

*Categorification and Formal Axiomatic method: Towards a Hermeneutic Model of Science*

By "categorification" I mean a reconstruction of mathematical concepts in category-theoretic terms which amounts to considering convenient mathematical objects like sets or topological spaces together with properly defined maps between these objects (functions and continuous transformations in the given example). I argue that categorification provides a viable alternative to (in fact a generalisation of) Formal Axiomatic method. My argument is the following. (1) The Formal Axiomatic method was designed by Hilbert as a response to the new situation in his contemporary mathematics where the notion of interpretation became to play a major role in this discipline. Think about interpretation of Lobachevskian geometry in Euclidean terms by Beltrami in 1968. (2) The Hilbert's response allowed for an adequate treatment of the new situation only in the case when interpretations in question are reversible, that is, when they are isomorphisms (think about substitutions of beer mugs for points). (3) But in the general case interpretations are not reversible. In particular Beltrami's "translation" of Lobachevskian geometry into terms of Euclidean geometry is an embedding (monomorphism) but not an isomorphism: Lobachevskian planimetry can be accounted for in terms of Euclidean stereometry (modulo certain reservations which I leave aside) but certainly not the other way round. (4) The notion of category captures the relevant notion of interpretation in the general case.

This generalisation of Formal approach made possible by Category theory has rather dramatic epistemological consequences which I'm going to explore. Just like in the case of Formal Axiomatic method these consequences extend far outside of pure mathematics. In particular Categorical approach suggests a replacement of traditional hierarchical structures associated with Aristotelian Model of science by "horizontal" structures directly linking different scientific domains. I shall point to an analogy between Categorical approach and Hermeneutic method, and argue that the former can be seen as a mathematicized version of the latter appropriate for exact sciences.