties, a fundamental principle of deontological ethics which can be traced back to Bentham. In logic, it appears as the correlativity of assertions and requests. Since nothing but assertions appear in the usual inference rules of logic, there arises the question of what the rules are that govern the correlative requests. In the case of constructive type theory, they turn out to be the rules which bring the meaning explanations for the various forms of assertion to formal expression. Thus, in analogy with Gentzen's dictum that the propositional operations, the connectives and the quantifiers, are defined by their introduction rules, we may say that the forms of assertion are defined by their associated request rules.

► DAG PRAWITZ, *Gentzen's justification of inferences and the ecumenical systems*.

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Some of the different proposals for how to make precise Gentzen's way of justifying the introduction and elimination rules of natural deduction are briefly surveyed. A crucial question is whether the justification is applicable only to inferences occurring in intuitionistic logic or can be extended also to inferences occurring in classical logic. I shall argue that it is extendible to classical inference rules but that for some logical constants the introduction rules must vary depending on whether the constant is read classically or intuitionistically—when the constant is read classically the rule must be weaker than when it is read intuitionistically.

Respecting this condition, it is possible to allow classical and intuitionistic logical constants in one and the same system, a system that we may call the ecumenical system. In this system the usual elimination rules for some logical constants do not hold when the constant is read classically. Modus ponens is an example—it is not valid generally when implication is read classically but remains valid when also all of the constants of the subformulas of the implication are read classically.

► SONJA SMETS, *The logical basis of a formal epistemology for social networks*.

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In this presentation I focus on a logical–philosophical study of group beliefs and collective “knowledge”, and their dynamics in communities of interconnected agents capable of reflection, communication, reasoning, argumentation, etc. In particular, the aim is to study belief formation and belief diffusion (doxastic influence) in social networks, and to characterize a group's “epistemic potential”. This covers cases in which a group's ability to track the truth is higher than that of each of its members (the “wisdom of the crowds”: distributed knowledge, epistemic democracy, and other beneficial forms of belief aggregation and deliberation), as well as situations in which the group's dynamics leads to informational distortions (the “madness of the crowds”: informational cascades, “groupthink”, the curse of the committee, pluralistic ignorance, group polarization, etc). I look at several logical formalisms that make explicit various factors affecting the epistemic potential of a group: the agents' degree of interconnectedness, their degree of mutual trust, their different epistemic interests (their “questions”), their different attitudes towards the available evidence and its sources, etc. In this presentation I refer to a number of recent articles (1,2,3,4,5,6), that

make use a variety of formal tools ranging from dynamic epistemic logics and probabilistic logics. I conclude with some philosophical reflections about the nature and meaning of group knowledge, as well as about the epistemic opportunities and dangers posed by informational interdependence.

[1] A. ACHIMESCU, A. BALTAG, and J. SACK, *The probabilistic logic of communication and change*. *The Journal of Logic and Computation*, (2016), no. 07.

[2] A. BALTAG, R. BODDY, and S. SMETS, *Group Knowledge in Interrogative Epistemology*, Outstanding Contributions to Logic Series, volume dedicated to J. Hintikka, Springer, forthcoming, 2017.

[3] A. BALTAG, Z. CHRISTOFF, J. U. HANSEN, and S. SMETS, *Logical models of informational cascades*, *Logic Across the University: Foundations and Applications, Proceedings of the Tsinghua Logic Conference, vol. 47* (J. van Benthem and F. Liu, editors), College Publications, Beijing, 2013, pp. 405–432.

[4] A. BALTAG, Z. CHRISTOFF, R. K. RENDSVIG, and S. SMETS, *Dynamic epistemic logics of diffusion and prediction in social networks (extended abstract)*, *Pre-Proceedings of LOFT, 2016* (G. Bonanno, W. van der Hoek, and A. Perea, editors), Maastricht, 2016.

[5] F. LIU, J. SELIGMAN, and P. GIRARD, *Logical dynamics of belief change in the community*. *Synthese*, vol. 191 (2014), no. 11, pp. 2403–2431.

[6] S. SMETS and F. R. VELASQUEZ-QUESADA, *How to make friends: A logical approach to social group creation*, *Proceedings of the Sixth International Workshop LORI, 2017* (A. Baltag and J. Seligman, editors), Maastricht, 2017.

Abstracts of Tutorials

- PATRICIA BOUYER, *On the verification of timed systems—and beyond*.
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Towards the development of more reliable computerized systems, expressive models are designed, targeting application to automatic verification (model-checking). As part of this effort, timed automata have been proposed in the early nineties [2] as a powerful and suitable model to reason about (the correctness of) real-time computerized systems. Timed automata extend finite-state automata with several clocks, which can be used to enforce timing constraints between various events in the system. They provide a convenient formalism and enjoy reasonably efficient algorithms (e.g., reachability can be decided using polynomial space), which explains the enormous interest that they provoked in the community of formal methods. Timed games [4] extend timed automata with a way of modelling systems interacting with external, uncontrollable components: some transitions of the automaton cannot be forced or prevented to happen. The reachability problem then asks whether there is a strategy (or controller) to reach a given state, whatever the (uncontrollable) environment does. This problem can also be decided, in exponential time.

Timed automata and games are not powerful enough for representing quantities like resources, prices, temperature, etc. The more general model of hybrid automata [14] allows for accurate modelling of such quantities using hybrid variables. The evolution of these variables follow differential equations, depending on the state of the system, and this unfortunately makes the reachability problem undecidable, even in the very restricted case of stopwatches (stopwatches are clocks that can be stopped, and hence, automata with stopwatches only are the simplest hybrid automata one can think of).

Weighted (or priced) timed automata [3,5] and games [1,9,16] have been proposed in the early 2000's as an intermediary model for modelling resource consumption or allocation problems in real-time systems (e.g., optimal scheduling [6]). As opposed to (linear) hybrid systems, an execution in a weighted timed model is simply one in the underlying timed model: the extra quantitative information is just an observer of the system, and it does not modify the possible behaviours of the system.

In this tutorial, we will present basic results concerning timed automata and games, and we will further investigate the models of weighted timed automata and games. We will present in particular the important optimal reachability problem: given a target location, we want to compute the optimal (i.e., smallest) cost for reaching a target location, and a corresponding strategy. We will survey the main results that have been obtained on that problem, from the primary results of [3,5,7,8,13,15,17] to the most recent developments [10,11]. We will also mention our new tool TiAMo, which can be downloaded at <https://git.lsv.fr/colange/tiamo>. We will finally show that weighted timed automata and games have applications beyond that of model-checking [12].

[1] R. ALUR, M. BERNADSKY, and P. MADHUSUDAN, *Optimal reachability in weighted timed games*, *Proceedings of the 31st International Colloquium on Automata, Languages and Programming (ICALP'04)* (J. Díaz, J. Karhumäki, A. Lepistö, and D. Sannella, editors), Lecture Notes in Computer Science, vol. 3142, Springer, Turku, Finland, 2004, pp. 122–133.

[2] R. ALUR and D. L. DILL, *A theory of timed automata*. *Theoretical Computer Science*, vol. 126 (1994), no. 2, pp. 183–235.

[3] R. ALUR, S. LA TORRE, and G. J. PAPPAS, *Optimal paths in weighted timed automata*, *Proceedings of the 4th International Workshop on Hybrid Systems: Computation and Control (HSCC'01)* (M. D. Di Benedetto and A. L. Sangiovanni-Vincentelli, editors), Lecture Notes in Computer Science, vol. 2034, Springer, Rome, Italy, 2001, pp. 49–62.

[4] E. ASARIN, O. MALER, A. PNUELI, and J. SIFAKIS, *Controller synthesis for timed automata*, *Proceedings of the IFAC Symposium on System Structure and Control*, Elsevier Science, 1998, pp. 469–474.

[5] G. BEHRMANN, A. FEHNER, T. HUNE, K. G. LARSEN, P. PETERSSON, J. ROMIJN, and F. VAANDRAGER, *Minimum-cost reachability for priced timed automata*, *Proceedings of the 4th International Workshop on Hybrid Systems: Computation and Control (HSCC'01)* (M. D. Di Benedetto and A. L. Sangiovanni-Vincentelli, editors), Lecture Notes in Computer Science, vol. 2034, Springer, Rome, Italy, 2001, pp. 147–161.

[6] G. BEHRMANN, K. G. LARSEN, and J. I. RASMUSSEN, *Optimal scheduling using priced timed automata*. *ACM Sigmetrics Performance Evaluation Review*, vol. 32 (2005), no. 4, pp. 34–40.

[7] P. BOUYER, T. BRIHAYE, V. BRUYÈRE, and J.-F. RASKIN, *On the optimal reachability problem*. *Formal Methods in System Design*, vol. 31 (2007), no. 2, pp. 135–175.

[8] P. BOUYER, T. BRIHAYE, and N. MARKEY, *Improved undecidability results on weighted timed automata*. *Information Processing Letters*, vol. 98 (2006), no. 5, pp. 188–194.

[9] P. BOUYER, F. CASSEZ, E. FLEURY, and K. G. LARSEN, *Optimal strategies in priced timed game automata*, *In Proceedings of the 24th Conference on Foundations of Software Technology and Theoretical Computer Science (FSTTCS'04)* (K. Lodaya and M. Mahajan, editors), Lecture Notes in Computer Science, vol. 3328, Springer, Chennai, India, 2001, pp. 148–160.

[10] P. BOUYER, M. COLANGE, and N. MARKEY, *Symbolic optimal reachability in weighted timed automata*, *Proceedings of the 28th International Conference on Computer Aided Verification (CAV'16)—Part I* (S. Chaudhuri and A. Farzan, editors), Lecture Notes in Computer Science, vol. 9779, Springer, Toronto, Canada, 2016, pp. 513–530.

[11] P. BOUYER, S. JAZIRI, and N. MARKEY, *On the value problem in weighted timed games*, *Proceedings of the 26th International Conference on Concurrency Theory (CONCUR'15)* (L. Aceto and D. de Frutos-Escrig, editors), LIPIcs, vol. 42, Leibniz-Zentrum für Informatik, Madrid, Spain, 2015, pp. 311–324.

[12] P. BOUYER, N. MARKEY, N. PERRIN, and P. SCHLEHUBER, *Timed automata abstraction of switched dynamical systems using control funnels*, *Proceedings of the 13th International Conference on Formal Modeling and Analysis of Timed Systems (FORMATS'15)* (S. Sankaranarayanan and E. Vicario, editors), Lecture Notes in Computer Science, vol. 9268, Springer, Madrid, Spain, 2015, pp. 60–75.

[13] T. BRIHAYE, V. BRUYÈRE, and J.-F. RASKIN, *On optimal timed strategies*, *Proceedings of the 3rd International Conference on Formal Modeling and Analysis of Timed Systems (FORMATS'05)* (P. Pettersson and W. Yi, editors), Lecture Notes in Computer Science, vol. 3821, Springer, Uppsala, Sweden, 2005, pp. 49–64.

[14] T. A. HENZINGER, P. W. KOPKE, A. PURI, and P. VARAIYA, *What's decidable about hybrid automata?* *Journal of Computer and System Sciences*, vol. 57 (1998), no. 1, pp. 94–124.

[15] K. G. LARSEN, G. BEHRMANN, E. BRINKSMA, A. FEHNER, T. HUNE, P. PETTERSSON, and J. ROMIJN, *As cheap as possible: Efficient cost-optimal reachability for priced timed automata*, *Proceedings of the 13th International Conference on Computer Aided Verification (CAV'01)* (G. Berry, H. Comon, and A. Finkel, editors), Lecture Notes in Computer Science, vol. 2102, Springer, Paris, France, 2001, pp. 493–505.

[16] S. LA TORRE, S. MUKHOPADHYAY, and A. MURANO, *Optimal-reachability and control for acyclic weighted timed automata*, *Proceedings of the 2nd IFIP International Conference on Theoretical Computer Science (TCS 2002)* (R. A. Baeza-Yates, U. Montanari, and N. Santoro, editors), IFIP Conference Proceedings, vol. 223, Kluwer, Montréal, Canada, 2007, pp. 485–497.

[17] J. I. RASMUSSEN, K. G. LARSEN, and K. SUBRAMANI, *On using priced timed automata to achieve optimal scheduling*, *Formal Methods in System Design*, vol. 29 (2006), no. 1, pp. 97–114.

- MAI GEHRKE, *On Stone duality in logic and computer science*.

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Stone duality shows that the category of bounded distributive lattices* is dually equivalent to a certain category of topological spaces. This duality underlies many connections between algebra and geometry or topology in mathematics. In logic, it is central to correspondences between syntax and semantics. More recently, it has been realised that Stone duality plays a central role in more algorithmic questions such as the decidability of certain classes of languages in automata theory.

In this three part tutorial, the first lecture will provide an introduction to Stone duality with an overview of its different versions and their applications. The second lecture will focus on applications in semantics and will introduce duals of certain functors, such as the Vietoris functor, which corresponds to classical quantification. The third lecture will concentrate on applications in the theory of formal languages and, in particular, on the notion of ultrafilter equations as a tool for separating complexity classes. The articles [1] and [2], which are geared to computer scientists rather than logicians, provide a survey on Stone duality and a gentle introduction to the applications in formal language theory, respectively.

*The algebras corresponding to the “and”, “or”, “true”, and “false” fragment of classical propositional logic.

[1] M. GEHRKE, *Duality in computer science*, *Logic in Computer Science* (M. Grohe, E. Koskinen, and N. Shankar, editors), ACM, Columbia University, New York City, NY, USA, 2016, pp. 12–26.

[2] M. GEHRKE and A. KREBS, *Stone duality for languages and complexity*, *Association for Computing Machinery Special Interest Group on Logic (ACM SigLog) and Computation News*, vol. 4, no. 2, pp. 23–53.

Abstracts of the Joint Session of CSL2017 and LC2017

- VERÓNICA BECHER, *Normal numbers, logic and automata*.

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Flip a coin a large number of times and roughly half of the flips will come up heads and half will come up tails. *Normality* makes analogous assertions about the digits in the expansion of a real number. Precisely, let b be an integer greater than or equal to 2. A real number is *normal*

to base b if each of the digits $0, 1, \dots, b-1$ occurs in its expansion with the same asymptotic frequency $1/b$, each of the blocks of two digits occurs with frequency $1/b^2$, each of the blocks of three digits occurs with frequency $1/b^3$, and so on, for every block length. A number is *absolutely normal* if it is normal to every base. Émile Borel [10] defined normality more than one hundred years ago to formalize a basic feature of randomness for real numbers. Many of his questions are still open, such as whether any of π , e , or $\sqrt{2}$ is normal in some base, as well as his conjecture that the irrational algebraic numbers are absolutely normal [11].

In this talk I will highlight some theorems on normal numbers proved with tools from *computability theory*, *automata theory*, and *descriptive set theory* and I will point out some open questions.

From computability theory: Alan Turing was the first. He gave an effective version of Borel's theorem showing that almost all numbers (in the sense of Lebesgue measure) are absolutely normal. Based on this construction Turing gave the first algorithm to compute an absolutely normal number [4,23]. A current research line aims to effectivize results in number theory and give algorithms to compute absolutely normal numbers that have also some other mathematical properties [6,12,18,22]. It is an open question whether there exists a fast algorithm that computes an absolutely normal number with fast speed of convergence to normality [7,17,20].

From automata theory: To regard normality from the point of view of finite automata we must consider expansions in a single base. So, we fix a base and we speak of normal sequences. V. Agafonov [1] established that a sequence is normal exactly when any subsequence selected by a finite automata is normal (see also [19]). Besides, normal sequences admit characterizations analogous to those for Martin-Löf random sequences [15], but using finite automata instead of Turing machines. C.P. Schnorr and H. Stimm [21] established that a sequence is normal exactly when no martingale defined by a finite automaton can make infinite profit. Dai, Lathrop, Lutz, and Mayordomo [14] obtained that a sequence is normal exactly when it can not be compressed by one-to-one finite automata with input and output (finite transducer). This characterization holds for various enrichments of finite automata [3,13]. An open question is whether normal sequences can be compressed by deterministic push-down automata.

From descriptive set theory. The set of real numbers normal to base 2, as a subset of the set of all real numbers, is complete at the third effective level of the Borel Hierarchy [16]. So is the set of absolutely normal numbers [5]. This gives another proof that the set of absolutely normal real numbers is different from the set of Martin-Löf random numbers, since this is just complete at the second level of the Borel Hierarchy. The set of real numbers that are normal to some base is complete at the fourth level of the Borel Hierarchy, both effective and noneffective [8]. This implies that there is no logical dependence between normality to different bases, other than multiplicative dependence. Recently Airey, Mance, and Jackson [2] proved that the subset of real numbers that preserve normality to a given base under addition is complete at the third level of the Borel Hierarchy.

[1] V. N. AGAFONOV, *Normal sequences and finite automata*. *Soviet Mathematics Doklady*, vol. 9 (1968), pp. 324–325.

[2] D. AIREY, B. MANCE, and S. JACKSON, *Some complexity results on sets related to normal numbers*, 2016, [arXiv:1609.08702](https://arxiv.org/abs/1609.08702).

[3] V. BECHER, O. CARTON, and P. A. HEIBER, *Normality and automata*. *Journal of Computer and System Sciences*, vol. 81 (2015), no. 8, pp. 1592–1613.

[4] V. BECHER, S. FIGUEIRA, and R. PICCHI, *Turing's unpublished algorithm for normal numbers*. *Theoretical Computer Science*, vol. 377 (2007), pp. 126–138.

[5] V. BECHER, P. A. HEIBER, and T. SLAMAN, *Normal numbers and the Borel hierarchy*. *Fundamenta Mathematicae*, vol. 226 (2014), pp. 63–77.

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**Abstracts of the Special Session on
Category theory and type theory in honor of
Per Martin-Löf on his 75th birthday**

- ▶ THIERRY COQUAND, *Univalent Type Theory*.
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The notion of sheaf models, which can be traced back to the studies of Beth and Kripke, is an important tool for metamathematical analysis of higher order logic. The problem for the generalization of such interpretations to dependent types is for the interpretation of universes, and this is precisely for this reason, in another context, that the notion of stacks was introduced. I will present a possible generalization for models of univalent type theory, i.e., dependent type theory where the univalence axiom holds, and where we have an operation of propositional truncation. This can be used in particular to show that such a type theory is compatible with continuity principles, and that it does not prove the principle of countable choice.

- ▶ RICHARD GARNER, *Polynomials and theories*.
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A basic tenet of Martin-Löf type theory [2] is that types are inductively generated by their elements. This idea finds clearest expression in the *W*-types and the more general *tree types* [3]; categorically, these admit characterisation as initial algebras for certain *polynomial endofunctors* of the category of types over a given context. The calculus of polynomial endofunctors is interesting in its own right, with application in combinatorics, algebraic topology, and computer science; a key organising principle is that polynomial functors between the slices of a locally cartesian closed category form into a bicategory whose composition is given by substitution of multivariate polynomials [1].

The notion of bicategory also crops up in a very deep observation of Walters [4]: namely, that the theory of categories enriched over a monoidal category admits generalisation to a theory of categories enriched over a bicategory. This is closely bound up with what is sometimes called indexed or variable category theory, that is, category theory relative to a base category that acts as a surrogate for the category of sets. In this talk, we consider the natural question: what are categories enriched over the bicategory of polynomials? The answer turns out to be quite interesting: they encode notions of Lawvere theory and **PROP** appropriate to the indexed setting.

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[4] R. F. C. WALTERS, *Sheaves and Cauchy-complete categories*, *Cahiers de Topologie et Géométrie Différentielle Catégoriques*, vol. 22 (1981), pp. 283–286.

- ▶ ANDRÉ JOYAL, *On some categorical aspects of homotopy type theory*.
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Few things can better illustrate the unity of mathematics than the homotopy interpretation of Martin-Löf type theory (Awodey-Warren, Voevodsky) and the discovery of the univalence axiom (Voevodsky). *Homotopy Type Theory* is the new field of mathematics springing from these discoveries. There are good evidences that **Hott** can contribute effectively to homotopy theory and to higher topos theory: nontrivial homotopy groups of spheres were computed by Brunerie and a new proof of a fundamental result of homotopy theory (the Blakers–Massey theorem) was discovered (and verified in *Agda*) by Favonia, Finster, Licata, and Lumsdane. The theorem was generalised by Anel, Biedermann, Finster, and the author, and applied to Goodwillie's calculus [arxiv/1703.09050/1703.09632]. The [ABFJ] articles are written in *mathematical creole*, a blend of homotopy theory, infty-category theory, category theory and type theory, but a formal verification in *Agda* by Finster and Licata is on the way. It is clear that category theory serves as an intermediate between type theory and homotopy theory [ALV/arxiv/1705.04310][CCHM/arxiv/1611.02108][LS/arxiv/1705.07088]. The basic aspects of the theory of infty-categories were recently formalised in **Hott** by Riehl and Shulman. The syntactic category of type theory happens to be a *path category* in the sense of Van den Berg. The notion of *tribe*, introduced independently by Shulman and the author, is somewhat simpler, but not every path category is a tribe. However, every fibration category is equivalent (in the sense of Dwyer–Kan) to a tribe by a construction of Cisinski and by the work of Szumilo and of Kapulkin. I will sketch the homotopy theory of tribes and of *simplicial tribes*.

- ▶ VLADIMIR VOEVODSKY, *Models, interpretations and the initiality conjectures*.
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Work on proving consistency of the intensional Martin-Löf type theory with a sequence of univalent universes (MLTT+UA) led to the understanding that in type theory we do not know how to construct an interpretation of syntax from a model of inference rules. That is, we now have the concept of a model of inference rules and the concept of an interpretation of the syntax and a conjecture that implies that the former always defines the latter. This conjecture, stated as the statement that the term model is an initial object in the category of all models of a given kind, is called the Initiality Conjecture. In my talk I will outline the various parts of this new vision of the theory of syntax and semantics of dependent type theories.

Abstracts of invited talks in the Special Session on Computability

- ▶ VERÓNICA BECHER, JAN REIMANN, AND THEODORE A. SLAMAN, *Irrationality exponents and effective Hausdorff dimension.*

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We generalize the classical theorem by Jarník and Besicovitch on the irrationality exponents of real numbers and Hausdorff dimension. Let a be any real number greater than or equal to 2 and let b be any non-negative real less than or equal to $2/a$. We show that there is a Cantor-like set with Hausdorff dimension equal to b such that, with respect to its uniform measure, almost all real numbers have irrationality exponent equal to a . We give an analogous result relating the irrationality exponent and the effective Hausdorff dimension of individual real numbers. We prove that there is a Cantor-like set such that, with respect to its uniform measure, almost all elements in the set have effective Hausdorff dimension equal to b and irrationality exponent equal to a . In each case, we obtain the desired set as a distinguished path in a tree of Cantor sets.

- ▶ KLAUS MEER, *Generalized finite automata over the real numbers.*

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Gandhi, Khousainov, and Liu introduced and studied a generalized model of finite automata able to work over arbitrary structures. The model mimics finite automata over finite structures but has an additional ability to perform in a restricted way operations attached to the structure under consideration. As one relevant area of investigations for this model Gandhi et al. identified studying the new automata over uncountable structures such as the real and complex numbers.

In the talk we pick up this suggestion and consider their automata model as a finite automata variant in the BSS model of real number computation. We study structural properties as well as (un-)decidability results for several questions inspired by the classical finite automata model.

This is joint work with A. Naif.

- ▶ ARNO PAULY, *Applications of computability theory in topology.*

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The notion of the *point degree spectrum* links σ -homeomorphism types of second-countable spaces to substructures of the enumeration degrees. Using the framework of computable analysis, we can extend Turing reducibility from Cantor space to represented spaces:

DEFINITION 1. We say that $x \in X$ is reducible to $y \in Y$ (denoted $x^X \leq_T y^Y$), iff there exists a partial computable function $f: \subseteq Y \rightarrow X$ with $f(y) = x$.

If X is second-countable, then the degrees of its points form a substructure of the enumeration degrees, and this substructure (up to products with \mathbb{N} and relativization) characterizes the σ -homeomorphism type of X :

DEFINITION 2. We say that X and Y are σ -homeomorphic, if there are partitions $X = \bigcup_{i \in \mathbb{N}} X_i$ and $Y = \bigcup_{i \in \mathbb{N}} Y_i$ such that X_i and Y_i are homeomorphic for all $i \in \mathbb{N}$.

Motivated by a connection to Banach space theory, Jayne had raised the question how many σ -homeomorphism types of uncountable Polish spaces there are. Arguments from dimension theory establish that Cantor space 2^ω and the Hilbert cube $[0, 1]^\omega$ are not σ -homeomorphic, and all other well-known uncountable Polish spaces are σ -homeomorphic to one of these. Whether there are more σ -homeomorphism types has been illusive for a long time. Using recursion-theoretic arguments and the point degree spectrum connection, we can establish:

THEOREM 3. *There are uncountably many σ -homeomorphism types of uncountable Polish spaces.*

The framework of point degree spectra enables further applications of computability theory to topology, and also applications in the reverse direction.

This is joint work with Takayuki Kihara. A preprint is available as [2]. A precursor of this approach is found in [3] by Joseph S. Miller.

[1] J. E. JAYNE, *The space of class a Baire functions*. *Bulletin of the American Mathematical Society*, vol. 80 (1974), pp. 1151–1156.

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[3] J. S. MILLER, *Degrees of unsolvability of continuous functions*. *The Journal of Symbolic Logic*, vol. 69 (2004), no. 2, pp. 555–584.

► MARIYA I. SOSKOVA, *Characterizing the continuous degrees*.

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The continuous degrees were introduced by J. Miller [3] as a way to capture the effective content of elements of computable metric spaces. They properly extend the Turing degrees and naturally embed into the enumeration degrees. Although nontotal (i.e., non-Turing) continuous degrees exist, they are difficult to construct: every proof we know invokes a nontrivial topological theorem.

In 2014 Cai, Lempp, Miller, and Soskova discovered an unusual structural property of the continuous degrees: if we join a continuous degree with a total degree that is not below it then the result is always a total degree. We call degrees with this curious property *almost total*. We prove that the almost total degrees coincide with the continuous degrees. Since the total degrees are definable in the partial order of the enumeration degrees [1], this implies that the continuous degrees are also definable. Applying earlier work of J. Miller [3] on the continuous degrees, this shows that the relation “PA above” on the total degrees is definable in the enumeration degrees.

In order to prove that every almost total degree is continuous, we pass through another characterization of the continuous degrees that slightly simplifies one of Kihara and Pauly [2]. Like them, we identify our almost total degree as the degree of a point in a computably regular space with a computable dense sequence, and then we apply the effective version of Urysohn’s metrization theorem (Schröder [4]) to reveal our space as a computable metric space.

This is joint work with Uri Andrews, Greg Igusa, and Joseph Miller.

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[2] T. KIHARA and A. PAULY, *Point degree spectra of represented spaces*, submitted.

[3] J. S. MILLER, *Degrees of unsolvability of continuous functions*. *The Journal of Symbolic Logic*, vol. 69 (2004), no. 2, pp. 555–584.

[4] M. SCHRÖDER, *Effective metrization of regular spaces*, *Computability and Complexity in Analysis*, vol. 235 (J. Wiedermann, K.-I. Ko, A. Nerode, M. B. Pour-El, and K. Weihrauch, editors), Informatik Berichte, 1998, pp. 63–80.

- KEITA YOKOYAMA, *On the first-order strength of Ramsey's theorem in reverse mathematics*. School of Information Science, Japan Advanced Institute of Science and Technology, 1-1 Asahidai, Nomi, Ishikawa 923-1292, Japan.

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Deciding the first-order part of Ramsey's theorem for pairs is one of the important problems in reverse mathematics. In this talk, I will overview the recent developments of this study. To decide the first-order part, a standard approach is proving Π_1^1 -conservation over some induction or bounding axiom by showing ω -extension property. In [1], Cholak/Jockusch/Slaman showed $WKL_0 + RT_2^2 + I\Sigma_2^0$ is a Π_1^1 -conservative extension of $I\Sigma_2^0$ and $WKL_0 + RT^2 + I\Sigma_3^0$ is a Π_1^1 -conservative extension of $I\Sigma_3^0$, and they posed whether they are Π_1^1 -conservative over $B\Sigma_2^0$ and $B\Sigma_3^0$, respectively. For RT^2 , the answer is yes, which is shown by sharpening the argument in [1] (see [4]). For RT_2^2 , the question is more difficult. Chong/Slaman/Yang [2] showed that a slightly weaker principle CAC is Π_1^1 -conservative over $B\Sigma_2^0$ by using ω -extension property. On the other hand, it is now known that $WKL_0 + RT_2^2$ is actually Π_3^0 -conservative over $B\Sigma_2^0$ by using the indicator argument [3]. In fact, one can characterize the first-order part of $WKL_0 + RT_2^2$ by generalizing the indicator argument used in [3].

[1] P. A. CHOLAK, C. G. JOCKUSCH, and T. A. SLAMAN, *On the strength of Ramsey's theorem for pairs*. *The Journal of Symbolic Logic*, vol. 66 (2001), no. 1, pp. 1–55.

[2] C.-T. CHONG, T. A. SLAMAN, and Y. YANG, Π_1^1 -conservation of combinatorial principles weaker than Ramsey's Theorem for pairs. *Advances in Mathematics*, vol. 230 (2012), pp. 1060–1077.

[3] L. PATEY and K. YOKOYAMA, *The proof-theoretic strength of Ramsey's theorem for pairs and two colors*, submitted.

[4] T. A. SLAMAN and K. YOKOYAMA, *The strength of Ramsey's theorem for pairs and arbitrary many colors*, draft.

Abstracts of invited talks in the Special Session on History of Logic

- PETER ØHRSTRØM, *The rise of temporal logic*. Department of Communication and Psychology, Aalborg University, Rendsburggade 14, 9000 Aalborg, Denmark.

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A. N. Prior (1914–1969) was the founder of modern temporal logic. In the 1950s and 1960s he showed that tense-logic can be used in order to keep track of the past and of the future possibilities in a way which makes it possible to reason systematically on temporal matters. From the early 1930s Prior had been an active member of the Presbyterian community in New Zealand. He became a specialist in the debates regarding the logical tension between the doctrine of divine foreknowledge and the doctrine of human freedom. He demonstrated how this logical problem can be formalized and analysed in terms of his tense-logic. He found great inspiration in the studies of Aristotle, Diodorus, Thomas Aquinas, William of Ockham, C.S. Peirce, Jan Łukasiewicz, Saul Kripke, and several others. He argued that in the discussion concerning divine foreknowledge and human freedom there are just a few

reasonable positions. In general Prior demonstrated that temporal logic can be used to analyze the notion of time itself as well as fundamental existential problems, such as the problem of determinism versus freedom of choice.

- ▶ JAN VON PLATO, *Gödel's reading of Gentzen's first consistency proof for arithmetic*.
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A shorthand notebook of Gödel's from late 1935 shows that he read Gentzen's original, unpublished consistency proof for arithmetic. By 1941, many such notebooks were filled with various formulations of the result, one with explicit use of choice sequences, and a generalization based on an induction principle for functionals of finite type over Baire space. Gödel's main aim was to extend Gentzen's result into a consistency proof for analysis. In the lecture, an overview of these so far unknown results about consistency proofs for arithmetic will be presented.

Abstracts of invited talks in the Special Session on Model Theory

- ▶ ZANIAR GHADERNEZHAD, *Nonamenability of automorphism groups of generic structures*.
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A group G is amenable if every G -flow has an invariant Borel probability measure. Well-known examples of amenable groups are finite groups, solvable groups, and locally compact abelian groups. Kechris, Pestov, and Todorćević established a very general correspondence which equates a stronger form of amenability, called extreme amenability, of the automorphism group of an ordered Fraïssé structure with the Ramsey property of its finite substructures. In the same spirit Moore showed a correspondence between the automorphism groups of countable structure and a structural Ramsey property, which englobes Følner's existing treatment. In this talk we will consider automorphism groups of certain Hrushovski's generic structures. We will show that they are not amenable by exhibiting a combinatorial/geometrical criterion which forbids amenability.

- ▶ FRANZISKA JAHNKE, *NIP fields and henselianity*.
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NIP is a model-theoretic dividing line which was introduced in Shelah's classification theory programme. As with any combinatorial property, it is a natural question to ask whether it corresponds to some well-known algebraic notion when one considers the class of NIP fields. An open conjecture by Shelah states that every NIP field is either real closed or separably closed or 'like the p -adic numbers'. In this talk, I will explain the conjecture and discuss some recent developments around it.

- ▶ VINCENZO MANTOVA, *Transseries as surreal analytic functions*.
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Transseries such as LE series arise when dealing with certain asymptotic expansions of real analytic function. Most transseries, though, are not convergent and cannot represent real analytic functions, if only just for cardinality reasons.

On the other hand, we can show that LE series do induce germs of nonstandard analytic functions on the *surreal line*. More generally, call "omega-series" the surreal numbers that can be generated from the real numbers and the ordinal omega by closing under exponentiation, logarithm, and infinite sum. Then omega-series form a proper class of transseries including LE series.

It turns out that all omega-series induce (germs of) surreal analytic functions. Moreover, they can be composed and differentiated in a way that is consistent with their interpretation as functions, extending the already known composition and derivation of LE series, and the derivation coincides with the simplest one on surreal numbers.

This is joint work with A. Berarducci.

- ▶ IVAN TOMAŠIĆ, *Enriching our view of model theory of fields with operators*.
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A naïve approach to developing the methods of homological algebra for difference and differential fields, rings and modules quickly encounters numerous obstacles, such as the failure of the hom-tensor duality.

We will describe a categorical framework that overcomes these difficulties, allowing us to transfer most classical techniques over to the difference/differential context.

We will conclude by applying these techniques to study the cohomology of different algebraic groups and discuss potential model-theoretic consequences.

Abstracts of invited talks in the Special Session on Philosophical Logic

- ▶ ALEXANDER C. BLOCK, LUCA INCURVATI, AND BENEDIKT LÖWE, *Maddian interpretations and their derived notions of restrictiveness*.
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Penelope Maddy's naturalistic approach to philosophy of mathematics aims to explain why the research community embraces some candidates for axiomatic foundations of mathematics and rejects others. In [5], she argued that since set theory aims to be a foundation for mathematics, it should conform to the methodological maxim MAXIMIZE and therefore, axiomatic set theories that are *restrictive* ought to be rejected. She then went on to define a formal notion of restrictiveness, based on a fixed class of interpretations. In [1,3,4], this notion was discussed and a number of technical and substantial issues were raised. Following [2], this talk will present the general framework for interpretations and their derived notions of restrictiveness and then go on to discuss a *symmetrised* version of Maddy's original notion that takes both inner model and outer model constructions into account and can deal with the substantial issues raised in [4].

[1] J. D. HAMKINS, *A multiverse perspective on the axiom of constructibility, Infinity and Truth* (C. Chong, Q. Feng, T. A. Slaman, and W. H. Woodin, editors), Lecture Notes Series, Institute for Mathematical Sciences, National University of Singapore, vol. 25, World Scientific, Singapore, 2013, pp. 25–45.

[2] L. INCURVATI and B. LÖWE, *Restrictiveness relative to notions of interpretation. The Review of Symbolic Logic*, vol. 9 (2016), no. 2, pp. 238–250.

[3] B. LÖWE, *A first glance at non-restrictiveness. Philosophia Mathematica*, vol. 9 (2001), pp. 347–354.

[4] ———, *A second glance at non-restrictiveness. Philosophia Mathematica*, vol. 11 (2003), pp. 323–331.

[5] P. MADDY, *Naturalism in Mathematics*, Clarendon Press, 1997.

- ▶ GIAMBATTISTA FORMICA, *On Hilbert's axiomatic method*.

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Hilbert's methodological reflection certainly shaped a new image of the axiomatic method. However, the discussion on the nature of this method is still open. There are (1) those who have seen it as a synthetic method, i.e., a method to derive theorems from axioms already and arbitrarily established; (2) others have counter-argued in favor of its analytical nature, i.e., given a particular scientific field the method is useful to reach the conditions (axioms) for the known results of the field (theorems) and to rightly place both in a well-structured theory; (3) still others have underlined the metatheoretical nature of the axiomatic reflection, i.e., the axiomatic method is the method to verify whether axioms already identified satisfy properties such as completeness, independence, and consistency.

Each of these views has highlighted aspects of the way Hilbert conceived and practiced the axiomatic method, so they can be harmonized into an image better suited to the function the method was called to fulfill: i.e., deepening the foundations of given scientific fields, to recall one of his well-known expressions. Considering some textual evidence from early and late writings, I shall argue that the axiomatic method is in Hilbert's hands a very flexible tool of inquiry and that to lead analytically to an axiomatic well-structured theory it needs to include dynamically both synthetic procedures and metatheoretical reflections. Therefore, in Hilbert's concern the expression "deepening the foundations" denotes the whole set of considerations, permitted by the axiomatic method, that allow the theoretician first to identify and then to present systems of axioms for given scientific fields.

- ▶ MICHÈLE FRIEND, *Reasoning abhorrently*.

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We reason in different ways on different occasions. Sometimes it is suitable to reason classically, sometimes constructively, and sometimes paraconsistently. We might insist on, prefer, be trained in, or find familiar, some forms of reasoning. Each form will enjoy its own suite of formal representations. Some formal representations are clearly extensions of others, for example, we can add modal operators to classical propositional logic. But sometimes we are called upon to reason in a way that is to-our-lights: incorrect, unfamiliar, disagreeable, or perverse; call these 'abhorrent' for short. At such times, to allay the threat of incorrectness, triviality, or absurdity, we reason hypothetically, or "in quotation marks". We compartmentalise the reasoning in some way. The tractable technical question is how to formally represent how we do this in such a way as to ultimately fend from whatever we find abhorrent. The deeper, philosophical question is how to understand what it is that we are doing when in the process of orchestrating such reasoning and carrying out such reasoning. After all, it is only later that we model such reasoning using a formal or semiformal representation that reconstructs the reasoning to show that it was legitimate.

- ▶ JULIETTE KENNEDY, *Squeezing arguments and strong logics*.

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G. Kreisel has suggested that squeezing arguments, originally formulated for the informal concept of first order validity, should be extendable to second order logic, although he points out obvious obstacles. We develop this idea in the light of more recent advances and delineate the difficulties across the spectrum of extensions of first order logics by generalised quantifiers and infinitary logics. In particular we argue that if the relevant

informal concept is read as informal in the precise sense of being untethered to a particular semantics, then the squeezing argument goes through in the second order case. Consideration of weak forms of Kreisel's squeezing argument leads naturally to reflection principles of set theory.

This is joint work with Jouko Väänänen.

- ▶ SARA NEGRI, *Reasoning with counterfactual scenarios: from models to proofs*. Department of Philosophy, University of Helsinki, Unioninkatu 40A, Finland.
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Ever since the sophisticated analysis provided by David Lewis, counterfactuals have been a challenge to logicians because they were shown to escape both the traditional truth-valued semantics and the standard possible worlds semantics. Lewis' semantics will be here generalized and shown capable of covering, in a modular way, all the systems for counterfactuals presented in [2]. On its basis, and along the methodology of [4], proof systems are developed that feature a transparent justification of their rules [3], good structural properties, analyticity, direct completeness, and decidability proofs [1,5,6].

[1] M. GIRLANDO, S. NEGRI, and N. OLIVETTI, *Labelled sequent calculi based on neighbourhood semantics for PCL and its extensions*, ms.

[2] D. LEWIS, *Counterfactuals*, Blackwell, 1973.

[3] S. NEGRI, *Non-normal modal logics: A challenge to proof theory*, *The Logica Yearbook 2016* (P. Arazim and T. Lavička, editors), College Publications, 2017.

[4] ———, *Proof theory for non-normal modal logics: The neighbourhood formalism and basic results*. *IfCoLog Journal of Logics and their Applications*, vol. 4 (2017), pp. 1241–1286.

[5] S. NEGRI and N. OLIVETTI, *A sequent calculus for preferential conditional logic based on neighbourhood semantics*, *Automated Reasoning with Analytic Tableaux and Related Methods* (H. De Nivelle, editor), Lecture Notes in Computer Science, vol. 9323, Springer, 2015, pp. 115–134.

[6] S. NEGRI and G. SBARDOLINI, *Proof analysis for Lewis counterfactuals*. *The Review of Symbolic Logic*, vol. 9 (2016), no. 1, pp. 44–75.

- ▶ DAVIDE RIZZA, *How to make an infinite decision*. School of Politics, Philosophy, Language and Communication Studies, University of East Anglia, Norwich, UK.
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Infinite exchange problems arise when certain computable features of finite iterations of decisions are studied over an actual (usually countable) infinity of acts. In presence of standard sequential reasoning, as familiar from real analysis, it looks as if infinite iterations lead to the loss of features typical of finite iterations. This conclusion, however, depends on the lack of a proper, computationally amenable, approach to actually infinite iterations of decisions. Once it is possible to offer a numerical specification of the infinitely large number of iterations concerned, it becomes possible to compute features that sequential reasoning could not represent. This gives rise to a uniform, elementary resolution of puzzles concerning infinite decisions (notably those in [2]). In this article I present a fruitful approach that achieves this goal, due to Yaroslav Sergeyev, informally presented in [3] and axiomatised in the context of second order predicative arithmetic in [1].

[1] G. LOLLI, *Metamathematical investigations on the theory of grossone*. *Applied Mathematics and Computation*, vol. 255 (2015), pp. 3–14.

[2] M. SCOTT and A. SCOTT, *Infinite exchange problems*. *Theory and Decision*, vol. 57 (2005), no. 4, pp. 397–406.

[3] Y. D. SERGEYEV, *A new applied approach for executing computations with infinite and infinitesimal quantities*. *Informatica*, vol. 19 (2008), no. 4, pp. 567–596.

Abstracts of invited talks in the Special Session on Proof Theory

- ▶ FERNANDO FERREIRA, *A herbrandized functional interpretation of classical first-order logic*.

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We define a (cumulative) functional interpretation of first-order classical logic and show that each theorem of first-order logic is naturally associated with a certain scheme of tautologies. Herbrand's theorem is obtained as a special case. The schemes are given through formulas of a language of finite-type logic defined with the help of an extended typed combinatory calculus that associates to each given type the type of its nonempty finite subsets. New combinators and reductions are defined, the properties of strong normalization and confluence still hold and, in reality, they play a crucial role in defining the above mentioned schemes. The functional interpretation is dubbed "cumulative" because it enjoys a monotonicity property now so characteristic of many recently defined functional interpretations.

Joint work with Gilda Ferreira in [1].

[1] F. FERREIRA and G. FERREIRA, *A herbrandized functional interpretation of classical first-order logic*. *Archive for Mathematical Logic*, vol. 56 (2017), no. 5–6, pp. 523–539.

- ▶ ANTON FREUND, *Type-two well-ordering principles and Π_1^1 -comprehension*.

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A well-ordering principle of type one consists of a construction which transforms linear orders into linear orders, together with the assertion that well-foundedness is preserved. It is known that many second order axioms of complexity Π_2^1 (e.g., arithmetical comprehension and arithmetical transfinite recursion) are equivalent to natural well-ordering principles of type one. Montalbán [2, Section 4.5] and Rathjen [3, Section 6] have conjectured that Π_1^1 -comprehension, which is a Π_3^1 -statement, corresponds to a well-ordering principle of type two: one that transforms each well-ordering principle of type one into a well-order. I will present recent progress [1] towards this conjecture.

[1] A. FREUND, *A higher Bachmann–Howard principle*, arXiv:1704.01662, preprint.

[2] A. MONTALBÁN, *Open questions in reverse mathematics*, this BULLETIN, vol. 17 (2011), pp. 431–454.

[3] M. RATHJEN, *Omega-models and well-ordering principles*, *Foundational Adventures: Essays in Honor of Harvey M. Friedman* (N. Tennant, editor), College Publications, London, 2014, pp. 179–212.

- ▶ ANNIKA KANCKOS, *Strong normalization for simply typed lambda calculus*.

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A solution is proposed to Gödel's Koan as the problem is stated in the TLCA list of open problems. As the problem is formulated it contains an element of vagueness as it is presented as the problem of finding a simple or easy ordinal assignment for strong normalization of the beta-reduction of simply typed lambda calculus. Whether a proof is sufficiently easy to categorize as a solution is thus a matter of opinion.

The solution is based on (Howard, 1970) and its improved notation in (Schütte, 1977). These normalization proofs also work for a system with a recursor. However, when the recursor is absent, as in our case, a further simplification is possible. The delta-operation becomes redundant (or at least highly simplified), as does the use of ordinal and vector variables, while the crucial Howard's permutation Lemma 2.6 is preserved with some alterations in the vector assigned to the abstracted term.

The proof also gives a unique ordinal assignment for strong normalization as opposed to the nonunique assignment of Howard. The limit ordinal of the assignment is ε_0 . That this is possible is more or less noted by Howard when he explains that the delta-operation is the point where the strong normalization proof breaks down for his unique assignment. The reason being that the division into cases in the definition of the delta-operation makes some vectors incomparable and it becomes impossible to prove that the inequalities are preserved when the delta-operator is applied. Therefore, Howard's unique assignment is limited to a weak normalization (however with the recursor included). As mentioned the presented result gives a unique assignment for strong normalization though the recursor is not included in order to fit the problem description of the Koan that was first proposed by Gödel.

- ▶ ANTON SETZER, *The extended predicative Mahlo Universe in Explicit Mathematics—model construction*.

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In [3] Setzer introduced the Mahlo universe V in type theory and determined its proof theoretic strength. This universe has a constructor, which depends on the totality of functions from families of sets in the universe into itself. Essentially for every such function f a subuniverse U_f of V was introduced, which is closed under f and represented in V . Because of the dependency on the totality of functions, not all type theorists agree that this is a valid principle, if one takes Martin-Löf type theory as a foundation of mathematics.

Feferman's theory of Explicit Mathematics [1] is a different framework for constructive mathematics, in which we have direct access to the set of partial functions. In such a setting, we can avoid the reference to the totality of functions on V . Instead, we can take arbitrary partial functions f , and try to form a subuniverse U_f closed under f . If f is total on U_f , then we add a code for it to V . In [2] we developed a universe based on this idea (using m as a name for V and sub as a name for U), and showed that we can embed the axiomatic Mahlo universe, an adaption of the Mahlo construction as in [3] to Explicit Mathematics, into this universe. We added as well an induction principle, expressing that the Mahlo universe is the least one. Since the addition of U_f to V depends only on elements of V present before U_f was added to V , it can be regarded as being predicative, and we called it therefore the extended predicative Mahlo universe.

In this talk we construct a model of the extended predicative Mahlo universe in a suitable extension of Kripke–Platek set theory, in order to determine an upper bound for its proof theoretic strength. The model construction adds only elements to the Mahlo universe which are justified by its introduction rules. The model makes use of a new monotonicity condition on family sets, the notion of a monotone operator for defining universes, and a special condition for closure operators. This is an alternative to Richter's $[\Gamma, \Gamma']$ operator for defining closure operators.

This is joint work with Reinhard Kahle, Lisbon.

[1] S. FEFERMAN, *A Language and Axioms for explicit Mathematics*, *Algebra and Logic* (J. Crossley, editor), Springer, 1975, pp. 87–139.

[2] R. KAHLE and A. SETZER, *An extended predicative definition of the Mahlo universe*, *Ways of Proof Theory* (R. Schindler, editor), Ontos Series in Mathematical Logic, Ontos Verlag, Frankfurt (Main), Germany, 2010, pp. 315–340.

[3] A. SETZER, *Extending Martin-Löf type theory by one Mahlo-universe*. *Archive for Mathematical Logic*, vol. 39 (2000), pp. 155–181.

Abstracts of invited talks in the Special Session on Set Theory

- ▶ WILLIAM CHEN, *Negative partition relations from cardinal invariants*.

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Classically, many partition relations involving ω_1 and countable ordinals were shown to fail from CH. In joint work with Shimon Garti and Thilo Weinert, we prove that having certain cardinal characteristics equal to \aleph_1 causes the failure of partition relations such as $\omega_1 \rightarrow (\omega_1, \omega + 2)_2^2$ and $\omega_1^2 \rightarrow (\omega_1\omega, 4)_2^2$. Most often, we use the hypothesis $\mathfrak{d} = \aleph_1$, but this seems quite strong. In an effort to use weaker hypotheses, we consider how partition relations behave under the stick principle, and with certain values of invariants for category, evasion, and prediction.

- ▶ BRENT CODY, *Adding a nonreflecting weakly compact set.*

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There is a strong analogy between stationary sets and weakly compact sets. However, by a theorem of Kunen there are models in which nonweakly compact sets can become weakly compact after forcing, whereas nonstationary sets can never be forced to become stationary. Thus, proofs about the ideal of nonweakly compact sets often require a finer analysis than their counterparts for the nonstationary ideal. Many questions whose analogues have been answered for the nonstationary ideal remain open for the weakly compact ideal, and higher order Π_1^1 -indescribability ideals. This talk will survey what is known in this area and will include a discussion of some recent results on the weakly compact reflection principle, which is a generalization of a certain stationary reflection principle. We say that the weakly compact reflection principle holds at κ and write $\text{Refl}_{\text{wc}}(\kappa)$ if and only if κ is a weakly compact cardinal and every weakly compact subset of κ has a weakly compact proper initial segment. The weakly compact reflection principle at κ implies that κ is ω -weakly compact, and in this talk we will discuss a proof that the converse of this statement can be false. Moreover, if κ is $(\alpha + 1)$ -weakly compact where $\alpha < \kappa^+$ then there is a forcing extension in which there is a weakly compact set $W \subseteq \kappa$ having no weakly compact proper initial segment, the class of weakly compact cardinals is preserved and κ remains $(\alpha + 1)$ -weakly compact.

- ▶ ASHUTOSH KUMAR, *Transversal of full outer measure.*

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For every partition of a set of reals into countable sets there is a transversal of full outer measure.

Joint work with S. Shelah.

[1] A. KUMAR and S. SHELAH, *A transversal of full outer measure*, preprint.

- ▶ YANN PEQUIGNOT, *Countable Borel chromatic numbers and Σ_2^1 sets.*

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Analytic sets enjoy a classical representation theorem based on well-founded relations. I will explain a similar representation theorem for Σ_2^1 sets due to Marcone [2,3] which is based on an intriguing topological graph: the Shift Graph. The chromatic number of this graph is 2, but its Borel chromatic number is infinite. We use this representation theorem to show that the Shift Graph is not minimal among the graphs of Borel functions which have infinite Borel chromatic number. While this answers negatively the primary outstanding question from [1], our proof surprisingly does not construct any explicit example of a Borel function whose graph has infinite Borel chromatic number and admits no homomorphism from the Shift Graph.

[1] A. S. KECHRIS, S. SOLECKI, and S. TODORČEVIĆ, *Borel chromatic numbers*. *Advances in Mathematics*, vol. 141 (1999), no. 1, pp. 1–44.

[2] A. MARCONE, *Foundations of bqo theory*. *Transactions of the American Mathematical Society*, vol. 345 (1994), no. 2, pp. 641–660.

[3] ———, *The set of better quasi orderings is Π_2^1 -complete*. *Mathematical Logic Quarterly*, vol. 41 (1995), pp. 373–383.

- SANDRA UHLENBROCK, *The hereditarily ordinal definable sets in inner models with finitely many Woodin cardinals*.

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An essential question regarding the theory of inner models is the analysis of the class of all hereditarily ordinal definable sets HOD inside various inner models M of the set theoretic universe V under appropriate determinacy hypotheses. Examples for such inner models M are $L(\mathbb{R})$, $L[x]$, and $M_n(x)$. Woodin showed that under determinacy hypotheses these models of the form HOD^M contain large cardinals, which motivates the question whether they are fine-structural as for example the models $L(\mathbb{R})$, $L[x]$, and $M_n(x)$ are. A positive answer to this question would yield that they are models of CH, \diamond , and other combinatorial principles.

The first model which was analyzed in this sense was $\text{HOD}^{L(\mathbb{R})}$ under the assumption that every set of reals in $L(\mathbb{R})$ is determined. In the 1990's Steel and Woodin were able to show that $\text{HOD}^{L(\mathbb{R})} = L[M_\infty, \Lambda]$, where M_∞ is a direct limit of iterates of the canonical mouse M_ω and Λ is a partial iteration strategy for M_∞ . Moreover Woodin obtained a similar result for the model $\text{HOD}^{L[x,G]}$ assuming Δ_2^1 determinacy, where x is a real of sufficiently high Turing degree, G is $\text{Col}(\omega, <\kappa_x)$ -generic over $L[x]$, and κ_x is the least inaccessible cardinal in $L[x]$.

In this talk I will give an overview of these results and outline how they can be extended to the model $\text{HOD}^{M_n(x,g)}$ assuming Π_{n+2}^1 determinacy, where x again is a real of sufficiently high Turing degree, g is $\text{Col}(\omega, <\kappa_x)$ -generic over $M_n(x)$ and κ_x is the least inaccessible cutpoint in $M_n(x)$ which is a limit of cutpoints in $M_n(x)$.

This is joint work with Grigor Sargsyan.

Abstracts of contributed talks

- BAHAREH AFSHARI, *Interpolation for modal μ -calculus*.

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Modal logics are known to widely enjoy interpolation and so does modal μ -calculus, the extension of modal logic by propositional fixed point quantifiers. D'Agostino and Hollenberg [2] utilise automata theory to show that bisimulation quantifiers are expressible in modal μ -calculus and can be used to define interpolants. I will present a constructive and purely syntactic proof of Lyndon (and hence also Craig) interpolation via a finitary sequent calculus of circular proofs introduced in [1].

[1] B. AFSHARI and G. E. LEIGH, *Cut-free completeness for modal μ -calculus*, *Proceeding of Thirty-Second Annual ACM/IEEE Symposium on Logic in Computer Science*, Reykjavik, Iceland, 2017, pp. 1–12.

[2] G. D'AGOSTINO and M. HOLLENBERG, *Logical questions concerning the μ -calculus*. *The Journal of Symbolic Logic*, vol. 65 (2000), no. 1, pp. 310–332.

- BAHAREH AFSHARI AND GRAHAM E. LEIGH, *Cut-free completeness for modal μ -calculus*.

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Modal μ -calculus is the extension of propositional modal logic by constructors for fixed points of inductive and co-inductive definitions. Kozen [1] proposed an axiomatisation for the logic which was proved to be complete by Walukiewicz [2]. Kozen's system contains cut and Walukiewicz' proof makes essential use of this rule. We present a cut-free sequent calculus for the logic that features a strengthening of the standard induction rule for greatest fixed point. The system is readily seen to be sound and its completeness is established by utilising a novel calculus of circular proofs. As a corollary we obtain a new, constructive, proof of completeness for Kozen's axiomatisation which avoids the usual detour through automata and games.

[1] D. KOZEN, *Results on the propositional μ -calculus*. *Theoretical Computer Science*, vol. 27 (1983), pp. 333–354.

[2] I. WALUKIEWICZ, *Completeness of Kozen's axiomatisation of the propositional μ -calculus*. *Information and Computation*, vol. 157 (2000), pp. 142–182.

- SERGEI ARTEMOV AND ELENA NOGINA, *On completeness of epistemic theories*.

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Semantic formalizations of epistemic situations as Kripke models produce complete descriptions: for each sentence F , they specify which of F or $\neg F$ holds. This renders semantic formalizations inadequate for incomplete scenarios. To represent all epistemic situations, complete and incomplete, we need *epistemic theories*, i.e., sets of epistemic formulas (cf. [1]), analogous to *mathematical theories*, many of which are incomplete (group theory, Peano Arithmetic, etc.).

We consider epistemic theories of card dealing and establish their completeness. One should not expect epistemic completeness to be maintained throughout the game: players could use private communications to learn facts which do not follow from the game description. For such situations, epistemic theories become essential.

Assume a deck of cards dealt to n players. Consider epistemic logic $S5_n$ with atomic propositions '*player i is dealt card j* '; for a given property P , let $\ulcorner P \urcorner$ denote its representation by a formula. Let Γ be set of formulas $S5_n + \ulcorner \text{rules of dealing} \urcorner + \ulcorner \text{each player knows her hand and deems possible any dealing consistent with it} \urcorner$. For each combination α of cards dealt, define theory

$$\Delta_\alpha = \ulcorner \text{common knowledge of } \Gamma \urcorner + \ulcorner \alpha \urcorner.$$

The standard model of card dealing (cf. [2]) has all possible dealings as worlds, indistinguishability as accessibility relations, and the natural evaluation of atomic propositions.

COMPLETENESS THEOREM. $\Delta_\alpha \vdash F$ iff $\alpha \Vdash F$ in the standard model.

[1] S. ARTEMOV, *Syntactic epistemic logic*, *Book of Abstracts. 15th Congress of Logic, Methodology and Philosophy of Science*, University of Helsinki, 2015, pp. 109–110.

[2] R. FAGIN, J. HALPERN et al., *Reasoning about Knowledge*, MIT Press, 1995.

- ASHOT BAGHDASARYAN AND HOVHANNES BOLIBEKYAN, *On some systems of minimal predicate logic with history mechanism*.

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Backwards proof search and theorem proving with a standard cut-free calculus for minimal logic is insufficient because of three problems. First, the proof search is not in general terminating caused by the possibility of looping. Second, it might generate proofs which are permutations of each other and represent the same natural deduction. Finally, during the proof some choice should be made to decide which rules to apply and where to use them. Several proof systems of I. Johansson's minimal logic of predicates were introduced

in [1]. Looping is the main issue in the system GM^- developed in [1]. Looping may easily be removed by checking if a sequent has already occurred in the branch. Though this is insufficient as it requires much information to be stored. Some looping mechanisms have been considered earlier in ([2,3]). One way to detect loops is adding history to each sequent.

We introduce two systems for first order minimal logic (SwMin and ScMin) which are slightly different. Both systems are based on the idea of adding context to the sequents. In one system, SwMin, the history is kept smaller, but ScMin detects loops more quickly. The heart of the difference between the two systems is that in the SwMin loop checking is done when a formula leaves the goal, whereas in the ScMin it is done when it becomes the goal.

THEOREM.

1. *The systems GM^- and SwMin are equivalent.*
2. *The systems GM^- and ScMin are equivalent.*

[1] H. R. BOLIBEKYAN and A. A. CHUBARYAN, *On some proof systems for I. Johansson's minimal logic of predicates*, *Proceedings of the Logic Colloquium*, 2003, p. 56.

[2] H. BOLIBEKYAN and T. MURADYAN, *On some loop detection strategies for minimal propositional logic*, *Proceedings of the Logic Colloquium*, 2011, pp. 45–46.

[3] D. GABBAY, *Algorithmic Proof with Diminishing Resources*, Lecture Notes in Computer Science, vol. 533, Springer, 1991, Part 1, pp. 156–173.

- ▶ NIKOLAY BAZHENOV, EKATERINA FOKINA, DINO ROSSEGGER, AND LUCA SAN MAURO, *Computable bi-embeddable categoricity of equivalence structures*. Sobolev Institute of Mathematics, and Novosibirsk State University, Novosibirsk, Russia.
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We study the algorithmic complexity of embeddings between bi-embeddable equivalence structures. To do this, we use the notions of Δ_α^0 bi-embeddable categoricity and relative Δ_α^0 bi-embeddable categoricity defined analogously to the standard concepts of Δ_α^0 categoricity and relative Δ_α^0 categoricity.

We give a characterization of Δ_1^0 bi-embeddably categorical equivalence structures, completely characterize Δ_2^0 bi-embeddably categorical and relatively Δ_2^0 bi-embeddably categorical equivalence structures, and show that all equivalence structures are relatively Δ_3^0 bi-embeddably categorical.

Furthermore, let the *degree of bi-embeddable categoricity* of a computable structure \mathcal{A} be the least Turing degree that, if it exists, computes embeddings between any computable bi-embeddable copies of \mathcal{A} . Then every computable equivalence structure has a degree of bi-embeddable categoricity, and the only possible degrees of bi-embeddable categoricity for equivalence structures are $\mathbf{0}$, $\mathbf{0}'$, and $\mathbf{0}''$.

- ▶ NIKOLAY BAZHENOV AND BIRZHAN KALMURZAYEV, *Weakly precomplete dark computably enumerable equivalence relations*. Department of Fundamental Mathematics, Al-Farabi Kazakh National University, 71 Al-Farabi avenue, Almaty 050038, Kazakhstan.
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We study computably enumerable equivalence relations (ceers). For the background, we refer the reader to [1].

A ceer E on ω is weakly precomplete if there exists a partial computable function fix such that for all e , if φ_e is total, then $fix(e) \downarrow$ and $\varphi_e(fix(e))Efix(e)$. We consider ceers relatively to the following well known reduction: a ceer R is said to be reducible to a ceer S (denoted by $R \leq_c S$) if there is a computable function f such that for all x and y , $xRy \Leftrightarrow f(x)Sf(y)$. A

ceer E is called dark if it is incomparable with Id under reduction \leq_c . We have the following result.

THEOREM 1. *For any dark ceer E there is a weakly precomplete dark ceer F such that $E <_c F$.*

S.A. Badaev showed that there is an infinite ω -chain of nonequivalent weakly precomplete ceers. Our result implies that for any dark ceer E , there is an infinite ω -chain of nonequivalent weakly precomplete dark ceers over E .

[1] U. ANDREWS, S. BADADEV, and A. SORBI, *A Survey on Universal Computably Enumerable Equivalence Relations*, Lecture Notes in Computer Science, vol. 10010, 2017, pp. 418–451.

- MARIO BENEVIDES, *Propositional Dynamic Logic for bisimilar programs with parallel operator and test.*

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In standard Propositional Dynamic Logic (PDL) literature [3], the semantics is given by Labeled Transition Systems, where for each program π we associate a binary relation R_π . Process Algebras also give semantics to process (terms) by means of Labeled Transition Systems. In both formalisms, PDL and Process Algebra, the key notion to compare processes is bisimulation. In PDL, we also have the notion of logic equivalence, that can be used to prove that two programs π_1 and π_2 are logically equivalent $\vdash \langle \pi_1 \rangle \varphi \leftrightarrow \langle \pi_2 \rangle \varphi$. Unfortunately, logic equivalence and bisimulation do not match in PDL. Bisimilar programs are logic equivalent but the converse does not hold.

This article proposes a semantics and an axiomatization for PDL that makes logically equivalent programs also bisimilar. This allows for developing Dynamic Logics to reasoning process algebras about specification, in particular about CCS (Calculus for Communicating Systems) [4]. As in CCS the bisimulation is the main tool to establish equivalence of programs, it is very important that these two relations coincide. We propose a new Propositional Dynamic Logic with a new nondeterministic choice operator, PDL+. We prove its soundness, completeness, finite model property, and EXPTIME-completeness for the satisfiability problem. We also add to PDL+ the parallel composition operator (PPDL+) and prove its soundness and completeness. We establish that the satisfiability problem for PPDL+ is in 2-EXPTIME. Finally, we define some fragments of PPDL+ and prove its EXPTIME-completeness. In ([1,2]) we do not deal with test operator. In this work we discuss some issues concerning test and point out some direction on how it can be handled.

[1] M. R. F. BENEVIDES, *Bisimilar and logically equivalent programs in PDL*. *Electronic Notes in Theoretical Computer Science*, vol. 305 (2014), pp. 5–18.

[2] ———, *Bisimilar and logically equivalent programs in PDL with parallel operator*, *Theoretical Computer Science*, published first online, 2017.

[3] D. HAREL, D. KOZEN, and J. TIURYN, *Dynamic Logic*, The MIT Press, 2000.

[4] R. MILNER, *Communication and Concurrency*, Prentice Hall International, London, UK, 1989.

- BRUNO BENTZEN, *A solution to Frege's puzzle in homotopy type theory.*

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One of the virtues of Voevodsky's celebrated univalence axiom is that it offers a formal justification for the common practice among mathematicians of identifying objects just in case they are isomorphic. Since in general there may be different isomorphisms between any two objects, it follows that a thing can be recognized as the same again in more than one way. Equipped with this axiom and other powerful features, homotopy type theory (The Univalent Foundations Program 2013) provides a novel notion of equality with a subtle structure that takes account of the different reasons a thing can be the same.

Over one hundred years ago, Frege (1982) drew attention to a puzzle concerning the slippery and multifaceted nature of equality. In a sense, he also arrived at the conclusion that there should be different ways for two objects to be identified—and he explained this by saying that two objects expressing a different sense denote the same referent. Now, a natural question is “can the homotopy-type theoretic notion of equality shed new light on Frege’s puzzle?”

In this work-in-progress talk, I shall propose a constructive solution for Frege’s puzzle based on elementary insights from homotopy type theory. I claim that, from the viewpoint of constructive semantics, Frege’s solution is unable to explain adequately the informative content of identity statements, since, as I shall argue, not only identity statements of the form “ $a = b$ ”, but also “ $a = a$ ” may contribute to extensions of our knowledge. More precisely, I hold that my approach can be seen as an extension of Frege’s ideas to account for constructive reasoning.

- ▶ ALEXANDR BESSONOV, *Gödel’s second incompleteness theorem cannot be used as an argument against Hilbert’s program.*

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Gödel’s second incompleteness theorem is commonly accepted as a decisive argument against realizability of Hilbert’s program of finitary grounding of mathematics in its original setting. We show that this widespread belief is wrong.

According to Gödel’s second incompleteness theorem, if the formal Dedekind–Peano arithmetic (PA) is consistent and the formula $\text{Prov}(x, y)$ that numeralwise expresses the provability predicate satisfies Hilbert–Bernays–Löb conditions, then the formula

$$\exists x \forall y \neg \text{Prov}(x, y) \tag{Consis}$$

that numeralwise expresses the consistency of PA is unprovable in PA. This readily implies that, for any formula A , the formula

$$\forall y \neg \text{Prov}(\ulcorner A \urcorner, y) \tag{*A}$$

that expresses the unprovability of A is unprovable in PA.

The argument against realizability of Hilbert’s program based on the second theorem is generally built as follows. Let PA be consistent. Suppose that there is an informal finitary consistency proof of PA. By von Neumann’s thesis (every finitary informal proof is formalizable in PA), such a proof would be formalizable in PA. As a result, a formula that expresses the consistency of PA would turn out to be provable, which would contradict the second incompleteness theorem (see, e.g., [1]).

We will show that such reasoning is incorrect. We know that the PA may be either consistent or inconsistent. *Tertium non datur*.

Let PA be inconsistent. In this case the second theorem cannot be applied because its formulation contains a presupposition of PA being consistent.

Let PA be consistent. Consider a formula $\neg(\mathbf{0} = \mathbf{0})$ and repeat von Neumann’s reasoning in relation to this formula. Suppose that there is an informal finitary proof that $\neg(\mathbf{0} = \mathbf{0})$ is unprovable in PA. Such a proof could be Gödel-style formalizable in PA. As a result, being an instance of $(*A)$, the formula $\forall y \neg \text{Prov}(\ulcorner \neg(\mathbf{0} = \mathbf{0}) \urcorner, y)$ that expresses the unprovability of $\neg(\mathbf{0} = \mathbf{0})$ would turn out to be provable, which would contradict the second incompleteness theorem. Thus we can conclude that an informal finitary proof of the unprovability of $\neg(\mathbf{0} = \mathbf{0})$ does not exist.

However, if PA is consistent, then such a finitary proof exists! Here is the proof: Suppose $\vdash_{\text{PA}} \neg(\mathbf{0} = \mathbf{0})$. In view of $\vdash_{\text{PA}} (\mathbf{0} = \mathbf{0})$, it would follow that $\vdash_{\text{PA}} (\mathbf{0} = \mathbf{0}) \ \& \ \neg(\mathbf{0} = \mathbf{0})$, and hence PA would be inconsistent, which contradicts our assumption. And this trivial proof is obviously finitary! We have thus arrived at a contradiction with von Neumann’s reasoning.

Thus the argument against realizability of Hilbert’s program based on the second theorem is incorrect from the outset.

This work was supported by the Russian Science Foundation (project 16-18-10359).

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- ▶ KRZYSZTOF BIELAS, PAWEŁ KLIMASARA, AND JERZY KRÓL, *Boolean-valued models of ZFC and forcing in geometry and physics*. Department of Astrophysics and Cosmology, University of Silesia in Katowice, Uniwersytecka 4, 40-007 Katowice, Poland.
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To every complex separable Hilbert space \mathcal{H} of quantum-mechanical (QM) states one can assign orthomodular lattice of projections $\mathbb{L}(\mathcal{H})$. Given a maximal complete Boolean algebra of projections $B \subset \mathbb{L}(\mathcal{H})$, it determines a Boolean-valued ZFC model V^B with real numbers corresponding bijectively to self-adjoint operators with spectral projections in B [2]. We provide the conditions for B to be atomless and the QM-meaning of the nontrivial forcing in V^B . For a generic ultrafilter G in V^B , the change of the real line R in 2-valued model V into $R[G]$ in V^B/G helps to solve some problems in cosmology.

Another change of the real line concerns the level of the formal language, i.e., $R[G] \rightarrow \mathbb{R}$ where $R[G]$ is the 1st order set of real numbers and \mathbb{R} is the unique (up to isomorphism) model of the 2nd order theory of Dedekind-complete ordered field. This shift is expected to take place in the cosmological model of expanding universe [1]. We show that this shift is derivable from $\mathbb{L}(\mathcal{H})$ and leads to a change in smoothness structure of spacetime manifold which must be an exotic R^4 . The embedding into the standard smooth \mathbb{R}^4 allows prediction of the cosmological constant value purely topologically.

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- ▶ FRODE ALFSON BJØRDAL, *Voluntionary deliberations*. Programa de Pós-Graduação em Filosofia, UFRN, Natal, Brasil.
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As a continuation of divulgations at *Trends in Logic XVI* and after in seminars and at *Encontro Brasileiro de Lógica XVIII I* I relate the *voluntionary* point of view \mathfrak{P} (“ruble”) which shifts attention to the set \mathbf{J} (“eet”) of sentences whose negation are not theses of the presupposed formal arithmetic \mathbf{T} as traditionally conceived; we assume \mathbf{T} is axiomatized so only *sentences* are derivable and only modus ponens is a primitive inference rule. Voluntionism alters how we think about fundamental matters, e.g., in that the standard Gödel sentence of \mathbf{T} is taken as a textbook liar sentence, and gives occasion to reinterpret issues concerning decidability and computability as other sentences independent of \mathbf{T} are treated similarly. Voluntionary systems are not like traditional paraconsistent approaches as classical logical theses are included and not contradicted even in the presence of comprehension; nevertheless: if γ is the standard Gödel sentence for \mathbf{T} both γ and $\neg\gamma$ are theses of \mathbf{J} so *modus ponens* does not, but exotic induced inferential principles hold for \mathbf{J} . Compare \mathfrak{P} with the author’s librationist set theory \mathcal{L} as in part set out in [1], [2], and [3].

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- CAROLINA BLASIO, JOÃO MARCOS, AND HEINRICH WANSING, *Monotonic functions are logically four-valued.*

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A monotonic function on a set \mathcal{S} is a \subseteq -preserving mapping on $2^{\mathcal{S}}$, that is, a function C such that $C(A) \subseteq C(A \cup B)$, for every $A, B \subseteq \mathcal{S}$. Tarski's fixpoint theorem guarantees the existence of the least and of the greatest fixpoints for monotonic functions. The latter have a variety of applications, in particular in providing a foundation for inductive and co-inductive definitions, and the proof methods associated therewith. A Tarskian closure operator on \mathcal{S} is a monotonic function on \mathcal{S} that is also inflationary (i.e., $A \subseteq C(A)$) and idempotent (i.e., $C(C(A)) = C(A)$); it is a generalization of the notion of topological closure, axiomatized by Kuratowski. A closure operator on \mathcal{S} is called structural when it commutes with endomorphisms on \mathcal{S} . (Structural) Tarskian closure operators are known [4] to be characterizable by a family of so-called logical matrices, viz. structures containing sets of 'algebraic' truth-values, some of which are distinguished. Their inflationary and idempotent character also guarantees that they may be characterized by (at most) two 'logical' values (cf. Chapter 4 of [3]). In the present contribution we will show how a generalized notion of closure and a two-dimensional notion of logical matrix (resp. B-closure and B-matrix) may be used to characterize any given monotonic function on a set \mathcal{S} , recovering a theme earlier explored at [2] in the context of symmetrical consequence relations involving two potentially distinct languages. We will also show that any B-matrix may be alternatively characterized by (at most) four logical values [1]. A brief discussion of inferential many-valuedness and its connections with bilattice-based reasoning, from a metalogical perspective, will ensue.

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- FEDERICO BOBBIO AND JIANYING CUI, *A plausibility model for regret games.*

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In this article, we develop a plausibility model by defining a new notion of rationality based on the assumption that a player believes that she doesn't play a weakly regret dominated strategy. Especially, we show that the interactive epistemic outcomes from the common belief of this type of rationality are in line with the solutions of the Iterated Regret Minimalization (IRM) algorithm. So, we state that one can achieve a characterization of the IRM algorithm in light of common belief of this type of rationality. A benefit of our characterization is that it provides the epistemic foundation to the IRM algorithm. Meanwhile, we also link solutions of the algorithm to modal μ -calculus to deepen our understanding of the epistemic characterization.

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- ENRIQUE CASANOVAS AND SAHARON SHELAH, *Universal theories and compactly expandable models*.

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A model M of countable vocabulary τ and cardinality κ is *expandable* if for every vocabulary $\tau' \supseteq \tau$ of cardinality $\leq \kappa$, if Σ is a first-order set of sentences of vocabulary τ' consistent with the first-order theory $\text{Th}(M)$ of M , then there is some expansion M' of M to τ' such that $M' \models \Sigma$. Call a set of first-order sentences Σ of vocabulary $\tau' \supseteq \tau$ *finitely satisfiable in M* if for every finite subset $\Sigma_0 \subseteq \Sigma$ there is an expansion of M that satisfies Σ_0 . M is *compactly expandable* if for every vocabulary $\tau' \supseteq \tau$ of cardinality $\leq \kappa$, if Σ is a first-order set of sentences of vocabulary τ' finitely satisfiable in M , then there is some expansion M' of M to τ' such that $M' \models \Sigma$. We present the result proved in [2], which shows that there are compactly expandable models which are not expandable, solving an open problem of [1]. The proof depends on some new result we have obtained on the logic $L(Q_{\aleph_0}^{\text{cf}})$ (see [3]), first-order logic with the additional quantifier $Q_{\aleph_0}^{\text{cf}}$ of cofinality \aleph_0 , namely the existence of κ -universal theories of $L(Q_{\aleph_0}^{\text{cf}})$ for any cardinal $\kappa = 2^{<\kappa} > \aleph_0$.

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- ANAHIT CHUBARYAN AND ARTUR KHAMISYAN, *Application of Kalmar's proof of deducibility in two valued propositional logic for many valued logic*.

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We focus on the problem of constructing of some standard Hilbert style proof systems for any version of many valued propositional logic. The generalization of well-known Kalmar's proof of deducibility for two valued tautologies inside classical propositional logic [1] gives us a possibility to suggest some method for defining of two types of axiomatic systems for any version of 3-valued logic, completeness of which is easy proved direct, without of loading into two valued logic.

First of constructed system bases on the logic with one designated value and *conjunction*, *disjunction*, *implication*, defined by Gödel, and *negation*, defined by permuting of truth values

cyclically. For every formula A, B, C of 3-valued logic, each σ_1, σ_2 from the set $\{0, 1/2, 1\}$ and $*$ $\in \{\&, \vee, \supset\}$, the following formulas are axioms schemes:

1. $A \supset (B \supset A)$,
2. $(A \supset B) \supset ((A \supset (B \supset C)) \supset (A \supset C))$,
3. $A^{\sigma_1} \supset (B^{\sigma_2} \supset (A * B)^{\varphi_{*(A,B,\sigma_1,\sigma_2)}}$,
4. $A^\sigma \supset (\neg A)^\delta$,
5. $(A \supset B) \supset ((\bar{A} \supset B) \supset ((\bar{A} \supset B) \supset B))$, where

$$\begin{aligned} \varphi_{\supset}(A, B, \sigma_1, \sigma_2) &= (\sigma_1 \supset \sigma_2) \& (\neg(A \vee \bar{A}) \vee (\bar{B} \supset B)) \vee (\neg(A \vee \bar{A}) \& \neg(B \vee \bar{B})), \\ \varphi_{\vee}(A, B, \sigma_1, \sigma_2) &= (\sigma_1 \vee \sigma_2) \vee (A \supset \bar{A}) \& \neg(\bar{B} \vee \bar{B}) \vee (\neg(\bar{A} \vee \bar{A}) \& (B \supset \bar{B})), \\ \varphi_{\&}(A, B, \sigma_1, \sigma_2) &= (\sigma_1 \& \sigma_2) \vee ((A \& \bar{A}) \vee (B \& \bar{B})) \vee ((A \& \bar{A}) \vee (B \& \bar{B})), \end{aligned}$$

and for $\delta = \frac{i}{2}$ ($0 \leq i \leq 2$) A^δ is A with $2 - i$ negations. Inference rule is *modus ponens*.

Note that axioms 3. and 4. are generalizations of formulas, using in Kalmar's proof of deducibility for two valued tautologies, therefore the completeness of this system is proved very easily. This method (i) can be base for direct proving of completeness for all well-known axiomatic systems of k -valued ($k \geq 3$) logics and may be for fuzzy logic also and (ii) can be base for constructing of new Hilbert-style axiomatic systems for all mentioned logics.

Second system obtained from first one by some restrictions, which allow to obtain the same by order bounds of main proof complexity characteristics for large sets of k -tautologies.

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- ▶ ANAHIT CHUBARYAN, ARMAN KARABAKHTSYAN, HAKOB NALBANDYAN, AND GARIK PETROSYAN, *Propositional sequent systems of two valued classical logic and many valued logics are no monotonous*.

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The minimal tautologies, i.e., tautologies, which are not a substitution of a shorter tautology, play main role in proof complexity area. Really all propositional formulae, proof complexities of which are investigated in many well known articles, are minimal tautologies. There is traditional assumption that minimal tautology must be no harder than any substitution in it. This idea was revised at first by Anikeev in [1]. He has introduced the notion of monotonous proof system and has given two types of no complete propositional proof systems: monotonous system, in which the proof lines of all minimal tautologies are no more, than the proof lines for results of a substitutions in them, and no monotonous system, the proof lines of substituted formulas in which can be less than the proof lines of corresponding minimal tautologies. First it was proved in [2] that Frege systems are monotonous neither by lines nor by size.

We introduce the analogous notion of minimal sequent (two or many valued) and show that well known propositional sequent systems of two valued classical logic (with and without cut rule) as well as the sequent systems, which are constructed by us for some versions of many valued logic are also monotonous neither by lines nor by size of proofs.

Acknowledgments. This work arose in the context of propositional proof complexity research supported by the Russian–Armenian University from funds of MESRF.

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- ▶ ANAHIT CHUBARYAN AND GARİK PETROSYAN, *On proof complexities for some classes of tautologies in Frege systems*.

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One of the most fundamental problems of the proof complexity theory is to find for classical propositional calculus a proof system, which has a polynomial size $p(n)$ proof for every tautology of size n . Cook and Reckhow named such a system a *super system* and showed in [1] that $NP = coNP$ iff a super system exists. Lately it is proved in [2] that $NP = coNP = PSPACE$, hence a super system must be. It is well known that many systems are not super. This question about Frege system, the most natural calculi for propositional logic, is still open.

Some results about Frege proof complexities are presented here. We introduce the notion of *specific* tautologies A in the form: $A = p \supset (A_1 \vee A_2 \vee \dots \vee A_k)$ ($k \geq 1$), where p is a literal (variable or negation of variable), neither $A_1 \vee A_2 \vee \dots \vee A_k$ nor every A_i ($1 \leq i \leq k$) are tautology or contradiction and $|A_i| \leq \frac{|A_1|}{2^{i-1}}$, and show that Frege systems are super systems iff there is a polynomial $p()$ such that all specific tautologies of size n have a proofs, size of which are bounded by $p(n)$. Then we show, that all *balanced* tautologies (every variable of which has only one positive and one negative occurrences), given in disjunctive normal form, also have Frege proofs with polynomial bounded sizes. Lastly we give some notes about relations between the proof complexities of tautologies A_n and B_n and proof complexities of the tautologies in a form $A_n * B_n$, where $*$ is \wedge, \vee, \supset . In particular we show that for some tautologies A_n and B_n proofs of formulas $A_n \vee B_n$ can be more easier than proofs every of A_n and B_n .

Acknowledgments. This work arose in the context of propositional proof complexity research supported by the Russian–Armenian University from funds of MESRF.

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- ▶ ANDRÉS CORDÓN-FRANCO AND F. FÉLIX LARA-MARTÍN, *On local induction rules: collapse and conservation properties*.

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Local induction schemes are variations of the classical induction schemes axiomatizing Peano arithmetic (Σ_n -induction or Π_n -induction). These local schemes are obtained by restricting the conclusion of the induction axioms to some class of definable elements. Given $n, m \geq 1$, the scheme $I(\Sigma_n, \mathcal{K}_m)$ is defined in this way, when the conclusion of the classical Σ_n -induction scheme is restricted to Σ_m -definable elements.

For $m = n$, the schemes $I(\Sigma_n, \mathcal{K}_n)$ and the corresponding induction rules associated to them, $(\Sigma_n, \mathcal{K}_n)$ -IR, have showed to be useful tools in the analysis of the conservation properties of parameter free Π_n -induction schemes and local reflection principles (see [1] and [2]). An especially interesting feature of $(\Sigma_n, \mathcal{K}_n)$ -IR is the “collapse” property (i.e., reduction of nested applications of the rule to unnested applications) that distinguishes this rule from the classical Σ_n -IR.

In this work we extend our previous work in [1] and focus on collapse and conservation properties of the rules $(\Sigma_n, \mathcal{K}_m)$ -IR and their parameter free counterparts. Namely:

1. For $n = m$, we discuss general collapse results for $(\Sigma_n, \mathcal{K}_n)$ -IR.
2. For $n > m \geq 1$, we discuss results á la Kreisel–Levy relating (parameter free) local induction rules and local reflection principles.
3. For $1 \leq n < m$ we discuss noncollapse and conservation results among rules $(\Sigma_n, \mathcal{K}_m)$ -IR.

Acknowledgments. Work partially supported by grant MTM2014-59178-P, Ministerio de Economía y Competitividad, Spanish Government.

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► LONGYUN DING, *On decomposing Borel functions*.

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The study of decomposing Borel functions originated by a question asked by Luzin: is every Borel function decomposable into countably many continuous functions? This question was answered negatively. So we turn to focus on: what kind of Borel functions is decomposable into countably many continuous functions?

Jayne–Rogers’ theorem shows that, a function of Baire class 1 is decomposable into countably many continuous functions with closed domains iff the preimage of any F_σ set is still F_σ . The generalization of Jayne–Rogers’ theorem is named The Decomposability Conjecture.

In this talk, we will introduce the recent developments on the decomposability conjecture.

► PHILIP EHRLICH, *Are points (necessarily) unextended?*

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Ever since Euclid defined a point as that which has no part it has been widely assumed that points are necessarily unextended. It has also been assumed that, analytically speaking, this is equivalent to saying that points or, more properly speaking, degenerate segments—i.e., segments containing a single point have length zero. In our talk we will challenge these assumptions. We will argue that neither degenerate segments having null lengths nor points satisfying the axioms of Euclidean geometry implies that points lack extension. To make our case, we will provide models of ordinary Euclidean geometry where the points are extended despite the fact that the corresponding degenerate segments have null lengths, as is required by the geometric axioms. The first model will be used to illustrate the fact that points can be quite large—indeed, as large as all of Newtonian space—and the other models will be used to draw attention to other philosophically pregnant mathematical facts that have heretofore been little appreciated.

► DMITRY EMEL'YANOV, BEIBUT KULPESHOV, AND SERGEY SUDOPLATOV, *On algebras of distributions for binary formulas of quite o-minimal theories with nonmaximum many countable models*.

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We apply a general approach for distributions of binary formulas [3] to the class of quite o-minimal theories with nonmaximum many countable models [2]. Using Cayley tables for countably categorical weakly o-minimal theories [1] we explicitly define the classes of commutative monoids \mathfrak{A}_n , respectively, $\mathfrak{A}_n^{\text{QR}}$, $\mathfrak{A}_n^{\text{QL}}$, $\mathfrak{A}_n^{\text{I}}$, of isolating formulas for isolated, respectively, quasirational to the right, quasirational to the left, irrational, 1-type p of quite o-minimal theories with nonmaximum many countable models, with convexity rank $\text{RC}(p) = n$. For an algebra $\mathfrak{P}_{v(p)}$ of binary isolating formulas of 1-type p , we have

THEOREM 1. *Let T be a quite o-minimal theory with nonmaximum many countable models, $p \in S_1(\emptyset)$ be a nonalgebraic type. Then there exists $n < \omega$ such that*

- (1) if p is isolated then $\mathfrak{P}_{v(p)} \simeq \mathfrak{A}_n$;
- (2) if p is quasirational to the right (left) then $\mathfrak{P}_{v(p)} \simeq \mathfrak{A}_n^{\text{QR}}$ ($\mathfrak{P}_{v(p)} \simeq \mathfrak{A}_n^{\text{QL}}$);
- (3) if p is irrational then $\mathfrak{P}_{v(p)} \simeq \mathfrak{A}_n^{\text{I}}$.

COROLLARY 2. *Let T be a quite o-minimal theory with nonmaximum many countable models, $p, q \in S_1(\emptyset)$ be nonalgebraic types. Then $\mathfrak{P}_{v(p)} \simeq \mathfrak{P}_{v(q)}$ if and only if $\text{RC}(p) = \text{RC}(q)$ and the types p and q are simultaneously either isolated, or quasirational, or irrational.*

DEFINITION 3 ([1]). We say that an algebra $\mathfrak{P}_{v(\{p,q\})}$ is *generalized commutative* if there is a bijection $\pi: \rho_{v(p)} \rightarrow \rho_{v(q)}$ witnessing that the algebras $\mathfrak{P}_{v(p)}$ and $\mathfrak{P}_{v(q)}$ are isomorphic (i.e., that their Cayley tables are equal up to π) and for any labels $l \in \rho_{v(p,q)}$, $m \in \rho_{v(q,p)}$, we have $\pi(l \cdot m) = m \cdot l$.

THEOREM 4. *Let T be a quite o-minimal theory with nonmaximum many countable models, $p, q \in S_1(\emptyset)$ be nonalgebraic nonweakly orthogonal types. Then the algebra $\mathfrak{P}_{v(\{p,q\})}$ is a generalized commutative monoid.*

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- LUIS ESTRADA-GONZÁLEZ AND JOSÉ DAVID GARCÍA-CRUZ, *Logical connectives as modalities.*

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Local operators (also known as Lawvere–Tierney topologies in the context of topos theory, or modal operators in other categorical contexts) have been useful in proving independence results in categorial set theory and more recently in providing categorial interpretations for quantum predicates. Our aim here is to use local operators and their duals to highlight a neglected feature of the usual logical connectives, namely their modal character. Disjunction and conditional have already been recognized as species of possibility; our contribution is the use of dual local operators to show that conjunction and subtraction are species of necessity. More exactly, disjunction is a possibility connective, conditional is a contingency connective, conjunction is a necessity connective, and subtraction is an impossibility connective. The modal characters of unary and zero-ary connectives are also discussed.

- LUIS ESTRADA-GONZÁLEZ AND ALEJANDRO SOLARES-ROJAS, *How could a logician help solving the $P \stackrel{?}{=} NP$ problem?*

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As Terence Tao has recalled several times, mathematics can benefit not only from correct proofs, or proofs that require some changes to be correct, but also from outlines of strategies for a proof, whether for opening lines of research or closing them definitely. Here, we discuss how a certain kind of logician could argue for $P = NP$ following a translation between logics approach. As $P = NP$ amounts to $FOL(LFPO) = SOL$, one could argue for the latter by providing a suitable translation between those logics. Though we do not provide any such a translation, we show that such an approach regarding those logics is not *a priori* ruled out. Thus, the broad strategy is as follows:

1. Follow the identities provided by descriptive complexity theory (see Immerman 1998).
2. Compare the expressive powers of $FOL(LFPO)$ and SOL via logical translations (see Manzano 1996).
3. Give reassurance of three kinds: (a) Conceptual: the corresponding translations do not distort the studied phenomena. (b) Mathematical: the translations do not imply any obvious contradiction with well-established mathematical results. (c) Philosophical: the translations do not imply any gratuitous counterintuitive claim regarding logic, mathematics or human nature (cf. Aaronson 2016).

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► LUIS ESTRADA-GONZÁLEZ AND MANUEL TAPIA-NAVARRO, *When Curry met Abel*.

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Curry's paradox represents a problem for uniform approaches to self-referential paradoxes, as seemingly no negation is involved in it and triviality is reached without the explosion principle, unlike most of the other paradoxes. In particular, purely paraconsistent approaches will not serve to block or solve the paradox. Using some ideas from abstract algebra and Abelian logic, in this article we argue that the strategy of blocking Curry's paradox by rejecting Detachment can be seen as a generalization of the rejection of the explosion principle, and thus of the paraconsistent strategy. This would imply that a uniform approach to all the self-referential paradoxes, at least those where object-language connectives are involved, is possible.

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► DAVID FERNÁNDEZ-DUQUE, PAUL SHAFER, HENRY TOWNSNER, AND KEITA YOKOYAMA, *Caristi's fixed point theorem and strong systems of arithmetic*.

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A *Caristi system* is a triple (X, f, V) , where X is a complete metric space, $V: X \rightarrow (0, \infty)$ is a lower semicontinuous function, and $f: X \rightarrow X$ is an arbitrary function such that, for all $x \in X$,

$$d(x, f(x)) \leq V(x) - V(f(x)).$$

Caristi's fixed point theorem states that any Caristi system has a fixed point; that is, there is $x_* \in X$ such that $f(x_*) = x_*$. This has been proven in the literature using the *critical point theorem*, which states that V has a pseudo-minimal point, and using *Caristi sequences*, which are transfinite sequences $(x_\xi)_{\xi < \Omega} \subseteq X$ such that $x_{\xi+1} = f(x_\xi)$ for all ξ , the sequence converges at limit ordinals, and $\Omega \leq \omega_1$ is a large enough ordinal.

We analyze Caristi's theorem and its known proofs in the context of reverse mathematics, where metric spaces are assumed separable and coded in the standard way. Among the results obtained, we have that, over RCA_0 :

- WKL_0 is equivalent to Caristi's theorem restricted to compact spaces with continuous V .
- ACA_0 is equivalent to Caristi's theorem restricted to compact spaces with lower semicontinuous V .
- TLPP_0 (the Σ_α -relative leftmost path principle for every well-ordering α) is equivalent to Caristi's theorem for Baire or Borel f .
- $\Pi_1^1\text{-CA}_0$ is equivalent to the critical point theorem for lower semicontinuous functions.
- $\Pi_\omega^0\text{-IFP}_0$ (the arithmetical inflationary fixed point scheme) is equivalent to the statement that if f is arithmetically defined, any point $x_0 \in X$ can be extended to a Caristi sequence $(x_\xi)_{\xi < \Omega} \subseteq X$ containing a fixed point of f .

These theories are all defined over the language of second-order arithmetic and we mention them in strictly increasing order of strength. In order to formalize these results, we also develop techniques for coding lower semicontinuous functions in this setting.

- MICHAŁ TOMASZ GODZISZEWSKI AND JOEL DAVID HAMKINS, *Computable quotient presentations of models of arithmetic and set theory*.

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We prove various extensions of the Tennenbaum phenomenon to the case of computable quotient presentations of models of arithmetic and set theory. Specifically, no nonstandard model of arithmetic has a computable quotient presentation by a c.e. equivalence relation. No Σ_1 -sound nonstandard model of arithmetic has a computable quotient presentation by a c.e. equivalence relation. No nonstandard model of arithmetic in the language $\{+, \cdot, \leq\}$ has a computably enumerable quotient presentation by any equivalence relation of any complexity. No model of ZFC or even much weaker set theories has a computable quotient presentation by any equivalence relation of any complexity. And similarly no nonstandard model of finite set theory has a computable quotient presentation.

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- ▶ DANNY DE JESÚS GÓMEZ-RAMÍREZ, *Mathematics: a meta-isomorphic version of classic mathematics based on proper classes*.

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An implicit working principle in Von Neumann–Bernays–Gödel set theory (NBG) is that small classes (or ‘sets’) are more suitable objects to start and work with for developing a general foundational framework for standard mathematics. On the other hand, proper classes are just ‘too big’ and formally ‘too dangerous’ in order to be able to ground any classic mathematical theory.

In this work, we will mainly show that these typical quantitative considerations about proper and small classes are just tangential facts regarding the consistency of Zermelo–Fraenkel set theory with Choice (ZFC). Effectively, we will construct a first-order logic theory D-ZFC (Dual theory of ZFC) strictly based on (a particular subcollection of) proper classes with a corresponding special membership relation, such that ZFC and D-ZFC are meta-isomorphic frameworks. More specifically, for any standard formal definition, axiom and theorem that can be described and deduced in ZFC, there exists a corresponding ‘dual’ version in D-ZFC and vice versa.

Finally, we prove the metafact that (classic) mathematics (i.e., theories grounded on ZFC) and mathematics (i.e., dual theories grounded on D-ZFC) are meta-isomorphic, i.e., for any concept, theory and conjecture in (classic) mathematics there exists a symmetric d-concept, d-theory, and d-conjecture in mathematics with equivalent formal properties, and vice versa.

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- ▶ VALENTIN GORANKO, ANTTI KUUSISTO, AND RAINE RÖNNHOLM, *Compositional vs game-theoretic semantics for alternating-time temporal logics*.

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The Alternating-Time Temporal Logic ATL is a multi-agent extension of the branching-time temporal logic CTL and one of the most popular logical formalisms for reasoning about strategic abilities of agents in synchronous multi-agent systems. The semantics of ATL is defined over multi-agent transition systems, also known as *concurrent game models*, in which agents take simultaneous actions at the current state and the resulting collective action determines the subsequent transition to a successor state.

We have introduced in [1] versions of game-theoretic semantics (GTS) for ATL. In GTS, truth is defined in terms of existence of a winning strategy in a semantic evaluation game, and thus the game-theoretic perspective appears in the framework of ATL on two semantic levels: on the object level in the standard semantics of the strategic operators, and on the

metalevel where game-theoretic logical semantics is applied to ATL. We unify these two perspectives into semantic evaluation games specially designed for ATL. The game-theoretic perspective enables us to identify new variants of the semantics of ATL based on limiting the time resources available to the verifier and falsifier in the semantic evaluation game. We introduce and analyse an *unbounded* and (*ordinal*) *bounded* GTS and prove these to be equivalent to the standard (Tarski-style) compositional semantics. We show that in both versions of GTS, truth of ATL formulae can always be determined in finite time, i.e., without constructing infinite paths. We also introduce a nonequivalent *finitely bounded* semantics and argue that it is natural from both logical and game-theoretic perspectives. In [2] we extend the GTS for ATL to the richer language ATL^+ and apply it to identify a hierarchy of extensions of ATL with tractable model checking and to obtain some new results on expressiveness and complexity of model checking.

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- ▶ HENSON GRAVES, *Axiomatic toposes for descriptive modeling*.
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Engineers and scientists are reinventing topos constructions for their modeling languages. Modeling languages in the UML family have constructions for products, powers, as well as subtypes. These language constructions are incomplete and do not have any accepted formal semantics. However, together with special purpose sublanguages the engineering modeling languages are used to design and analyze complex systems. With an axiomatic semantics topos based modeling languages can serve as the foundation for a new generation of modeling language tools which integrate automated reasoning with simulation.

Axiomatic topos theory as developed by Lawvere with rule axioms for products and powers goes a long way to providing an axiomatic modeling language suitable for science and engineering. However, subobjects (subtypes) play an extensive role in system modeling. A constructive axiom for canonical subtypes is given to replace the traditional subobject classification axiom in the context of axiomatic Cartesian closed categories with powers. The axiom sets which use the language axioms are toposes with canonical subobjects which serves as a replacement for set theory as a modeling language. A descriptive model is an axiom set which includes the language axioms.

An aircraft flying over terrain can be modeled in this formalism using maps whose domain is linear time to types representing the aircraft, its components and interconnections. These maps are represented as sheaves on the algebra of subtypes of time. The sheaf maps represent the time evolution of a system with its components. This gives a point free algebraic representation. Time subtypes can be represented as subsets of the spectrum of time type. The interpretations of these models are strict logical functors to *Set*. This provides a formal basis for simulation correctness, as a simulation is an interpretation.

- ▶ LAURI HELLA AND MIIKKA VILANDER, *Formula size games for modal logics*.
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Succinctness is an important research topic that has been quite active in modal logic recently. If two logics \mathcal{L} and \mathcal{L}' have equal expressive power, it is natural to ask, whether there are properties that can be expressed in \mathcal{L} by a substantially shorter formula than in \mathcal{L}' .

One of the most common methods in the literature for proving lower bounds on the length of formulas expressing given properties is the *Adler–Immerman game* ([1]). We propose (see [2]) another type of formula size game for modal logic. In the Adler–Immerman game the players produce the whole syntax tree of the separating formula. In our game we use

parameters m and k referring to the number of modal operators and binary connectives in a formula, thus enabling a game where only a part of the separating formula is constructed in any single play.

We illustrate the use of our game by proving a nonelementary succinctness gap between first-order logic FO and modal logic ML. More precisely, we define a bisimulation invariant property of pointed Kripke models by a first-order formula of size $\mathcal{O}(2^n)$, and show that this property cannot be defined by any ML-formula of size less than the exponential tower of height $n - 1$.

We are currently working on an adaptation of our formula size game for the modal μ -calculus. Questions of succinctness and definability for the modal μ -calculus are largely unexplored and none of the other methods mentioned here have been used in this context. We intend to use our new game to investigate these questions.

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► KOICHIRO IKEDA, *A note on small stable theories*.

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A type $p \in S(T)$ is called special, if there are $a, b \models p$ such that $\text{tp}(b/a)$ is isolated and nonalgebraic, and $\text{tp}(a/b)$ is nonisolated. The Lachlan conjecture says that if there is no stable Ehrenfeucht theory. It can be seen that if there is a counterexample of the Lachlan conjecture then the theory has a special type. Modifying Hrushovski's generic pseudoplane [2], Herwig constructed a small stable theory with a type of infinite weight [1]. His example may be close to a counterexample of the Lachlan conjecture, but it does not have a special type. In this talk, I will introduce some result on a relation between generic structures and theories with a special type.

[1] B. HERWIG, *Weight ω in stable theories with few types*. *The Journal of Symbolic Logic*, vol. 60 (1995), pp. 353–373.

[2] E. HRUSHOVSKI, *A stable \aleph_0 -categorical pseudoplane*, preprint, 1988.

► MIRJANA ILIĆ, *A normalizing system of natural deduction for relevant logic*.

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Several natural deduction calculi are known for relevant logics, see Anderson and Belnap [1], Dunn [5], Brady [3], and Meyer and McRobbie [9]. Some of them are with the explicit distribution rule, such as Anderson–Belnap's and Meyer–McRobbie's, some of them have normalization theorems, such as Brady's, however, all of them, use a kind of relevance numerals in order to keep track of the use of hypotheses.

On the other hand, relevant numerals are not needed in sequent calculi of relevant logics, see e.g., Dunn [4,5], Minc [10], and Bimbo [2]. We formulate a natural deduction calculus, of a particular relevant logic, by defining the translation from its sequent calculus formulation into natural deduction. We consider the contraction-less relevant logic RW_+° and we take its sequent calculus GRW_+° , admitting cut-elimination, presented in [7]. The resulting natural deduction calculus is a normalizing natural deduction system, without explicit distribution rule and free from relevant numerals. Our translations from sequent to natural deduction calculus and vice versa are similar to Negri's translations between those calculi for intuitionistic linear logic [11]; however, due to the presence of two types of multisets of formulae, intensional and extensional ones, needed for the proof of the distribution of conjunction over disjunction in relevant logics, see Dunn [4] and Minc [10], our translations are significantly different from Negri's translations.

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► ASSYLBEK ISSAKHOV AND FARIZA RAKYMZHANKYZY, *Hyperimmunity and A-computable numberings*.

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Let \mathcal{F} be a family of total functions which is computable by an oracle A , where A is an arbitrary set. A numbering $\alpha : \omega \mapsto \mathcal{F}$ is called A -computable if the binary function $\alpha(n)(x)$ is A -computable. [1].

LEMMA 1. *Let \mathcal{F} be an infinite A -computable family of total functions, where A is an arbitrary set. Then \mathcal{F} has an A -computable Friedberg numbering.*

A degree a is hyperimmune if a contains a hyperimmune set, and a is hyperimmune free otherwise. Every nonzero degree comparable with $0'$ is hyperimmune. Dekker showed that for every nonrecursive c.e. set A there is a hyperimmune set B such that $B \equiv_T A$, which means that every nonrecursive c.e. degree contains a hyperimmune set.

LEMMA 2. *For every hyperimmune set A there exists a nonrecursive A -computable set B .*

It is known [2], that if A is an arbitrary set, \mathcal{F} is an infinite A -computable family of total functions and \mathcal{F} has at least two nonequivalent A -computable Friedberg numberings, then \mathcal{F} has infinitely many pairwise nonequivalent A -computable Friedberg numberings. And also [3], if \mathcal{F} is an infinite A -computable family of total functions, where $0' \leq_T A$, then \mathcal{F} has infinitely many pairwise nonequivalent A -computable Friedberg numberings.

We extend these results:

THEOREM 3. *Let \mathcal{F} be an infinite A -computable family of total functions, where A is a hyperimmune set. Then \mathcal{F} has infinitely many pairwise nonequivalent A -computable Friedberg numberings.*

Note that, [4], if an A -computable family \mathcal{F} of total functions contains at least two elements, where A is a hyperimmune set, then \mathcal{F} has no A -computable principal numbering.

THEOREM 4 (Issakhov). *Let \mathcal{F} be a finite A -computable family of total functions, where Turing degree of the set A is hyperimmune free. Then \mathcal{F} has an A -computable principal numbering.*

QUESTION. *Is it true the formulation of previous theorem for infinite family?*

The main talk will be around this question.

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- ▶ ERIC JOHANNESSON AND ANDERS LUNDSTEDT, *When one must strengthen one’s induction hypothesis*.

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Sometimes when trying to prove a fact by induction, one gets “stuck” at the induction step. The solution is often to use a “stronger” induction hypothesis, that is to prove a “stronger” result by induction. But in such cases, can we say that “strengthening the induction hypothesis” is necessary in order to prove the fact?

The general problem of when one must, in order to prove a fact X , first prove another fact Y , seems very hard. Interestingly, the special case of when one must strengthen one’s induction hypothesis turns out to be more manageable. We provide the following characterization of when one in fact must strengthen one’s induction hypothesis.

Let $\text{Th}(\mathcal{N})$ be the set of sentences of first-order arithmetic that are true in the standard model. Let $T \subseteq \text{Th}(\mathcal{N})$ and let $\varphi(x)$ and $\psi(x)$ be formulas both with at most one free variable x . Say that $\psi(x)$ witnesses that T proves $\forall x\varphi(x)$ with and only with strengthened induction hypothesis if and only if

- (1) $T \cup \{\varphi(0) \wedge \forall x (\varphi(x) \rightarrow \varphi(x + 1)) \rightarrow \forall x \varphi(x)\} \not\vdash \forall x \varphi(x)$,
- (2) $T \vdash \varphi(0)$,
- (3) $T \vdash \psi(0)$,
- (4) $T \vdash \forall x (\psi(x) \rightarrow \psi(x + 1))$,
- (5) $T \vdash \forall x \psi(x) \rightarrow \forall x \varphi(x)$.

We show that this definition applies to a number of natural examples. By reflecting on mathematical practice, we argue that this definition does capture the notion of “proof by strengthened induction hypothesis”.

- ▶ DIANA KABYLZHANOVA, *A note on computably enumerable preorders*.

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A preorder is a reflexive and transitive binary relation. We are interested in computably enumerable (c.e.) preorders, in particular, in weakly precomplete c.e. preorders, [2]. Let P and Q be c.e. preorders. We say that P is computably reducible to Q ($P \leq_c Q$) if there is a computable function f such that xPy iff $f(x)Qf(y)$ for every $x, y \in \omega$. A c.e. preorder P is light if there exists a c.e. preorder Q in which all classes are singletons such that $Q \leq_c P$, and c.e. preorder P is called dark if P is not light and has no computable classes, [1]. A c.e. preorder P is finite if P has a finite number of classes. We say that c.e. preorder P is weakly precomplete if for every total function φ_e there exist x_e such that $\varphi_e(x_e) \sim_P x_e$.

THEOREM 1. *Let P be a nonuniversal c.e. preorder. Then there exists a weakly precomplete, nonuniversal c.e. preorder Q , such that $P \leq_c Q$.*

THEOREM 2. *For every finite c.e. preorder P there are infinitely many minimal dark c.e. preorders P_d such that $P \leq_c P_d$.*

[1] U. ANDREWS and A. SORBI, *Joins and meets in the structure of ceers*, in preparation.

[2] S. BADAEV and A. SORBI, *Weakly precomplete computably enumerable equivalence relations*. *Mathematical Logic Quarterly*, vol. 62 (2016), no. 1–2, pp. 111–127.

- YECHIEL M. KIMCHI, *Partition relations equiconsistent with $o(o(\dots o(\kappa)\dots)) = 2$* .

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PREAMBLE. We try to associate the consistency strength of statements like $o(\kappa) = \alpha$ (for κ measurable) with various partition relations of the form $\kappa \rightarrow (\mu)_\lambda^\alpha$. Here, we restrict ourselves to partitions of the form $\aleph_1 \rightarrow (\omega^\alpha)_{\aleph_0}^{\omega^\alpha}$. Since we work under *ZFC*, the partition properties are limited to definable functions.

In [3], M. Spector proved that for $\alpha = 1$

$$CON(\exists \kappa (o(\kappa) = \alpha)) \iff CON(\aleph_1 \rightarrow (\omega^\alpha)_{\aleph_0}^{\omega^\alpha}).$$

In [1] we have shown that it can be generalized to $\alpha = 2$ only (which serves as the basis for the current presentation). In order to resurrect the nice equiconsistency we defined the notion of *weak-homogeneity*, and recently, in [2], we extended the result to

$$CON(\exists \kappa (o(\kappa) = \kappa^+)) \iff CON(\aleph_1 \xrightarrow{WH} (\aleph_1)_{\aleph_0}^{\aleph_1}).$$

The failure of the original equiconsistency for $\alpha = 3$, lead us in the past to prove

$$CON(\aleph_1 \rightarrow (\omega^3)_{\aleph_0}^{\omega^3}) \iff CON(\exists \kappa (o(o(\kappa)) = 2)).$$

In this presentation we extend the latter for all $\alpha < \omega$, and for that we need two simple definitions. The first one is just notational: $\kappa \xrightarrow{cl} (\mu)_\lambda^\alpha$ means that both the homogeneous sequence of o.t. μ and the sequences of o.t. α in the domain of the functions, are restricted to closed sequences. The second iterates the $o(\kappa)$ function:

DEFINITION. $o^n(\mu)$ is defined by induction on $n \in \omega$ for any ordinal μ :

- (i) $o^0(\mu) = \mu$,
- (ii) $o^{n+1}(\mu) = o(o^n(\mu))$.

We are now able to state the following two related theorems:

THEOREM 1. For any $n \in \omega (n \geq 2)$, $CON(\aleph_1 \xrightarrow{cl} (\omega^n)_{\aleph_0}^{\omega^n}) \iff CON(\exists \kappa (o^n(\kappa) = 2))$.

THEOREM 2. For $n \in \omega (n \geq 1)$, $CON(\aleph_1 \rightarrow (\omega^{n+1})_{\aleph_0}^{\omega^{n+1}}) \iff CON(\exists \kappa (o^n(\kappa) = 2))$.

Note 1: The new result is the forward direction (from left to right).

Note 2: The exact consistency strength of the statement $\aleph_1 \rightarrow (\omega^\omega)_{\aleph_0}^{\omega^\omega}$ is still not known. All we know (cf. [1]) is that it implies the consistency of the statement $\exists \kappa (o(\kappa) > \kappa)$, witnessing yet another jump in the relationship between partition properties and measurable cardinals.

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- PHOKION G. KOLAITIS, *Schema mappings: structural properties and limits*.

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A schema mapping is a high-level specification of the relationship between two database schemas. For the past fifteen years, schema mappings have played an essential role in the modeling and analysis of important data interoperability tasks, such as data exchange and data integration. Syntactically, schema mappings are expressed in some schema-mapping language, which, typically, is a fragment of first-order logic or second-order logic. In the first part of the talk, we will introduce the main schema-mapping languages, will discuss the fundamental structural properties of these languages, and will then use these structural

properties to obtain characterizations of various schema-mapping languages in the spirit of abstract model theory. In the second part of the talk, we will examine schema mappings from a dynamic viewpoint by considering sequences of schema mappings and studying the convergence properties of such sequences. To this effect, we will introduce a metric space that is based on a natural notion of distance between sets of database instances and will investigate pointwise limits and uniform limits of sequences of schema mappings. Among other findings, it will turn out that the completion of this metric space can be described in terms of graph limits arising from converging sequences of homomorphism densities.

- ▶ ANGELIKI KOUTSOUKOU-ARGYRAKI, *An invitation to proof mining: two applications in nonlinear operator theory.*

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The revival of Kreisel's program of *unwinding of proofs* by Kohlenbach as *proof mining* has been very fruitful for applications in many mathematical disciplines, especially within analysis. The scope of the program is the extraction of constructive information (e.g., computable bounds) from nonconstructive mathematical proofs. This can be a priori guaranteed by certain logical metatheorems. The quantitative content emerges through the discovery of quantifiers that were implicit in the original proof. The bounds obtained are explicit, highly uniform and of low complexity. We present here: (i) Bounds extracted for the computation of approximate common fixed points of one-parameter nonexpansive semigroups on a subset of a Banach space, obtained via proof mining on a proof by Suzuki. The bounds differ from those that had been obtained in [1] via proof mining on a completely different proof by Suzuki of a generalised version of the studied statement. (ii) Computable rates for the convergence of the resolvents of set-valued operators on a real Banach space that fulfill certain accretivity conditions to the zero of each operator, that were extracted via proof mining on a proof by García-Falset. The above results are, among others, included in [2] and can be of interest for optimization theory.

[1] U. KOHLENBACH and A. KOUTSOUKOU-ARGYRAKI, *Effective asymptotic regularity for one-parameter nonexpansive semigroups.* *Journal of Mathematical Analysis and Applications*, vol. 433 (2016), no. 2, pp. 1883–1903.

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- ▶ BEIBUT KULPESHOV AND SERGEY SUDOPLATOV, *On distributions for countable models of quite o-minimal theories with nonmaximum many countable models.*

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Quite o-minimal theories (which were introduced in [1]) form a subclass of the class of weakly o-minimal theories preserving a series of properties of o-minimal theories. Using structural results on quite o-minimal Ehrenfeucht theories and solving the Vaught's conjecture [2] similar to [3], a general approach to the classification of countable models of complete theories [4] is applied to the class of quite o-minimal theories with nonmaximum many countable models.

We use the following theorem and the general decomposition formula [4] for the number $I(T, \omega)$ of countable models of theory T , the finite Rudin–Keisler preorder $\text{RK}(T)$ of almost prime models of T , and the distribution function IL of limit models with respect to $\text{RK}(T)$:

$$I(T, \omega) = |\text{RK}(T)| + \sum_{i=0}^{|\text{RK}(T)/\sim_{\text{RK}}|-1} \text{IL}(\widetilde{M}_i). \tag{1}$$

THEOREM 1 ([2]). *Let T be a quite o-minimal theory in a countable language. Then either T has 2^ω countable models or T has exactly $3^k \cdot 6^s$ countable models, where k and s are natural numbers. Moreover, for any $k, s \in \omega$ there is a quite o-minimal theory T with exactly $3^k \cdot 6^s$ countable models.*

The Rudin–Keisler preorders $\text{RK}(T)$ as well as the distribution functions IL are described for quite o-minimal theories T with nonmaximum many countable models. The decomposition formula (1) is represented in the following form:

$$3^k \cdot 6^s = 2^k \cdot 3^s + \sum_{t=0}^k \sum_{m=0}^s 2^{s-m} \cdot (2^t \cdot 4^m - 1) \cdot C_k^t \cdot C_s^m.$$

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[2] B. S. KULPESHOV and S. V. SUDOPLATOV, *Vaught’s conjecture for quite o-minimal theories*, *Annals of Pure and Applied Logic*, vol. 168 (2017), no. 1, pp. 129–149.

[3] L. L. MAYER, *Vaught’s conjecture for o-minimal theories*, *The Journal of Symbolic Logic*, vol. 53 (1988), no. 1, pp. 146–159.

[4] S. V. SUDOPLATOV, *Classification of Countable Models of Complete Theories*, Novosibirsk, Edition of NSTU, 2014.

► **TAISHI KURAHASHI**, *Two theorems on provability logics*.

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We say that a formula $\tau(v)$ is a numeration of a theory T if $\{n \in \omega : \text{PA} \vdash \tau(\bar{n})\}$ is exactly the set of all Gödel numbers of the axioms of T . For each numeration $\tau(v)$ of T , the provability predicate $\text{Pr}_\tau(x)$ of T is naturally constructed. An arithmetical interpretation f is a mapping from the set of all propositional variables to the set of sentences of arithmetic. Each arithmetical interpretation f is uniquely extended to the mapping f_τ from the set of all modal formulas to the set of sentences of arithmetic so that f_τ commutes with every propositional connective, and $f_\tau(\Box A)$ is $\text{Pr}_\tau(\ulcorner f_\tau(A) \urcorner)$. The provability logic $\text{PL}_\tau(U)$ of $\tau(v)$ relative to a theory U is the set $\{A : U \vdash f_\tau(A)\}$ for all arithmetical interpretations f of modal formulas (see [1,2]).

We proved the following two theorems.

THEOREM 1. *Let U be any recursively axiomatized consistent extension of PA. If L is one of the logics GL_α , D_β , S_β and GL_β^- where $\alpha \subseteq \omega$ is recursively enumerable and $\beta \subseteq \omega$ is cofinite, then there exists a Σ_1 numeration $\tau(v)$ of some extension of IS_1 such that $\text{PL}_\tau(U)$ is exactly L .*

THEOREM 2. *Let T be any recursively axiomatized consistent extension of PA. If L is one of the logics K and $\text{K} + \Box(\Box^n p \rightarrow p) \rightarrow \Box p$ ($n \geq 2$), then there exists a Σ_2 numeration $\tau(v)$ of T such that $\text{PL}_\tau(T)$ is exactly L .*

The logics $\text{K} + \Box(\Box^n p \rightarrow p) \rightarrow \Box p$ ($n \geq 2$) were introduced by Sacchetti [3].

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► **MICHAEL LIEBERMAN, JIŘÍ ROSICKÝ, AND SEBASTIEN VASEY**, *Set-theoretic pathologies in accessible categories*.

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Recent work in abstract model theory (see [2,3,4]) has highlighted the highly desirable properties of abstract classes under large cardinals axioms, chiefly the assumption of a proper class of strongly (or almost strongly) compact cardinals. There are parallel results for accessible categories (see [5,6]), in addition to earlier work of [1] concerning Vopěnka’s Principle. We here consider the other end of the spectrum: pathological behavior of accessible categories assuming that there is only a set of measurable cardinals or, indeed, that $V = L$. The pathological examples, which are built directly out of the cumulative set-theoretic hierarchies, include the non-co-well-powered accessible category considered in [1] and [7], as well as an example tucked away in [8], which we have newly adapted to this context.

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[2] J. BALDWIN and W. BONEY, *Hanf numbers and presentation theorems in AECs*, *Beyond First Order Model Theory* (J. Iovino, editor), CRC Press, 2017, to appear.

[3] W. BONEY, *Tameness from large cardinal axioms*. *The Journal of Symbolic Logic*, vol. 163 (2012), pp. 2008–2017.

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[7] M. MAKKAI and R. PARÉ, *Accessible Categories: The Foundations of Categorical Model Theory*, Contemporary Mathematics, vol. 104, AMS, 1989.

[8] S. SHELAH, *Model theory for a compact cardinal*, arXiv:1303.5247v3.

- ROUSSANKA LOUKANOVA, *Type Theory of Restricted Algorithms and Neural Networks*. Department of Mathematics, Stockholm University, Sweden.
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Moschovakis [1] introduced a new approach to the mathematical concept of algorithm. In [2], he extended the approach to typed acyclic recursion, by a formal language L_{ar}^λ equipped with a reduction calculus. The theory L_{ar}^λ represents crucial semantic distinctions in formal and natural languages. We present our development of L_{ar}^λ to Type Theory of Restricted Algorithms (TTofRAI g), as a mathematical theory of the notion of algorithm, by adding a restrictor as an operator. The purpose is to model procedural memory and functionality of biological entities, in particular neurons and neural networks.

Like L_{ar}^λ , TTofRAI g has two kinds of typed variables: *pure variables*, for λ -abstraction operator and *memory (recursion) variables*, for storing information. The terms of TTofRAI g are generated by the rules:

$$A ::= c^\tau : \tau \mid x^\tau : \tau \mid B^{(\sigma \rightarrow \tau)}(C^\sigma) : \tau \mid \lambda(v^\sigma)(B^\tau) : (\sigma \rightarrow \tau) \tag{1a}$$

$$\mid (A_0^{\sigma_0} \text{ where } \{p_1^{\sigma_1} := A_1^{\sigma_1}, \dots, p_n^{\sigma_n} := A_n^{\sigma_n}\}) : \sigma_0 \tag{1b}$$

$$\mid (A_0^{\sigma_0} \text{ such that } \{C_1^{\tau_1}, \dots, C_m^{\tau_m}\}) : \sigma_0, \tag{1c}$$

given that c is a constant, x is a variable of ether kind, and p_i , are recursion variables of respective types, and each τ_i is either the type t of truth values, or the type \tilde{t} of state dependent truth values.

A *recursion term* A of the form (1b) designates a recursor, i.e., an algorithm for computing the denotation of A . A term A of the form (1c) designates a restrictor that constrains the denotation of A with constraints $C_1^{\tau_1}, \dots, C_m^{\tau_m}$.

Reduction calculus. We introduce a reduction calculus of TTofRAI g , which extends the reduction system of L_{ar}^λ . Each term has a unique, up to congruence, canonical form. The recursion terms in canonical forms represent algorithms for mutually recursive computations, which, in addition, can be restricted by constraints of the form (1c). Assignments

of terms to memory variables in recursion terms (1b) represent saving objects and outcomes of computations in memory cells. Semantically, the memory variables, which occur in a TToFRAIlg term, represent memory cells of a computational entity, which are engaged in algorithmic computations. The subclass of TToFRAIlg, which is limited to recursion terms (1b) with acyclic assignments, represents acyclic algorithms that always end their computations.

Neural networks. Memory cells in specialised assemblies can establish networks of memory cells. A formal language of functional neural nets (NNets) is a specialised version of the language TToFRAIlg. We define terms designating *neural nets* as complex units of restricted memory variables and terms. A neural net consists of memory components, which are restricted simultaneously by complex constraints, and can involve recursive computations.

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[2] ———, *A logical calculus of meaning and synonymy*, *Linguistics and Philosophy*, vol. 29, pp. 27–89.

- ▶ ROBERT LUBARSKY, *Determinacy of Boolean combinations of Σ_3^0 games*. Department of Mathematical Sciences, Florida Atlantic University, 777 Glades Rd., Boca Raton, FL 33431, USA.
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Welch characterized the ordinal at which winning strategies for all Σ_3^0 games appear, via Σ_2 reflection; namely, it is the least ordinal which is the ordinal standard part of a nonstandard model which has an infinite nested sequence of pairs of ordinals, the smaller of which is a Σ_2 substructure of the larger. This reflection property is strictly between Σ_2 admissibility and Σ_2 nonprojectibility. Montalban and Shore showed that this is the beginning of a hierarchy, in that the least ordinal for winning strategies for all games which are alternating differences of m -many Σ_3^0 sets is strictly between the least $m + 1$ -admissible and $m + 1$ -nonprojectible. Here we show the straightforward generalization of Welch's result, that this ordinal is the least standard part of a model with an infinite nesting of Σ_{m+1} -elementary pairs. This talk will be an introduction to the subject.

[1] A. MONTALBAN and R. SHORE, *The limits of determinacy in second order arithmetic*, *Proceedings of the London Mathematical Society*, vol. 104 (2012), pp. 223–252.

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[3] P. WELCH, *Weak systems of determinacy and arithmetical quasi-inductive definitions*, *The Journal of Symbolic Logic*, vol. 76 (2011), pp. 418–436.

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A linear order L is *strongly surjective* if there exists an order preserving surjection from L onto each of its suborders. For example, an ordinal is strongly surjective if and only if it is of the form $\omega^\alpha m$, for some $\alpha < \omega_1$ and $m > 0$.

Our main result is that the set StS of countable strongly surjective linear orders is a $\check{D}_2(\Pi_1^1)$ -complete set. This means that StS is the union of an analytic and a coanalytic set, and is complete for the class of sets that can be written in this way. More in detail, we show that the countable strongly surjective linear orders which are scattered form a Π_1^1 -complete set, while the countable strongly surjective linear orders which are not scattered form a Σ_1^1 -complete set. Our proof of the upper bound for scattered strongly surjective orders makes an essential use of both effective descriptive set theory and the fact that order preserving surjections well quasi-order the countable linear orders ([1,3]).

Even if the study of the first two levels of the projective hierarchy is a long-standing topic, examples of sets that are true Δ_2^1 are very rare. In fact, as far as we know, StS is the first concrete example of a “natural” $\check{D}_2(\Pi_1^1)$ -complete set.

If time permits, I'll also discuss uncountable strongly surjective linear orders. We can prove their existence under either PFA or \diamond^+ , while the provability in ZFC of the existence of these orders is an interesting open problem.

This is joint work with Riccardo Camerlo and Raphaël Carroy ([2]).

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[2] ———, *Linear orders: When embeddability and epimorphism agree*, arXiv:1701.02020.

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- ▶ JUAN CARLOS MARTÍNEZ, *On pcf spaces which are not Fréchet–Urysohn*. Faculty of Mathematics, University of Barcelona, 08007 Barcelona, Spain.
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An *admissible poset* is a triple $\langle T, \prec, i \rangle$ such that T is a nonempty set, \prec is a well-founded ordering on T and $i: [T]^2 \rightarrow [T]^{<\omega}$ satisfying the following two properties:

- (1) For all $u, s, t \in T$, $u \preceq s$ and $u \preceq t$ iff $u \preceq v$ for some $v \in i\{s, t\}$.
- (2) For all $t \in T$ and all α less than the \prec -rank of t , $\{s \in T: s \prec t\} \cap \{s \in T: \text{rank}(s) = \alpha\}$ is infinite.

An admissible poset $\langle T, \prec, i \rangle$ has associated with it a locally compact, Hausdorff and scattered space X of underlying set T whose basic open sets are of the form $b_t \setminus (b_{u_0} \cup \dots \cup b_{u_n})$, where $b_t = \{s \in T: s \preceq t\}$ for each $t \in T$. If Y is a subset of T , \overline{Y} denotes the closure of Y in X .

A *pcf structure* is an admissible poset $\langle \theta + 1, \prec, i \rangle$ where θ is an infinite ordinal such that the following conditions are satisfied:

- (PCF1) If $v \prec \mu$ then $v \in \mu$.
- (PCF2) $\overline{\theta} = \theta + 1$.
- (PCF3) If $I \subseteq \theta + 1$ is an interval, then \overline{I} is also an interval.
- (PCF4) $\xi \prec \theta$ for every $\xi \in \theta$.
- (PCF5) For each $v \in \theta$ of uncountable cofinality there is a club C_v of v such that $\overline{C_v} \subseteq v + 1$.

The compact, Hausdorff, scattered space X associated with a pcf structure is called a *pcf space*, whose *height* is defined as the least ordinal α such that the α th Cantor–Bendixson level of X is empty. In [1], it was shown by means of a forcing argument that if CH holds then there is a pcf space of height $\omega_1 + 1$ which is not Fréchet–Urysohn, answering in a partial way a question posed by Todorćević. Then, we will give here a simpler proof of Pereira’s theorem by means of a forcing-free argument and we will extend his result to pcf spaces of any height $\delta + 1$ where $\delta < \omega_2$ with $\text{cf}(\delta) = \omega_1$.

[1] L. PEREIRA, *Applications of the topological representation of the pcf-structure*. **Archive for Mathematical Logic**, vol. 47 (2008), no. 5, pp. 517–527.

- ▶ JOSÉ M. MÉNDEZ, GEMMA ROBLES, SANDRA M. LÓPEZ, AND MARCOS M. RECIO, *Belnap–Dunn semantics for natural implicative expansions of Kleene’s strong three-valued matrix*.
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Belnap–Dunn type bivalent semantics is the semantics originally defined for interpreting Anderson and Belnap’s “First Degree Entailment Logic” (cf. [1] and references therein). On the other hand, the notion of a “natural implication” is understood as it is defined in [2]. According to this notion, there are exactly 24 natural implicative expansions of Kleene’s strong three-valued matrix with 1 and 1/2 as designated values. Some of these expansions characterize interesting logics such as paraconsistent expansions of the three-valued extensions of the positive fragments of Lewis’ S5 and three-valued Gödel logic G3.

The aim of this article is to define a Belnap–Dunn type bivalent semantics for the logics determined by each one of these 24 implicative expansions.

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[1] A. R. ANDERSON and N. D. BELNAP, Jr., *Entailment. The Logic of Relevance and Necessity, vol. 1*, Princeton University Press, 1975.

[2] N. TOMOVA, *A Lattice of implicative extensions of regular Kleene’s logics. Reports on Mathematical Logic*, vol. 47 (2012), pp. 173–182.

- ▶ RUSSELL MILLER, *Topology of isomorphism types of countable structures*. Mathematics Dept., Queens College & CUNY Graduate Center, 65-30 Kissena Blvd, Queens NY 11367, USA.

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Let \mathfrak{C} be a class of countable structures, closed under isomorphism. The collection of all members of \mathfrak{C} with domain ω forms a subspace of Cantor space: the atomic diagram of each structure becomes a subset of ω , using a Gödel coding of the atomic formulas in the language of \mathfrak{C} with extra constants from ω . We give this space the subspace topology, and then endow the quotient space $I(\mathfrak{C}) = \mathfrak{C}/\cong$, under the relation of isomorphism, with the quotient topology. The result is that we view the isomorphism types of elements of \mathfrak{C} as elements of this topological space $I(\mathfrak{C})$.

The isomorphism relation on \mathfrak{C} often resembles various of the well-known Borel equivalence relations on either Cantor space 2^ω or Baire space ω^ω . Determining which Borel equivalence relations yield spaces homeomorphic to $I(\mathfrak{C})$ requires the use of techniques from computable structure theory, along with reductions of the sort used in Borel reducibility, only stronger. These reductions may be regarded as type-2 computable functions. Often the main goal is to determine which definable relations on the members of \mathfrak{C} , if added to the language, turn $I(\mathfrak{C})$ into a recognizable space: when this happens, we may say that the elements of \mathfrak{C} are *classified* up to isomorphism by the members of the recognizable space.

The talk will consist largely of examples of these phenomena, mostly using classes in which isomorphism is an arithmetic relation, such as algebraic fields, finite-valence graphs, torsion-free abelian groups, and equivalence structures.

- ▶ RYSZARD MIREK, *Euclidean geometry in Renaissance*. Department of Philosophy, Pedagogical University of Krakow, Podchorazych 1, Poland.
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In Euclidean Elements in Book IV, Proposition 16, one can find how to inscribe an equilateral and equiangular fifteen-angled figure in a given circle. This construction was used both in theoretical and practical terms by Piero della Francesca. For instance in the setting of his painting *Baptism of Christ* one can find the first part of the construction. In the top side of the rectangle we construct an equilateral triangle, and we find that its apex falls at the point where the central vertical axis passes through the tip of Christ’s right foot. Then we locate the

center of the triangle and find it to be precisely at the fingertips of Christ's hands in prayer. In this way it is possible to set the center point of the painting. The result can be combined with Proposition 1.13 of his *De Prospectiva Pingendi*. In the second part of the treatise one can find more geometrical problems and theorems that have obvious relevance to Piero's work as a painter. There are problems of drawing a combination of prisms (Proposition 2.6), a beam of octagonal cross-section, lying on the ground plane (2.8), of drawing a cross-vaulted structure with a square ground plane (2.11).

My goal here is to describe the advanced geometrical exercises presented in the form of propositions. The treatise of Piero della Francesca is manifestation of a union of the fine arts and the mathematical sciences of arithmetic and geometry. The proofs of propositions are presented both in geometrical and mathematical form but from a logical point of view it is proposed by me a method of natural deduction that takes into account the importance of diagrams within formal proofs.

- ▶ ALIREZA MOFIDI, *Some VC-combinatorial aspects of definable set systems*.
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Several aspects of interactions between combinatorial features of definable set systems and model theoretic properties of them have been explored in different studies in recent years such as [1,2,3,4], etc. For example many connections between notions of VC-dimension, VC-density, (p,q)-theorems and compression schemes from combinatorial sides and NIP, forking and UDIFS from model theoretic side has been studied. Also some VC-combinatorial invariants are defined in [5]. We will talk about some further developments in these directions. We consider several new combinatorial assumptions on definable set systems, in particular some properties with an extremal combinatorial nature, and then explore their model theoretic impacts for example on complexities in stability hierarchy, spaces of types, etc. We also give several examples in each case. Meanwhile, we give characterizations of some stability theoretic dividing lines in terms of such combinatorial properties.

[1] M. ASCHENBRENNER, A. DOLICH, D. HASKELL, D. MACPHERSON, and S. STARCHENKO, *Vapnik–Chervonenkis density in some theories without the independence property I*. *Transactions of the American Mathematical Society*, vol. 368 (2016), no. 8, pp. 5889–5949.

[2] A. CHERNIKOV and P. SIMON, *Externally definable sets and dependent pairs II*. *Transactions of the American Mathematical Society*, vol. 367 (2015), pp. 5217–5235.

[3] V. GUINGONA and C. HILL, *On Vapnik–Chervonenkis density over indiscernible sequences*. *Mathematical Logic Quarterly*, vol. 1–2 (2014), pp. 59–65.

[4] H. JOHNSON, *Vapnik–Chervonenkis density on indiscernible sequences, stability, and the maximum property*. *Notre Dame Journal of Formal Logic*, vol. 56 (2015), no. 4, pp. 583–593.

[5] A. MOFIDI, *On some dynamical aspects of NIP theories*. *Archive for Mathematical Logic*, to appear.

- ▶ ANTONIO MONTALBAN AND JAMES WALSH, *Canonical aspects of reflection principles*.
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It is a well known empirical phenomenon that natural axiomatic theories are well-ordered by their consistency strength. One expression of this phenomenon comes from ordinal analysis, a research program whereby recursive ordinals are assigned to theories as a measurement of their consistency strength. One method for calculating the proof-theoretic ordinal of a

theory T involves demonstrating that T can be approximated over a weak base theory by reflection principles, such as consistency statements and their generalizations [1,2]. Why are natural theories amenable to such analysis? Fixing a base theory T that interprets elementary arithmetic, we study recursive monotonic functions on the Lindenbaum algebra of T . In this talk we discuss some results that demonstrate that consistency and other reflection principles are canonical among such functions. We also discuss how these results address our motivating questions.

[1] L. BEKLEMISHEV, *Proof-theoretic analysis by iterated reflection*. *Archive for Mathematical Logic*, vol. 6 (2003), no. 42, pp. 515–552.

[2] ———, *Provability algebras and proof-theoretic ordinals I*. *Annals of Pure and Applied Logic*, vol. 128 (2004), no. 1–3, pp. 103–123.

► JOACHIM MUELLER-THEYS, *On the provability of consistency*.

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A consistency sentence $\text{Con}_\Sigma^c := \neg \text{Prov}_\Sigma(\ulcorner \sigma \urcorner)$ states in the standard model that the decidable system Σ is consistent, viz. $\mathcal{N} \models \text{Con}_\Sigma^c$ iff $\Sigma \not\vdash \perp$. We showed at [1] that this is the case if $\Sigma \vdash \neg \sigma$ or $\Sigma \not\vdash \sigma$. So Gödel's Con_Σ is a consistency sentence indeed. By Löb's Theorem, $\Sigma \not\vdash \text{Con}_\Sigma^c$ if $\Sigma \vdash \text{PA}$ is consistent.

We have recently found an alternative consistency sentence, the unprovability of which can be shown much more easily and already for consistent $\Sigma \vdash Q$. The proof exploits that the provability predicate does *not* negatively represent Σ in itself, viz. there are σ_B such that $\Sigma \not\vdash \sigma_B$, but *non* $\Sigma \vdash \neg \text{Prov}_\Sigma(\ulcorner \sigma_B \urcorner)$, whence $\text{Con}'_\Sigma := \text{Con}_{\Sigma}^{\sigma_B}$ already does the job. [1]

Specifying a remark of Evgeny I. Gordon during LC '15, such *negative* consistency sentences do not show the unprovability of consistency *in general*; they only show the unprovability of consistency *by them*. Accordingly, there might be *positive* consistency sentences, which would—by the analogous argument—prove the consistency of Σ in Σ . If $\Sigma \not\vdash \sigma$ and $\Sigma \vdash \neg \text{Prov}_\Sigma(\ulcorner \sigma \urcorner)$, Con_Σ^c is a positive consistency sentence; and *total* negative self-irrepresentability seems to be unnatural and unlikely.

In search for suchlike sentences, we realised that $\Sigma \not\vdash \text{Con}_\Sigma^c$ for all $\Sigma \vdash \neg \sigma$, and, subsequently, that the required $\Sigma \not\vdash \text{Con}_\Sigma^c$ implies $\Sigma \not\vdash \text{Con}_\Sigma^c$ can be proven without any precondition on σ . This has the incredible consequence that $\Sigma \not\vdash \neg \text{Prov}_\Sigma(\ulcorner \sigma \urcorner)$ for all σ . In particular, all consistency sentences are negative. It follows either that there is *no* Con_Σ^c stating in the theory of Σ that Σ is consistent.

Note. We obtained the theorem first in a more complicated and less general way by $\neg \Box p \notin \text{GL}$ (which we had gained from a lemma for [2]) and Solovay's Theorem.

[1] J. MUELLER-THEYS, *Defining & simplifying Gödel's 2nd incompleteness theorem*, *ASL 2017 Spring Meeting*, Seattle.

[2] ———, *On uniform substitution*, this Bulletin, vol. 20 (2014), pp. 264–5.

► RAJA NATARAJAN, *Diagrammatic reasoning for Boolean equations*.

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Diagrammatic approaches to deductive and formal reasoning [1,2] have seen a resurgence in recent years. We propose a diagrammatic method for deciding whether Boolean equations over set-valued variables are tautologies or not. Conventional diagrammatic approaches to the above decision problem work reasonably well when the total number of sets under consideration is rather small. However, conventional approaches become cumbersome, if not completely unusable, while dealing with a large number of sets. We devise an algorithm for the above decision problem, and demonstrate that it scales well when the number of set variables in the equations increases rapidly.

[1] M. GARDNER, *Logic Machines and Diagrams*, second ed., The University of Chicago Press, 1982.

[2] D. D. ROBERTS, *The Existential Graphs of Charles S. Peirce*, Mouton & Co. N.V., Publishers, The Hague, 1973.

► ITAY NEEMAN AND ZACH NORWOOD, *Happy and mad families*.

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In 2015, Törnquist [4] answered an old question of Mathias [1] by showing that there are no infinite mad families in the Solovay model. Mathias's original article explores a connection between mad families and the H -Ramsey property for H a happy family, but Törnquist's proof is purely combinatorial and does not exploit this connection. We prove the following theorem: in the Solovay model, every $X \subseteq [\omega]^\omega$ is H -Ramsey for every happy family H that also belongs to the Solovay model. This gives a new proof of Törnquist's theorem.

Törnquist also asked whether the Axiom of Determinacy (AD) implies that there are no infinite mad families. Using a new generic absoluteness result that builds on the absoluteness results of [3], we show how to give a positive answer under AD^+ , a well-studied strengthening of AD. (It is open whether AD and AD^+ are equivalent.) In fact, we show that under AD^+ every $X \subseteq [\omega]^\omega$ is H -Ramsey for every happy family H .

[1] A. R. D. MATHIAS, *Happy families*. *Annals of Mathematical Logic*, vol. 12 (1997), no. 1, pp. 59–111.

[2] I. NEEMAN and Z. NORWOOD, *Happy and MAD families in $L(\mathbb{R})$* , submitted.

[3] I. NEEMAN and J. ZAPLETAL, *Proper forcing and $L(\mathbb{R})$* . *The Journal of Symbolic Logic*, vol. 66 (2001), no. 2, pp. 801–810.

[4] A. TÖRNQUIST, *Definability and almost disjoint families*, submitted.

► VLADISLAV NENCHEV, *Definability between temporal relations in dynamic mereology*.

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This article explores definability dependencies between temporal and spatio-temporal relations in some dynamic mereological systems. These systems are part of a point-free approach to spatial and temporal theories. The approach in question describes space and time in terms of “regions”, which are tangible and/or regular parts of space or time (“periods” or “epochs” may be used for parts of time). The point-free theories forgo standard Euclidean notions like “point” or “line”, arguing that such objects are abstract and do not exist in reality. Space and time are built, instead, on regions, while points and lines are complex constructs of specific sets of regions (see [1] and [2] for recent studies in this area).

The current studies compare three types of systems, which are different types of dynamic spatio-temporal structures. The first two types are mereological reducts of dynamic structures from [2]: *Dynamic Mereological Algebras* (DMAs) are algebraic structures that use products of Boolean algebras to track changes in space and time, while *rich Dynamic Mereological Algebras* are a specific kind of DMAs that include special spatio-temporal regions, called “time representatives”. The third type of structures is the relational variants of DMAs from [1] that have much weaker language and conditions on their domains. All of these systems include the following four dynamic relations: *unstable part-of* (a dynamic region is sometimes part of another dynamic region), *stable overlap* (a dynamic region always overlaps with another), *stable underlap* (a pair of regions always do not exhaust the whole space), and *temporal contact* (a pair of regions exist simultaneously at some point).

The results in this article show that in rich DMAs all of the four relations are equivalent (each of them can define the other three), in general DMAs the first three are equivalent, while the temporal contact is independent, and in relational DMAs all four relations are completely independent from each other.

[1] V. NENCHEV, *Logics for stable and unstable mereological relations*. *Central European Journal of Mathematics*, vol. 9 (2011), no. 6, pp. 1354–1379.

[2] D. VAKARELOV, *Dynamic mereotopology. III. Whiteheadian type of integrated point-free theories of space and time. Part II*. *Algebra and Logic*, vol. 55 (2016), no. 1, pp. 9–23.

- ▶ TAHSIN ONER AND IBRAHIM SENTURK, *An analysis of Peterson's intermediate syllogisms with Carroll's diagrammatic method*.

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In this work, our purpose is to analyze the Peterson's Intermediate Syllogisms by means of Carroll's diagrammatic method. For this aim, we first construct a formal system PISLCD (Peterson's Intermediate Syllogistic Logic with Carroll Diagrams), which gives us a formal approach to logical reasoning with diagrams, for representations of the fundamental Intermediate propositions and show that they are closed under the intermediate syllogistic criterion of inference which is the deletion of middle term. Therefore, it is implemented to let the formalism comprise synchronically bilateral and trilateral diagrammatical appearance and a naive algorithmic nature. And also, there is no specific knowledge or exclusive ability is needed in order to understand it and use it.

In other respects, we examine algebraic properties of Peterson's intermediate syllogisms in PISLCD. To this end, we explain quantitative relation between two terms by means of bilateral diagrams. Thereupon, we enter the data, which are taken from bilateral diagrams, on the trilateral diagram. With the help of elimination method, we obtain a conclusion which is transformed from trilateral to bilateral diagram. A Peterson's intermediate syllogistic system consists of 4000 syllogistic moods. 105 of them are valid forms.

Finally, we show that syllogism is valid if and only if it is provable in PISLCD. This means that PISLCD is sound and complete.

[1] L. CAROLL, *Symbolic Logic* (C. N. Potter, editor), 1896.

[2] R. L. CIGNOLI, I. M. D'OTTAVIANO, and D. MUNDICI, *Algebraic Foundations of Many-Valued Reasoning*, Springer Science & Business Media, 2013.

[3] A. E. KULINKOVICH, *Algorithmization of reasoning in solving geological problems*, *Proceedings of the Methodology of Geographical Sciences*, Naukova Dumka, 1979, pp. 145–161.

[4] J. ŁUKASIEWICZ, *Aristotle's Syllogistic from the Standpoint of Modern Logic*, Clarendon Press, Oxford, 1951.

[5] U. NURIYEV, T. ONER, and I. SENTURK, *An algebraic approach to categorical syllogisms by using bilateral diagrams*, *Theoretical and Applied Aspects of Cybernetics. Proceedings of the 5th International Scientific Conference of Students and Young Scientists*, Kyiv-Ukraine, 2015, pp. 14–21.

[6] T. ONER and I. SENTURK, *A construction of Heyting algebra on categorical syllogisms*, *Matematichki Bilten*, vol. 40 (2016), no. 4, pp. 5–12.

[7] R. PAGNAN, *A diagrammatic calculus of syllogisms*, *Journal of Logic Language and Information*, vol. 21 (2012), pp. 347–364.

[8] E. TURUNEN, *An algebraic study of Peterson's Intermediate Syllogisms*, *Soft Computing*, vol. 18 (2014), no. 12, pp. 2431–2444.

- ▶ FRANCESCO PARENTE, *Keisler's order via Boolean ultrapowers*.

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In this talk, we shall present some applications of the Boolean ultrapower construction [2] to Keisler's order.

Over the last decade, Malliaris and Shelah proved a striking sequence of results in the intersection between model theory and set theory, solved a long-lasting problem [1], and developed surprising connections between classification theory and cardinal characteristics of the continuum. The main motivation of their work is the study of Keisler's order, introduced

originally in 1967 as a device to compare the complexity of complete theories by looking at saturated ultrapowers of their models.

Although the definition of Keisler’s order makes use of regular ultrafilters on power-set algebras, recently there has been a shift towards building ultrafilters on complete Boolean algebras. In particular, moral ultrafilters have emerged as the main tool to find dividing lines among unstable theories.

Motivated by this new Boolean-algebraic framework, in this talk we shall address the following question: what kind of classification can arise when we compare theories according to the saturation of Boolean ultrapowers of their models?

We shall show that most model-theoretic properties of κ -regular ultrafilters can be generalized smoothly to the context of κ -distributive Boolean algebras. On the other hand, we shall prove the existence of regular ultrafilters on the Cohen algebra \mathbb{C}_κ with unexpected model-theoretic features.

[1] M. MALLIARIS and S. SHELAH, *Cofinality spectrum theorems in model theory, set theory, and general topology*. *Journal of the American Mathematical Society*, vol. 29 (2016), no. 1, pp. 237–297.

[2] R. MANSFIELD, *The theory of Boolean ultrapowers*. *Annals of Mathematical Logic*, vol. 2 (1971), no. 3, pp. 297–323.

- ▶ FRANCO PARLAMENTO AND FLAVIO PREVIALE, *On the admissibility of the structural rules in Kanger’s sequent calculus with restricted equality rules*.

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Kanger’s sequent calculus for first order logic with equality, introduced in the classic [1], is a sequent calculus for classical first order logic with equality, free of structural rules, based on the following equality rules:

$$\frac{\Gamma_1\{v/r\}, s = r, \Gamma_2\{v/r\} \Rightarrow \Delta\{v/r\}}{\Gamma_1\{v/s\}, s = r, \Gamma_2\{v/s\} \Rightarrow \Delta\{v/s\}} P_3 \quad \frac{\Gamma_1\{v/r\}, r = s, \Gamma_2\{v/r\} \Rightarrow \Delta\{v/r\}}{\Gamma_1\{v/s\}, r = s, \Gamma_2\{v/s\} \Rightarrow \Delta\{v/s\}} P_4$$

where Γ_1, Γ_2 , and Δ are sequences of formulas and $\Gamma\{v/t\}$ denotes the result of substituting all the free occurrences of v in Γ by t . [1] restricts the applications of P_3 by the requirement that $rank(r) \leq rank(s)$ and those of P_4 by the requirement that $rank(r) < rank(s)$, and the applications of the γ -rules:

$$\frac{\Gamma_1, F\{x/t\}, \forall x F, \Gamma_2 \Rightarrow \Delta}{\Gamma_1, \forall x F, \Gamma_2 \Rightarrow \Delta} \quad \frac{\Gamma \Rightarrow \Delta_1, F\{x/t\}, \exists x F \Delta_2}{\Gamma \Rightarrow \Delta_1, \exists x F \Delta_2}$$

by the requirement that the term t be present free in the endsequent or be a fresh variable in case there are no free terms in the endsequent. If such restrictions on the equality and γ -rules are dropped, a syntactic proof of the admissibility of all the structural rules, including the cut rule, over the resulting calculus, as well as over its intuitionistic version, is known from [2]. We address that admissibility issue in case the restriction on the equality rules is maintained, and give a syntactic proof that the unrestricted equality rules are admissible over the restricted ones, from which it follows that cut elimination still holds. The proof is based on the admissibility of the contraction rule for equalities in the restricted calculus, for which a syntactic proof remains to be given. The result is obtained through a strengthening of Orevkov’s claim in [3] concerning the existence of nonlengthening derivations, that by itself would fall short of establishing the desired result, since nonlengthening in the specific case ensures only that we have the same restriction $rank(r) \leq rank(s)$ in both P_3 and P_4 (see also [4]).

[1] S. KANGER, *A simplified proof method for elementary logic*, *Computer Programming and Formal Systems* (P. Braffort and D. Hirshberg, editors), North-Holland, Amsterdam, 1963, pp. 87–94.

[2] F. MUNINI and F. PARLAMENTO, *Admissibility of the structural rules in Kanger's sequent calculus for first order logic with equality*, *Logic Colloquium 2015*, this BULLETIN, vol. 22 (2016), no. 3, p. 414.

[3] V. P. OREVKOV, *On nonlengthening applications of equality rules*. *Zapiski Nauchnyh Seminarov LOMI*, vol. 16 (1969), pp. 152–156 (In Russian) English translation in: *Seminars in Mathematics: Steklov Math. Inst.* (A. O. Slisenko, editor), Studies in Constructive Logic, vol. 16, Consultants Bureau, NY–London, 1971, pp. 77–79.

[4] F. PARLAMENTO and F. PREVIALE, *The cut elimination and nonlengthening property for the sequent calculus with equality*, *Logic Colloquium*, 2016, arXiv 1705.00693.

- THOMAS PIECHA AND PETER SCHROEDER-HEISTER, *Intuitionistic logic is not complete for standard proof-theoretic semantics*.

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Prawitz conjectured that intuitionistic first-order logic is complete with respect to a notion of proof-theoretic validity [1,2,3]. We show that this conjecture is false. The notion of validity obeys the following standard conditions, where S refers to atomic bases (systems of production rules):

1. $\models_S A \wedge B \iff \models_S A$ and $\models_S B$.
2. $\models_S A \vee B \iff \models_S A$ or $\models_S B$.
3. $\models_S A \rightarrow B \iff A \models_S B$.
4. $\Gamma \models A \iff$ For all S : ($\models_S \Gamma \implies \models_S A$).
5. If $\Gamma \models A$ and $\Gamma, A \models_S B$, then $\Gamma \models_S B$.

Any semantics obeying these conditions satisfies the generalized disjunction property:

For every S : if $\Gamma \models_S A \vee B$, where \vee does not occur positively in Γ , then either $\Gamma \models_S A$ or $\Gamma \models_S B$.

This implies the validity (\models) of Harrop's rule $\neg A \rightarrow (B \vee C) / (\neg A \rightarrow B) \vee (\neg A \rightarrow C)$, which is admissible but not derivable in intuitionistic logic.

[1] D. PRAWITZ, *Towards a foundation of a general proof theory*, *Logic, Methodology and Philosophy of Science IV* (P. Suppes et al., editors), North-Holland, 1973, pp. 225–250.

[2] ———, *An approach to general proof theory and a conjecture of a kind of completeness of intuitionistic logic revisited*, *Advances in Natural Deduction* (L. C. Pereira, E. H. Haeusler, and V. de Paiva, editors), Springer, Berlin, 2014, pp. 269–279.

[3] P. SCHROEDER-HEISTER, *Validity concepts in proof-theoretic semantics*. *Synthese*, vol. 148 (2006), pp. 525–571.

- EDOARDO RIVELLO, *On extending the general recursion theorem to non-wellfounded relations*.

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The principle of definition by recursion on a wellfounded relation [1], can be stated as follows: Let A be any set and let P be the set of all partial functions from A to some set B . Let $G: A \times P \rightarrow B$ be any function and let $R \subseteq A \times A$ be any binary relation.

Fact 1 (Montague): If R is wellfounded on A then there exists a unique function $f: A \rightarrow B$ such that

$$\forall x \in A (f(x) = G(x, f \upharpoonright x^R)), \quad (1)$$

where $x^R = \{y \in A \mid y R x\}$.

If R is not wellfounded on the entire domain A , an obvious way of extending this method of definition is to identify a proper subset W of A on which R is wellfounded and to apply the principle to this set. The usual choice for W is the *wellfounded part* of R , defined as the set of all R -wellfounded points of A .

In my talk, after examining several different strategies to prove Fact 1, I will present a new approach to extend this method of definition to all kinds of binary relations. We look at subsets X of A on which R is not necessarily wellfounded, yet there exists a unique function

$g: X \rightarrow B$ which satisfies (1) for all $x \in X$. Let us call such subsets *determined*. Then we can prove

THEOREM. *There exists a unique subset U of A such that (a) U is R -closed, i.e., $\forall x \in U, x^R \subseteq U$; (b) U is determined and all R -closed subsets of U are determined; (c) U is the largest subset of A satisfying (a) and (b). This theorem ensures, for any relation R , the existence and uniqueness of a function $g: U \rightarrow B$ which satisfies (1) on its domain and is defined on a domain U which extends the wellfounded part W of R .*

[1] R. MONTAGUE, *Well-founded relations: Generalizations of principles of induction and recursion*. *Bulletin American Mathematical Society*, vol. 61, p. 442.

- ▶ GEMMA ROBLES, FRANCISCO SALTO, AND JOSÉ M. BLANCO, *Routley–Meyer semantics for natural implicative expansions of Kleene’s strong three-valued matrix*.

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Routley–Meyer semantics, originally introduced for interpreting relevance logic, is a highly malleable semantics capable of modelling families of nonclassical logics very different from each other. Let us now understand the notion of a “natural implication” following [2]. Then, there are exactly six natural implicative expansions of Kleene’s strong three-valued matrix with 1 as the sole designated value.

The aim of this article is to endow each one of the logics characterized by these six expansions with a Routley–Meyer type ternary relational semantics. There are well-known logics among those determined by these six expansions. Łukasiewicz three-valued logic Ł3 is an example.

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[1] R. T. BRADY, R. K. MEYER, V. PLUMWOOD, and R. ROUTLEY, *Relevant Logics and Their Rivals*, vol. 1, Ridgeview Publishing Co., Atascadero, CA, 1982.

[2] N. TOMOVA, *A lattice of implicative extensions of regular Kleene’s logics*. *Reports on Mathematical Logic*, vol. 47 (2012), pp. 173–182.

- ▶ ANDREI RODIN, *Two “styles” of axiomatization: Rules versus axioms. A modern perspective*.

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In a Hilbert-style nonlogical axiomatic theory the semantics of logical symbols is rigidly fixed, while the interpretation of nonlogical symbols usually varies giving rise to different models of the given theory. All nonlogical content of such a theory is comprised in its nonlogical axioms (e.g., axioms of ZF) while rules, which generate from these axioms new theorems, belong to the logical part of the theory (aka underlying logic).

An alternative approach to axiomatization due to Gentzen amounts to a presentation of formal calculi in the form of systems of rules without axioms. Gentzen did not try to extend his approach to nonlogical theories by considering specific nonlogical rules as a replacement for nonlogical axioms. However the more recent work in Univalent Foundations of Mathematics [2] suggests that the Gentzen-style rule-based approach to formal presentation of theories may have important applications also outside the pure logic.

A reason why one may prefer a rule-based formal representation is that it is more computer-friendly. This, in particular, motivates the recent work on the constructive

justification of the Univalence Axiom via the introduction of new operations on types and contexts [1]. However this pragmatic argument does not meet the related epistemological worries. What kind of knowledge may represent a theory having the form of a bare system of rules? Is such a form of a theory appropriate for representing a knowledge of objective human-independent reality? How exactly truth features in rule-based nonlogical theories?

Using HoTT as a motivating example I provide some answers to these questions and show that the Gentzen-style rule-based approach provides a viable alternative to the standard axiomatic approach not only in logic but also in science more generally.

Acknowledgments. The work is supported by Russian Foundation for Basic Research, research grant 16-03-00364.

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► ALEKSANDRA SAMONEK, *Relation algebras, representability, and relevant logics*.

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This talk is an introduction to the problems concerning certain relevant logics and relation algebras.

[4] has shown how to obtain sound and complete semantics for RM , *i.e.*, the implicational fragment $R \rightarrow$ of R with the axiom mingle $A \rightarrow (A \rightarrow A)$. He also demonstrated how one can obtain a sound but not complete interpretation of R by replacing sets with commuting dense binary relations. But RM does not have a variable-sharing property (VSP) which R has. A modal restriction of RM in case of which the VSP is preserved was given in [5] together with the argument that from an intuitive semantical point of view, this modal restriction of RM is an alternative to Anderson and Belnap's logic of entailment E ([1]).

[6] has studied a version of positive minimal relevant logic B and [2] demonstrated that B is fully interpretable in the variety of weakly associative relation algebras which are not representable. [3] went on to show that if representability is dropped, one can obtain a complete interpretation of certain relevant logics in the language of relation algebras.

We will examine the mentioned results in order to clarify the connection between certain relation algebras and relevant logics like R and RM and see (i) whether such connection entails full interpretability of relevant logics in terms of relation algebras and (ii) what are the consequences of achieving this interpretability for representability and completeness.

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- DENIS I. SAVELIEV, *Systems of propositions referring to each other: A model-theoretic view*. Steklov Mathematical Institute of the Russian Academy of Sciences, 8 Gubkina St., Moscow, Russia.

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We investigate arbitrary sets of propositions such that some of them state that some of them (possibly, themselves) are wrong, and criterions of paradoxicality or nonparadoxicality of such systems. For this, we propose a finitely axiomatized first-order theory with one unary and one binary predicates, T and U . An heuristic meaning of the theory is as follows: variables mean propositions, Tx means that x is true, Uxy means that x states that y is wrong, and the axioms express natural relationships of propositions and their truth values. A model (X, U) is called nonparadoxical iff it can be enriched to some model (X, T, U) of this theory, and paradoxical otherwise. E.g., a model corresponding to the Liar paradox consists of one reflexive point, a model for the Yablo paradox is isomorphic to natural numbers with their usual ordering, and both these models are paradoxical.

We show that the theory belongs to the class Π_2^0 but not Σ_2^0 . We propose a natural classification of models of the theory based on a concept of a collapse of models. Furthermore, we show that the theory of nonparadoxical models, and hence, the theory of paradoxical models, belongs to the class Δ_1^1 but is not elementary. We consider also various special classes of models and establish their paradoxicality or nonparadoxicality. In particular, we show that models with reflexive relations, as well as models with transitive relations without maximal elements, are paradoxical; this general observation includes the instances of Liar and Yablo. On the other hand, models with conversely well-founded relations, and more generally, models with relations that are winning in sense of a certain membership game are nonparadoxical. Finally, we propose a natural classification of nonparadoxical models based on the above-mentioned classification of models of our theory.

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- DENIS I. SAVELIEV AND ILYA B. SHAPIROVSKY, *Defining modal logics of relations between models*.

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Let \mathcal{C} be a class of models in a fixed signature and \mathcal{R} a relation on \mathcal{C} ; e.g., $\mathfrak{A} \mathcal{R} \mathfrak{B}$ may mean “ \mathfrak{B} is a submodel of \mathfrak{A} ”, “ \mathfrak{B} is a homomorphic image of \mathfrak{A} ”, “ \mathfrak{B} is an extension (for models of arithmetic or set theory: an end-extension, a generic extension) of \mathfrak{A} ”, “ \mathfrak{B} is an existential closure of \mathfrak{A} ”, etc. We interpret modal formulas by sentences of a model-theoretic language \mathcal{L} such that $\diamond\varphi$ is true at a model \mathfrak{A} (“ φ is possible at \mathfrak{A} ”) iff φ is true at some model \mathfrak{B} with $\mathfrak{A} \mathcal{R} \mathfrak{B}$. A few recent instances of a similar approach deal with models of PA ([4,6]) and ZF ([1,2,3]). In these cases, the first-order languages are powerful enough to put the interpretation inside them. This is not true for arbitrary models: $\diamond\varphi$ may be not first-order expressible. However, once \mathcal{L} is chosen strong enough to overcome this, truth and validity of modal formulas can be defined in terms of general frame semantics, and the modal theory of $(\mathcal{C}, \mathcal{R})$ defined as the set of all valid modal formulas turns out to be a normal modal logic. This provides a general framework for defining and studying modal logics of model-theoretic relations.

We apply this approach to the case where $\mathfrak{A} \mathcal{R} \mathfrak{B}$ means “ \mathfrak{B} is a submodel of \mathfrak{A} ”. In general, even infinitary first-order languages are not powerful enough to express the satisfiability in submodels. However, for any signature with $< \kappa$ functional symbols (and arbitrarily many predicate symbols), the monadic fragment of the second-order language $\mathcal{L}_{\kappa, \omega}^2$ expresses the satisfiability of its own sentences in submodels. We prove that whenever the signature contains at least one functional symbol of arity ≥ 2 and \mathcal{C} is the class of all models in this signature, then the modal theory of $(\mathcal{C}, \mathcal{R})$ is S4 if the signature does not have constant symbols, and S4.1.2 otherwise.

Acknowledgments. The work is supported by grant 16-11-10252 of the Russian Science Foundation. A preliminary report can be found in [5].

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► GIORGIO SBARDOLINI, *The semanticist's guide to ramification*.

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I outline an account of intensional paradoxes in Ramified Higher Order Logic (RHOL). These paradoxes are intensional counterparts of the paradoxes derived by a syntactic truth predicate. One reason why the intensional paradoxes are especially interesting is that they arise from reasoning about domains of propositions. Thus, they are especially relevant for our understanding of the foundations of Semantic Theory.

In his work on intensional paradoxes, Kaplan (1995) sketches a version of RHOL. Ramification is one way of articulating a consistent metalanguage for Semantic Theory in which the rules for the logical operators are classical. Thus the resulting theory is compatible with standard Montague Grammar.

There are several different ways of ramifying, and there are different interpretations of the metaphysical underpinnings of ramification. Here I discuss a simple and user-friendly version of RHOL (in fact, so simple that it could be taught in undergraduate textbooks) in which predicative restrictions on the level of formulas are introduced only by generalization over propositional domains. In effect, on my favorite version, a Ramified Logic is one in which the inference from $\forall pSp$ to Sq sometimes fails. I argue that this version of RHOL is preferable to Kaplan's from the standpoint of the foundations of Semantics. A crucial premise for this argument is that on the former version, but not on Kaplan's, ramification allows enough impredicativity over the domain of propositions and attitude operators for the definition of a Stalnakerian Common Ground for arbitrary classes of propositions.

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► MICHAEL SHENEFELT AND HEIDI WHITE, *Why does formal deductive logic start with the classical Greeks?*

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Many ancient people studied logic in the broad sense of argumentation, but the study of formal deductive validity starts with the classical Greeks. For some reason, the only person to invent a study of validity in virtue of form was Aristotle, and all other logicians have had his example to follow. Why?

We contend that formal logic emerged as a result of two factors—one geographical, the other political.

First, unlike other regions of the ancient world, classical Greece had a geography that favored small states, dominated by urban crowds. The ease of navigating the Mediterranean caused the commercial classes to grow, and the small size of these states meant that these same commercial crowds dominated the politics of the classical age. As a result, political questions were settled, not by kings or small groups of nobles, but in mass meetings like the Athenian Assembly. The mechanics of these meetings put special emphasis on public argumentation.

Second, these same crowds, when called to make political decisions, often behaved irrationally. Such crowds had dominated the Athenian Assembly, but when Athens lost its war against Sparta, and then followed with the execution of Socrates, a reaction among intellectuals led to the development of formal logic. Philosophers focused increasingly on the difference between rational argumentation and irrational, and this theme, developed by Plato but later expanded by Aristotle, culminated in the first known system of formal logic.

We attribute the Greek relish for logical demonstration, even in mathematics, to an argumentative political environment, and we draw our argument from our book. *If A, Then B: How the World Discovered Logic* (Columbia University Press).

► ANDREI SIPOȘ, *Proof mining in convex optimization*.

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Proof mining is a research program introduced by U. Kohlenbach in the 1990s ([2] is a comprehensive reference, while [3] is a survey of recent results), which aims to obtain explicit quantitative information (witnesses and bounds) from proofs of an apparently ineffective nature. This offshoot of interpretative proof theory has successfully led so far to obtaining some previously unknown effective bounds, primarily in nonlinear analysis and ergodic theory. A large number of these are guaranteed to exist by a series of logical metatheorems which cover general classes of bounded or unbounded metric structures.

For the first time, this paradigm is applied to the field of convex optimization (for an introduction, see [1]). We focus our efforts on one of its central results, the proximal point algorithm. This algorithm, or more properly said this class of algorithms, consists, roughly, of an iterative procedure that converges (weakly or strongly) to a fixed point of a mapping, a zero of a maximally monotone operator or a minimizer of a convex function. Similarly to other cases previously considered in nonlinear analysis, we may obtain rates of metastability or rates of asymptotic regularity. What is interesting here, however, is that for a relevant subclass of inputs to the algorithm—“uniform” ones, like uniformly convex functions or uniformly monotone operators—we may obtain an effective rate of convergence. The notion of convergence, being represented by a Π_3 -sentence, has been usually excluded from the prospect of being quantitatively tractable, unless its proof exhibits a significant isolation of the use of *reductio ad absurdum* (see [4,5]). Here, however, a peculiarity of the input, namely its uniformity, translates into a logical form that makes possible this sort of extraction.

These results are joint work with Laurențiu Leuştean and Adriana Nicolae.

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- ▶ ALEXANDRA SOSKOVA, *Structural properties of the cototal enumeration degrees*. Faculty of Mathematics and Informatics, Sofia University, 5 James Bourchier Blvd., 1124 Sofia, Bulgaria.

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The talk will be an overview on the structural properties of the cototal enumeration degrees which form a proper substructure of the enumeration degrees. The cototal enumeration degrees properly extend the substructure of the total enumeration degrees. The skip is a monotone operator on enumeration degrees.

We study cototality, using the skip operator and give some examples of classes of enumeration degrees that either guarantee or prohibit cototality. The skip has many of the nice properties of the Turing jump, but not every e-degree is reducible to its skip. The e-degrees reducible to their skip are exactly the cototal degrees. The cototal enumeration degrees are characterized [1] as the enumeration degrees of complements of maximal independent sets for infinite computable graphs on the natural numbers. The image of the continuous degrees, introduced by Joseph Miller [5], is contained in the cototal enumeration degrees [1]. Further characterizations are given by Ethan McCarthy [4], Takayuki Kihara, Arno Pauly [2], and Takayuki Kihara by private conversation.

Recently Joseph Miller and Mariya Soskova [6] prove that the cototal enumeration degrees form a dense substructure of the enumeration degrees. Moreover they show that these are exactly the enumeration degrees which contain sets with good approximations in the sense of Alistair Lachlan and Richard Shore [3].

Acknowledgments. This is joint work with Uri Andrews, Hristo Ganchev, Rutger Kuyper, Steffen Lempp, Joseph Miller, and Mariya Soskova.

[1] U. ANDREWS, H. GANCHEV, R. KUYPER, S. LEMPP, J. MILLER, A. SOSKOVA, and M. SOSKOVA, *On cototality and the skip operator in the enumeration degrees*, submitted.

[2] T. KIHARA and A. PAULY, *Point degree spectra of represented spaces*, submitted.

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[6] J. MILLER and M. SOSKOVA, *Density of the cototal enumeration degrees*, submitted.

- ▶ YUTA TAKAHASHI, *A proof-theoretic semantics for disjunction*. Graduate School of Informatics, Nagoya University, Furo-cho, Chikusa-ku, Nagoya, Japan. *E-mail:* yuuta.taka84@gmail.com.

Okada and Takemura ([1]) introduced phase semantics for λ -terms of Laird's dual affine/intuitionistic λ -calculus, whose types are composed from intuitionistic implication \rightarrow , linear implication \multimap , and linear additive product $\&$. The validity in this semantics has several key features of the validity in proof-theoretic semantics (PTS), which was introduced by Prawitz ([2]) and analyzed by Schroeder-Heister ([4]), so one can provide Okada–Takemura's semantics with a PTS-style foundation. This poses the following question: Can one supply Okada–Takemura's semantics with an interpretation of disjunction, keeping the connection to PTS?

First, we introduce a Okada–Takemura-style semantics for the term-calculus $M_{\rightarrow\wedge\vee}$ of minimal propositional logic with the connectives \rightarrow , \wedge , and \vee . Our interpretation of disjunction \vee is inspired by Sandqvist's ([3]) and keeps the connection to PTS. Next, we prove the completeness of $M_{\rightarrow\wedge\vee}$ in the following sense: Every valid term in our semantics is typable. Finally, we note that strong normalization of $M_{\rightarrow\wedge\vee}$ follows from our proof for its completeness.

Acknowledgments. This is a joint work with Ryo Takemura. The author is supported by KAKENHI (Grant-in-Aid for JSPS Fellows) 16J04925.

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► BENNO VAN DEN BERG, *Models of set theory in path categories*.

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A classical result by Peter Aczel from 1978 [1] shows how one can interpret the constructive set theory **CZF** in Martin-Löf's constructive type theory, by regarding sets as well-founded trees modulo bisimulation. Moerdijk and Palmgren [4] showed that the same sets-as-trees idea can be used to build models of **CZF** in suitable “predicative toposes”. We revisit the work by Aczel, Moerdijk, and Palmgren in the light of recent developments in homotopy type theory. The claim is that the sets-as-trees interpretation never uses any definitional equalities and up-to-homotopy versions of the various type constructors suffice to interpret **CZF**. The main challenge is to avoid subtle mistakes involving universes and our main categorical tools are the notion of a path category and the theory of fibred categories.

This is joint work with Ieke Moerdijk and based on the preprints [2,3].

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[3] B. VAN DEN BERG and I. MOERDIJK, *Exact completion of path categories and algebraic set theory—Part I*, **arXiv:1603.02456**, October 2016.

[4] I. MOERDIJK and E. PALMGREN, *Type theories, toposes and constructive set theory: Predicative aspects of AST*, **Annals of Pure and Applied Logic**, vol. 114 (2002), pp. 155–201.

► WEI WANG, *On the computability of perfect subsets of sets with positive measure*.

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It is well-known that every set of reals with positive measure contains a perfect subset. In a joint project of Chong, Li, Yang, and Wei Wang, we study the computability of such perfect subsets. We show that every effectively closed set C with positive measure contains a low perfect subset. Moreover, the Turing degrees of perfect subsets of C contain all degrees above the halting problem. We also prove that every set with positive measure contains a perfect subset not computing any given noncomputable set.

► ANDREAS WEIERMANN, *On generalized Goodstein sequences*.

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We define generalized Goodstein sequences with respect to the Schwichtenberg–Wainer hierarchy of fast growing functions. The resulting Goodstein principles will then not be provable in the usual theory for noniterated inductive definitions.

The results are partly in joint work with T. Arai and S. Wainer.

► FELIX Q. WEITKÄMPER, *Constructing and classifying stability-preserving substructures*.

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Algebraic Stability Theory is the branch of Model Theory that applies concepts from stability theory to concrete mathematical structures. Its most fundamental problems are of the form “Given a mathematically interesting class of structures, which of them stand at a certain level of the stability hierarchy?”

Paradigms for results of this sort are Macintyre’s theorem that all ω -stable fields are algebraically closed on the one hand and Hrushovski and Itai’s theorem that there are many non-differentially-closed ω -stable differential fields on the other hand. Tackling these problems at any level of generality seems unfeasible, however, if one takes “(ω -)stable structure” to mean “structure with an (ω -)stable first-order theory”. This is because the existence of an ω -stable theory of differential fields, for example, requires the existence of a well-behaved saturated differential field, and determining saturated models will usually require a discussion of axiomatisability issues. Such issues, though, are highly dependent on the concrete algebraic properties of the class in which one is working. We argue that the more general context of Homogeneous Model Theory provides a more appropriate interpretation for questions of this type, in which “stable structure” is taken to mean “stable homogeneous structure” instead. In this framework, we provide a general construction scheme for substructures preserving degree of stability and discuss how understanding the close connection between these derived structures and their parent structure could help us ask more meaningful questions in this fundamental area of Algebraic Model Theory.

- ▶ HANWEN WU AND HONGWEI XI, *Multirole Logic*.

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We formulate *multirole logic* [1] as a new form of logic and naturally generalize Gentzen’s celebrated result of cut-elimination between two sequents into one between n sequents for any $n \geq 1$.

While the first and foremost inspiration for multirole logic came to us during a study on multiparty session types in distributed programming [2], it seems natural in retrospective to introduce multirole logic by exploring the well-known duality between conjunction and disjunction in classical logic. Let $\overline{\mathcal{D}}$ be a (possibly infinite) underlying set of integers, where each integer is referred to as a role. In multirole logic, each formula A can be annotated with a set R of roles to form the i -formula $[A]_R$. For each *ultrafilter* \mathcal{U} on the power set of $\overline{\mathcal{D}}$, there is a (binary) logical connective $\wedge_{\mathcal{U}}$ such that $[A_1 \wedge_{\mathcal{U}} A_2]_R$ is interpreted as the conjunction (disjunction) of $[A_1]_R$ and $[A_2]_R$ if $R \in \mathcal{U}$ ($R \notin \mathcal{U}$) holds. Furthermore, the notion of negation is generalized to *endomorphisms* on $\overline{\mathcal{D}}$. We formulate both multirole logic (MRL) and linear multirole logic (LMRL) as natural generalizations of classical logic (CL) and classical linear logic (CLL), respectively. Among various metaproperties established for MRL and LMRL, we obtain one named *multiparty cut-elimination* stating that every cut involving one or more sequents can be eliminated. For instance, the cut-rule in CL is generalized to the following one:

$$\frac{\Gamma_1, [A]_{R_1} \quad \dots \quad \Gamma_n, [A]_{R_n}}{\Gamma_1, \dots, \Gamma_n},$$

where $\overline{R_1} \uplus \dots \uplus \overline{R_n} = \overline{\mathcal{D}}$ is assumed. Note that Gentzen’s cut-elimination is the special case where $n = 2$.

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[2] ———, *Propositions in linear multirole logic as multiparty session types*, arXiv:1611.08888 [cs.PL], 2016.

- ▶ SUSUMU YAMASAKI, *A modal operator in multimodal mu-calculus and induced semiring structure*.

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The meanings of formulas in multimodal mu-calculus (as an extended version of action logic [2]) are presented by the author, where the states for interactive communication and function application are monitored (conditioned) by formulas. The syntax of the formulas is give in BNF: $\varphi ::= \text{tt} \mid p \mid \neg\varphi \mid \sim\varphi \mid \varphi \vee \varphi \mid \langle c \rangle \varphi \mid \mu x. \varphi \mid \varphi \rangle t$, with the truth tt, propositions p , two kinds of negation \neg and \sim , the disjunction operator \vee , the least fixed point operator μ , and prefix/postfix modal operators $\langle c \rangle$ (for communications) and $\rangle t$ (for terms).

This contribution is concerned with the case that the postfix modal operator $\rangle t$ (in the above logic) is provided with (another type of) logical formulas. The modal operator might be related to and motivated by declarative programming, originating from: (i) propositional formulas of the expression $\bigwedge_j (l_j^i \wedge \dots \wedge l_{n_j}^i \rightarrow l^j)$ (where l_j^i and l^j are literals) and (ii) their models (by taking correspondence with a Heyting algebra on $\{0, 1/2, 1\}$). Regarding the model, an extended version of fixed point theory (in [1]) is available so that the pair form (P -set, N -set) may be constructed, where P -set and N -set are positive and negative sets of propositions assigned to 1 and to 0, respectively.

As a next step to the modal operator, the model pair is abstracted into the form for sequential and alternative effects because the meaning of modal operator is involved in a transition system and thus contains state-transition effects of sequence and alternation. In this contribution, the sequential effect is restricted only to the positive set “ P -set”, such that a form (P -seq, N -set) is aimed at, where P -seq is a subset of the set containing all the finite proposition sequences (formed by *concatenation*) from the set of propositions. Taking the *alternation* (for concatenation formation) into consideration, the form $\Sigma_i(P\text{-seq}_i, N\text{-set}_i)$ (where Σ_i is a direct sum) would be defined.

The set of the forms $\Sigma_i(P\text{-seq}_i, N\text{-set}_i)$ might be finally constructed into a *semiring*, with the operations *addition* and *multiplication* in accordance to *alternation* and *concatenation*, respectively. This construction is well managed with the effects of the negative sets (N -set’s).

[1] A. VAN GELDER, K. A. ROSS, and J. S. SCHLIPF, *The well-founded semantics for general logic programs*. **Journal of the ACM**, vol. 38 (1991), no. 3, pp. 619–649.

[2] M. HENNESSY and R. MILNER, *Algebraic laws for nondeterminism and concurrency*. **Journal of the ACM**, vol. 32 (1985), no. 1, pp. 137–161.

► AIBAT YESHKEYEV, *Some properties of central types for EPSCJ theories*.

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This abstract is associated with the concepts of convexity theory in the class Existentially Prime Jonsson theories. We denote such theories as Existentially Prime Strongly Convex Jonsson (EPSCJ).

Also we have concentrating our attention to not arbitrary subsets but use have deal with Jonsson subsets ([1,2]) of some semantic model for fixing Jonsson theory.

First of all, we are interested in describing models of central types of Jonsson fragments [3] with respect to stability topics.

DEFINITION 1. Let T is an arbitrary Jonsson theory in the language of the first order signature σ . Let C is a semantic model of theory T . Let $A \subseteq C$ is a Jonsson set of theory T . Let $\sigma_\Gamma(A) = \sigma \cup \{c_a \mid a \in A\} \cup \Gamma$, $\Gamma = \{P\} \cup \{c\}$. Let $T_A^C = T \cup Th_{\forall\exists}(C, a)_{a \in A} \cup \{P(c_a) \mid a \in A\} \cup \{P(c)\} \cup \{''P \subseteq''\}$ where $\{''P \subseteq''\}$ is an infinite set of sentences expressing the fact that the interpretation of symbol P is existentially closed submodel in the language of the signatures $\sigma_\Gamma(A)$ and this model is a definable closure of the set A . It is understood that the consideration the set of sentences is Jonsson theory and this theory generally is not complete. Let T^* is the center of the Jonsson theory T_A^C and $T^* = Th(C')$ where C' is a semantic model of the theory T_A^C . By restriction theory T_A^C to signatures $\sigma_\Gamma(A) \setminus \{c\}$ the theory T^* becomes a complete type. This type we call as the central type of the theory T relatively the Jonsson set A and denoted by P_A^C .

Let L be an arbitrary language. Let T be perfect Jonsson theory, complete for existential sentences in the language L , and its semantic model is C . We say that two Jonsson (algebraically) sets (equivalent, cosemantic, categorical), if there are respectively, (Jonsson equivalent, cosemantic, categorical, syntactically similar, semantically similar, etc.) the models obtained by the corresponding closure of these sets. Consider, for example, cosemantic. Two Jonsson sets are cosemantic, if their respective closures are cosemantic, etc. [1].

Let us consider the stability for fragments of Jonsson sets.

Let X Jonsson set and M existentially closed model, where $dcl(X) = M$.

Consider the fragment of Jonsson set X as the theory $Th_{\forall\exists}(M) = T_M$. And we consider T_M instead of theory T in the definition 1. We have the following results:

LEMMA 1. T_M will Jonsson theory in the enrichment of above signature.

THEOREM 1. Let T_M , as described above an existentially complete perfect EPSCJ theory. If $\lambda \geq \omega$, then the following conditions are equivalent:

- (1) T^* is λ -stable, where T^* is the center of T ;
- (2) T_A^C is $J - \lambda$ -stable [1].

THEOREM 2. Let T_M existentially complete EPSCJ theory. Then the following conditions are equivalent:

- (1) T_M^* – ω -categorical;
- (2) T_A^C – ω -categorical.

[1] M. T. KASYMETOVA and A. R. YESHKEYEV, *Jonsson Theory and its Classes of Models*, Publisher of the Karaganda State University, 2016, p. 346.

[2] A. YESHKEYEV, *On Jonsson sets and some their properties*, *Abstract Booklet of Logic Colloquium*, Vienna Summer of Logic, 2014, pp. 108–109.

[3] ———, *Properties of central type for fragments of Jonsson sets*, *Abstract Booklet of Logic Colloquium*, Helsinki Summer of Logic, 2015, pp. 751–752.

► BYEONG-UK YI, *Plural categoricals and squares of opposition*.

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This article studies the logic of plural categoricals: (a) ‘Any unicorns are animals’, (a*) ‘Any of the unicorns are animals’, etc. To do so, it is important to distinguish between two groups thereof:

Group 1 (G1) **A**: Any Ps are Qs. **E**: No Ps are Qs.
 I: Some Ps are Qs. **O**: Some Ps are not Qs.

Group 2 (G2) **A***: Any of the Ps are Qs. **E***: None of the Ps are Qs.
 I*: Some of the Ps are Qs. **I***: Some of the Ps are not Qs.

G1 categoricals (e.g., (a)) are not logically equivalent to matching G2 categoricals (e.g., (a*)) (see [6]). Modern logic gives essentially correct accounts of G1 categoricals. Regarding G2 categoricals, however, traditional logic arguably yields correct accounts. Assume, following Strawson [4]–[5], that G2 categoricals presuppose that the plural terms replacing ‘the Ps’ (e.g., ‘the horses’) refer to some things (see [1]). Then all the theses in the traditional square of opposition (see [2]) hold. But **E*** and **I***, unlike **E** and **I**, are not convertible (see [3]).

[1] T. MCKAY, *Plural Predication*, Oxford University Press, 2006.

[2] T. PARSONS, *The traditional square of opposition*, *Stanford Encyclopedia of philosophy*, Summer 2017 ed. (E. Zalta, editor), URL = <https://plato.stanford.edu/archives/sum2017/entries/square/>, forthcoming.

[3] T. SMILEY, *Mr. Strawson on traditional logic*. *Mind*, vol. 76 (1967), no. 301, pp. 118–120.

[4] P. STRAWSON, *On referring*. *Mind*, vol. 59 (1950), no. 235, pp. 320–344.

[5] ———, *Introduction to Logical Theory*, Methuen, 1952.

[6] B.-U. YI, *Quantifiers, determiners, and plural constructions*, *Unity and Plurality: Logic, Philosophy, and Linguistics* (M. Carrara, F. Moltmann, and A. Arapinis, editors), Oxford University Press, Oxford, 2016, pp. 121–170.

Abstracts of papers submitted by title

- ▶ S. S. BAIZHANOV AND B. SH. KULPESHOV, *On preserving properties under expanding models of quite o-minimal theories.*

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Here we discuss properties that are preserved under expanding models of an \aleph_0 -categorical quite o-minimal theory by a convex unary predicate. We prove that the following properties as quite o-minimality [2], \aleph_0 -categoricity and convexity rank [3] are preserved under such expansions.

Let M be a weakly o-minimal structure, $A, B \subseteq M$, M be $|A|^+$ -saturated, and let $p, q \in S_1(A)$ be nonalgebraic types.

DEFINITION 1. [1] We say that p is not *weakly orthogonal* to q if there are an A -definable formula $H(x, y)$, $\alpha \in p(M)$, and $\beta_1, \beta_2 \in q(M)$ such that $\beta_1 \in H(M, \alpha)$ and $\beta_2 \notin H(M, \alpha)$.

DEFINITION 2. [2] We say that p is not *quite orthogonal* to q if there is an A -definable bijection $f: p(M) \rightarrow q(M)$. We say that a weakly o-minimal theory is *quite o-minimal* if the relations of weak and quite orthogonality for 1-type coincide.

Quite o-minimal theories form a subclass of the class of weakly o-minimal theories preserving a series of properties of o-minimal theories. For instance, in [4], \aleph_0 -categorical quite o-minimal theories were completely described. This description implies their binarity (the similar result holds for \aleph_0 -categorical o-minimal theories).

THEOREM 3. *Let M be a model of an \aleph_0 -categorical quite o-minimal theory, M' be an expansion of M by an arbitrary finite family of convex unary predicates. Then M' is a model of an \aleph_0 -categorical quite o-minimal theory of the same convexity rank.*

[1] B. S. BAIZHANOV, *Expansion of a model of a weakly o-minimal theory by a family of unary predicates. The Journal of Symbolic Logic*, vol. 66 (2001), no. 3, pp. 1382–1414.

[2] B. S. KULPESHOV, *Convexity rank and orthogonality in weakly o-minimal theories. News of the National Academy of Sciences of the Republic of Kazakhstan*, Physical and Mathematical Series, vol. 227, 2003, pp. 26–31.

[3] ———, *Weakly o-minimal structures and some of their properties. The Journal of Symbolic Logic*, vol. 63 (1998), no. 4, pp. 1511–1528.

[4] ———, *Countably categorical quite o-minimal theories. Journal of Mathematical Sciences*, vol. 188 (2013), no. 4, pp. 387–397.

- ▶ MARTIN MOSE BENTZEN, *Logic without unique readability—a study of semantic and syntactic ambiguity.*

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One of the main reasons for introducing a formal language is to remove ambiguity, the possibility of assigning several meanings to a linguistic expression. Typically, this is achieved through ensuring unique readability of formulas by using brackets (or another convention, such as Polish notation). Unique readability implies meaning uniqueness, exactly one valuation of a sentence given an interpretation of basic formulas and recursive truth conditions. Obviously, in natural language this one-to-one correspondence between syntax and semantics is absent, the unique readability assumption does not hold true universally. Whereas, e.g., scope ambiguities in natural languages have been studied extensively, ambiguous formal languages have not been the focus of in depth research. Here, we lift the assumption of unique readability by omitting the brackets from propositional logic, making it possible to formally distinguish between syntactic and semantic ambiguity. A valuation then amounts

to a semantic disambiguation, and rather than a unique valuation (truth value), there is a set of valuations corresponding to ways a formula could have been constructed. We show what happens to familiar concepts of logic such as definability, satisfiability, and validity. Here follows two simple examples illustrating the relation between syntactic and semantic ambiguity. In some cases unique readability can be regained through careful construction of formulas. E.g., although an attempt to define $p \rightarrow q$ as $\neg p \vee q$ would be syntactically and semantically ambiguous, one may define it as $q \vee \neg p$, which can be read only one way (but obviously this construction is not stable under substitution). Syntactic ambiguity does not imply semantic ambiguity, although it is typically the case. For instance, although the formula $p \wedge \neg p \wedge p$ can be read in three ways, it has only one possible meaning (a contradiction).

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The three Written English expressions ‘statement’, ‘proposition’, and ‘sentence’ used in logic and philosophy of logic are ambiguous (multisense, polysemic): people use each with multiple normal meanings (senses, definitions). Several of their meanings are vague (imprecise, indefinite): they admit borderline (marginal, fringe) cases. This article juxtaposes, distinguishes, and analyses several senses of these words focusing on a constellation of recommended senses.

As recommended, a *statement* is a unique event, a speech-act performed by a unique person at a unique time and place. By contrast, propositions and sentences are timeless and placeless abstractions. A *proposition* is an intensional object, a sense composed of senses (concepts). A *sentence* is a linguistic entity (string-type) composed of character-types. Sentences in themselves are meaningless.

It is only a proposition that is properly said to be true or to be false, although—with suitable qualification—statements, or even sentences, may be said to be true or false in appropriate derivative senses.

Persons use sentences to express the propositions they state in the statements they make. As examples make clear, one and the same sentence is routinely used on different occasions to express different propositions. Likewise clarified by examples is the fact that different sentences express one and the same proposition. Persons make statements; they don’t make sentences or propositions.

This article clarifies, qualifies, and, in a few cases, retracts various views previously expressed by the author. It is intended as a philosophical sequel to [1], [2], and [3]

[1] J. CORCORAN, *Meanings of implication. Diálogos*, vol. 9 (1973), pp. 59–76.

[2] ———, *Argumentations and logic. Argumentation*, vol. 3 (1989), pp. 17–43.

[3] ———, *Sentence, proposition, judgment, statement, and fact: Speaking about the Written English used in logic. The Many Sides of Logic* (W. Carnielli et al., editors), College Publications, 2009, pp. 71–103.

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The ambiguous verb ‘contradicts’ and its cognates play important roles in logic and logic-related literature. We study their uses. We build on [3]: a treatment of the verb ‘implies’ and its cognates. We exploit similarities with ‘implies’ as studied in [1], which observes that the verb ‘implies’ as a relation verb can express relations of various semantic categories: for example, person-to-proposition relations as well as proposition-to-proposition relations: as in ‘Cantor implies that omega exists’ and ‘Zorn’s Lemma implies the Choice Axiom’, respectively.

The verb ‘contradicts’ expresses person-to-person relations ([2, pp. 52 et al.]), person-to-proposition relations ([2, pp. 68, 108, et al.]), and proposition-to-proposition relations

([2, p. xxvii et al.]) (to mention only three): as in “Cantor contradicts Brouwer”, ‘Brouwer contradicts Bivalence’, and ‘Bivalence contradicts Trivalence’. Of course there are several more, e.g., the set-to-proposition relation: a given set of propositions contradicts a given proposition iff the set implies the negation of the given proposition: ‘Lobachevskian Geometry contradicts Euclid’s Parallel Postulate’.

The noun ‘contradiction’—besides being used as a proper name of various relations including the one that holds from a set to a proposition iff the former contradicts the latter—has various uses as a common noun applying to propositions, for example, to those whose negations are tautological. It also serves as a part of ambiguous expressions such as ‘the principle of contradiction’ [4].

The string ‘contradicting’ is found in different grammatical categories; [2] uses it in three categories on a single page, p. xxviii.

[1] K. BARBER and J. CORCORAN, *Agent and premise implication*, this BULLETIN, vol. 15 (2009), p. 235.

[2] M. COHEN and E. NAGEL, *Introduction to Logic*, second ed. (J. Corcoran, editor), Hackett, 1993.

[3] J. CORCORAN, *Meanings of implication*. *Diálogos*, vol.9 (1973), pp. 59–76.

[4] J. CORCORAN and J. LEGAULT, *Aristotle, Boole, and Tarski on contradiction*, this BULLETIN, vol. 19 (2013), p. 515.

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Where $S(x)$ and $P(x)$ are predicates in the sense of [1] and [2], the predicate $[S(x) \rightarrow P(x)]$ is called the *conditional [predicate] with antecedent $S(x)$ and consequent $P(x)$* . The sentence $\forall x[S(x) \rightarrow P(x)]$, which is the universalization of the conditional $[S(x) \rightarrow P(x)]$, is said to be *the universalized-conditional [sentence] with antecedent $S(x)$ and consequent $P(x)$* . Students should note that universalized-conditionals are universals; no universalized-conditional is a conditional.

The universalized-conditional $\forall x[(S(x) \ \& \ \sim P(x)) \rightarrow P(x)]$ is *the negated-consequent-qualification [NCQ] of the universalized-conditional $\forall x[S(x) \rightarrow P(x)]$* . It is easy to see that the NCQ of a universalized-conditional is logically-equivalent to the universalized-conditional itself.

Moreover, the existentialized-conjunction corresponding to an NCQ, say, $\exists x[(S(x) \ \& \ \sim P(x)) \ \& \ P(x)]$, is evidently inconsistent and thus not implied by its NCQ unless the latter is inconsistent. This shows that no consistent negated-consequent-qualification has existential import in the sense of [1] and [2].

But since every universalized-conditional is logically equivalent to its NCQ, we have the *Hazen Lemma*: every consistent universalized-conditional is logically-equivalent to a universalized-conditional without existential import.

One important question—answered definitively by Allen Hazen in correspondence with Corcoran and Masoud—concerns which universalized-conditionals with existential-import are logically-equivalent to universalized-conditionals without existential-import.

Hazen’s Theorem: A universalized-conditional with existential-import is logically equivalent to some universalized-conditional without existential-import iff it is consistent.

Hazen’s contributions nicely complement our results in [1] and [2]. They use none of our previously published conclusions: they are entirely new.

[1] J. CORCORAN and H. MASOUD, *Existential-import today*. *History and Philosophy of Logic*, vol. 36 (2015), pp. 39–61.

[2] ———, *Existential-import mathematics*, this BULLETIN, vol. 21 (2015), pp. 1–14.

► VALERIA DE PAIVA AND GISELLE REIS, *Benchmarking linear logic*.

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Benchmarking automated theorem proving (ATP) systems using standardised problem sets is a well-established method for measuring their performance, especially in the case of classical logical systems. However, the availability of such libraries for *nonclassical* logics is very limited. For intuitionistic logic several small collections of formulas have been published and used for testing ATP systems and Raths, Otten, and Kreitz [2] consolidated and extended these small sets to provide the ILTP Library <http://www.cs.uni-potsdam.de/ti/iltp/>. For quantified modal systems we have both Wisniewski, Steen and Benz Müller's as well as the Raths and Otten libraries of problems.

In this work we seek to provide a similar benchmark for Girard's Linear Logic [1] and some of its variants. For quick bootstrapping of the collection of problems we use Girard's translation of the collection of intuitionistic theorems discussed in the ILTP library. Eventually we hope to compare different Linear Logic provers over an augmented collection of problems.

[1] J.-Y. GIRARD, *Linear logic*. *Theoretical Computer Science*, vol. 50 (1987), no. 1, pp. 1–101.

[2] C. KREITZ, J. OTTEN, and T. RATHS. *The ILTP problem library for intuitionistic logic*. *Journal of Automated Reasoning*, vol. 38 (2007), no. 1–3, pp. 261–271.

► KIT FINE AND MARK JAGO, *Semantics for exact entailment*.

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An *exact truthmaker* for A is a state which, as well as guaranteeing A 's truth, is wholly relevant to it. States with parts irrelevant to whether A is true do not count as exact truthmakers for A . Giving semantics in this way produces a very unusual consequence relation, *exact entailment*, understood in terms of preservation of exact truthmakers from premises to conclusion. On this understanding, conjunctions do not exactly entail their conjuncts. This feature makes the resulting logic highly unusual.

In this article, we set out formal semantics for exact entailment in terms of mereological structures on a domain of states. The main result of the article is a characterisation theorem, which establishes the syntactic form premises and conclusions must take in an exact entailment. This gives us a conceptual handle on when an exact entailment holds. In intuitive terms, it holds when some ground for the conclusion lies 'in between' a ground for one premise and a ground for all premises taken together. Using this theorem, we show that exact entailment is compact and decidable.

We then investigate the effect of various restrictions on the semantics. The first is to *nonvacuous* models, wherein every atomic sentence letter has a truthmaker and a falsemaker somewhere in the model. The second is to *convex* models, whereby states lying in-between two truthmakers for some A must also be truthmakers for A . We show that neither restriction, in isolation, affects the entailment relation. But their combination produces a stronger logic, for which we provide a further characterisation theorem.

Finally, we formulate a sequent-style proof system for exact entailment and give soundness and completeness results.

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When placed within the context of proof-theoretic justification of deduction (Dummett, Prawitz) recent studies on the question of stability of intelim rules reveal the importance of the two distinct notions of intrinsic harmony, and, total harmony (or, what is the same thing: the satisfaction of the requirement of conservative extension (Belnap)) with respect to the

intelim rules. The inversion principle of Prawitz captures the notion of intrinsic harmony. But nonsubstructural weak disharmony (Dicher) can creep in even if the inversion principle is satisfied by a constant, as can be seen in the case of the constant called 'knot' (Dicher) which is the dual of 'tonk' (Belnap). Dicher's study hints that lack of nonsubstructural weak disharmony amounts to stability for intelim rules which are insulated from tinkering with structural rules. For Dummett, harmony along with this sort of stability make a constant self-justifying. So, intrinsic harmony does not entail the satisfaction of the requirement of conservative extension. Does the satisfaction of the requirement of conservative extension entail intrinsic harmony? The present article attempts to show that the intelim rules for constants of minimal logic (system M of Prawitz) when satisfy the requirement of conservative extension in the context of the language for deducibility-as-such (Belnap, 'Tonk, plonk and plink') also have intrinsic harmony, i.e., respect the inversion principle. It goes by contrapositively showing that within the specified context, if the inversion principle is violated then conservative extension is also violated. Dummett conjectured that 'intrinsic harmony implies total harmony in a context where stability prevails' (*The Logical Basis of Metaphysics*, HUP, 1991, p. 290). In such a case, given that total harmony entails intrinsic harmony in a specified context, intrinsic harmony coupled with stability (or, lack of nonsubstructural weak disharmony), and total harmony would coincide in that context.