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Collection and Connection

Abstract
Spatial thinking is inherent to the concept of **extensionality** and hence to the (extensional) set theory. To make this clear I provide a dual **intensional** reading of ZF which has a *temporal* meaning. This gives a reason to embed a topological structure into the theory (in both cases) instead of putting the structure on the top of it as it is usually made. Combined *spatiotemporal* systems are tentatively considered. Finally I consider a possibility to apply the same approach to logic (challenging Russell's logical atomism).

By a «set» we understand every collection to a whole M of definite, well-differentiated objects m of our intuition or our thought. We call these objects the «elements» of M.
Cantor, 1895
.. all we are interested in with sets is what members they have. ...
Drake, 1974

$x\in y =_{Def}$ set x is **element** of set y; set y is **host** of set x

Why think of sets in terms of their elements but not in terms of their hosts?

ZF	**ZF***
$\forall x\forall y(\forall z(z\in x\leftrightarrow z\in y)\rightarrow x=y)$ **extensionality**:	$\forall x\forall y(\forall z(x\in z\leftrightarrow y\in z)\rightarrow x=y)$ **intensionality** (identity of indiscernibles)
x is **atom** $=_{Def} \neg\exists y(y\in x)$; given extensionality the atome is unique	x is **world** $=_{Def} \neg\exists y(x\in y)$ given intensionality the world is unique
$\forall a\forall b(a\neq b\rightarrow\exists p\forall x(x\in p\leftrightarrow(x=a\lor x=b)))$ **pairing**: shared host: space	$\forall a\forall b(a\neq b\rightarrow\exists p\forall x(p\in x\leftrightarrow(x=a\lor x=b)))$ **connection**: shared element: time
$\forall a(\exists b(b\in a)\rightarrow\exists y\forall x(x\in y \leftrightarrow\exists z\ (x\in z\ \&\ z\in a)))$ **union**: elements of elements	$\forall a(\exists b(a\in b)\rightarrow\exists y\forall x(y\in x \leftrightarrow\exists z\ (z\in x\ \&\ a\in z)))$ **intersection**: hosts of hosts
y is **subset** of z $=_{Def}\forall x(x\in y\rightarrow x\in z)$ every element of y is an element of z $\forall x(x\subseteq x)$	y is **superelement** of z $=_{Def}\forall x(y\in x\rightarrow z\in x)$ every host of y is a host of z $\forall x(x\supseteq x)$
$\forall a\exists y\forall x(x\in y\leftrightarrow x\subseteq a)$ **power**: set of (all the) subsets given powering no worlds	$\forall a\exists y\forall x(y\in x\leftrightarrow a\supseteq x)$ **root**: element of (all the) superelements given rooting no atoms
predicates generate **subsets** $\forall a\exists y\forall x\ (x\in y\leftrightarrow x\in a\ \&\ \varphi(x))$ $(x\neq x)$: the atom (the empty set) exists $\forall x(\varnothing\subseteq x)$	*abstractors* generate **superelements** $\forall a\exists y\forall x\ (y\in x\leftrightarrow a\in x\ \&\ \varphi(x))$ the world exists $\forall x(W\supseteq x)$
$\exists y(y\in x)\rightarrow\exists y(y\in x\ \&\forall z\neg(z\in x\&z\in y)$ **foundation**	$\exists y(x\in y)\rightarrow\exists y(x\in y\ \&\forall z\neg(x\in z\&y\in z)$ **upside down foundation (cofoundation)**