Constructive Identities for Physics

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Abstract

In his classical paper "On Sense and Reference" [1] Frege asks: In which sense the Morning Star (MS) and the Evening Star (ES) are the same planet Venus? As Frege observes the assertion of identity MS=ES unlike the assertion of identity MS=MS (or MS=MorningStar) has empirical content and in this respect is not trivial. So it is unclear how the same notion of identity may apply in these two very different cases. Frege solves the problem by distinguishing between the sense and the reference of a given linguistic expression: although sentences MS=MS and MS=ES have the same reference true they still have different senses (i.e., different meaning).

Notice that Frege's example is not purely linguistic: in his time similar questions concerning identities of some comets and asteroids remained open; in today's astronomy the identification of new astronomical sources is never given for free but always has a complicated procedural character. Although Frege managed to explain how the identity MS=ES can hold in principle, his theory of identity provides no clue to how this or any other identity occurring in empirical sciences can be possibly grounded and justified. Since Frege's theory does not take the issue of empirical justification into account it remains largely irrelevant to the practice of empirical sciences. Given that the naive pretheoretical understanding of identity proves insufficient at least in some areas of the modern science (think about the particle physics) the need of a new formal approach to identity in physics and other empirical sciences seems me obvious.

In this paper I develop a constructive approach to identity in physics based on Martin-Löf's Constructive Type theory [2] and Voevodsky's Homotopy Type theory [3]. While earlier attempts to modify the classical identity were mostly motivated by the idea of its weakening (as in the case of Krause's theory of quasi-sets, for example), the Homotopy Type theory allows for a view on identity as a construction from available empirical data and theoretical predictions/retrodictions. Suppose after Frege that identities of MS and ES are somehow fixed beforehand. This means that one is in a position to identify two independent observations of MS as observations of one and the same object; similarly for ES. In Martin-Löf's theory such presupposed identities are called definitional, while the

non-trivial identity MS=ES counts as propositional. In order to establish proposition MS=ES one uses available observational data (along with a theory allowing for predictions and retrodictions of future and past positions positions of celestial bodies) and reconstructs a continuous path (trajectory) from MS to ES. In the classical celestial mechanics such a continuous trajectory indeed qualifies as the wanted identity proof, namely, as the wanted evidence of the fact that MS and ES is in fact one and the same planet continuously moving from its morning position to its evening position.

Modern physics provides contexts where such identity paths are multiple and support non-trivial homotopic structures. Two obvious examples are gravitational lensing and Feynman path integrals (interpreted in terms of multiple paths of the same particle). In both these cases the identification (of sources and particles correspondingly) involves not only paths but also their homotopies (i.e. paths between paths or 2-paths). Thus in these cases the 2nd order identities (in the sense of Martin-Löf's theory) also acquire a physical meaning. Whether still higher-order identities may equally have some physical meaning remains a research question. A recent work by Schreiber [4] where higher-order identities are understood as gauge transformations suggests the answer in positive. Pushing this line one may tentatively consider the Homotopy type theory as a general logico-mathematical framework for representing physical objects: in this framework objects are represented by higher-order homotopy groupoids, which determine the objects' identity types and also their topological properties.

Tentative applications of higher identity types in empirical contexts help one to clarify the distinction between the definitional and the propositional identity Martin-Löf's theory from an epistemological standpoint. Admittedly one cannot proceed a scientific reasoning without taking some notion of identity for granted. In Martin-Löf's theory this role is played by the definitional identity. However there is no reason to consider the definitional identity as fundamental. We treat the identity MS=MorningStar as definitional simply because we have decided (after Frege) not to analyze the way in which different phenomena observed at different times by different people are identified as observations of the same Morning Star. In a different context the non-trivial empirical character of this identity can be similarly taken into account. Thus a definitional identity has the character of explicit assumption (that can be questioned and analyzed if needed) rather than that of ultimate foundation.

References

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