New-Old Axiomatic Method

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Hilbert&Bernays 1934

The term axiomatic will be used partly in a broader and partly in a narrower sense.We will call the development of a theory axiomatic in the broadest sense if the basic notions and presuppositions are stated first, and then the further content of the theory is logically derived with the help of definitions and proofs. In this sense, Euclid provided an axiomatic grounding for geometry, Newton for mechanics, and Clausius for thermodynamics.

Hilbert&Bernays 1934

[F]or axiomatics in the narrowest sense, the *existential form* comes in as an additional factor. This marks the difference between the *axiomatic method* and the *constructive* or *genetic* method of grounding a theory. While the constructive method introduces the objects of a theory [..], an axiomatic theory [in the narrow sense of "axiomatic"] refers to a fixed system of things (or several such systems) [i.e. to one or several models].[..] This is an idealizing assumption that properly augments [?] the assumptions formulated in the axioms.

Hilbert&Bernays 1934

When we now approach the task of such an impossibility proof [= proof of consistency], we have to be aware of the fact that we cannot again execute this proof with the method of axiomatic-existential inference. Rather, we may only apply modes of inference that are free from idealizing existence assumptions.

Hilbert&Bernays 1934

Yet, as a result of this deliberation, the following idea suggests itself right away: If we can conduct the impossibility proof without making any axiomatic-existential assumptions, should it then not be possible to provide a grounding for the whole of arithmetic directly in this way, whereby that impossibility proof would become entirely superfluous?

Hilbert's answer is in negative because of his worries about infinity. His argument does not appear to me as conclusive. This is a topic for another research.

Some reasons to be dissatisfied with FAM

(1) FAM does not apply straightforwardly in the mainstream 20th c. maths.

Example: Group theory is a model theory of the axiomatic group theory, i.e., the theory determined by the three group axioms. These axioms serve *only for defining* the concept of group. Most of theorems of groups theory (like Lagrange theorem) do not follow directly from these three axioms (just like the angle sum theorem of the Euclidean geometry does not follow directly from the definition of triangle).

Some reasons to be dissatisfied with FAM

(2) The impact of FAM on Set theory is unclear.
Example: The Independence of CH from ZF is well-established mathematical fact; the proof of this theorem (Gödel-Cohen) is not a formal axiomatic proof - notwithstanding the fact that this theorem treats a formal theory, namely ZF as its object (its subject-matter). This Independence result neither proves nor refutes CH. It does not allow to rule out CH as ill-posed either (after the example of Euclid's 5th Postulate). The full-scale relativism about mathematical statements is not consistent with the claim that the Independence of CH from ZF is well-established.

Some reasons to be dissatisfied with FAM

(3) The 20th c. showed no significant progress in the axiomatization of physics (Hilbert's 6th Problem). During this century FAM played <u>no role at all</u> in the mainstream research in physics and other natural sciences.
This one, in my view, is the strongest reason (however in my book I don't focus on it).

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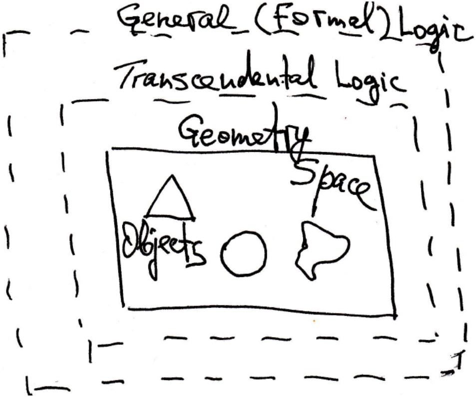
Conclusions on FAM

- Axiomatic method does not reduce to FAM. Giving up FAM is not giving up the axiomatic method as such.
- Hilbert's notion of FAM has several independent ingredients. In particular the "idealizing existential assumption" is wholly independent from the issue of symbolism (the former is more essential for FAM since FAM may work also in non-symbolic form as in *Foundations* of 1899).
- The predicate "formal" must be carefully interpreted in each particular context.

Classical Picture (Newton-Kant)

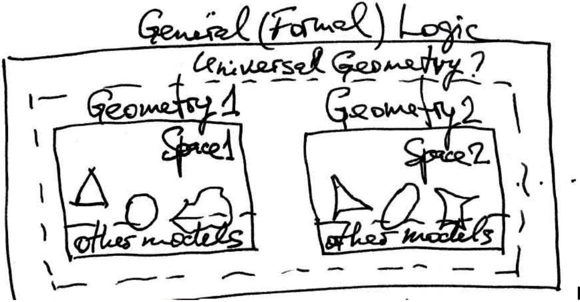


(inclusion means subsumption rather than being a part of)

Friedman on Kant on Euclid

Euclidean geometry [..] is not to be compared with Hilbert's axiomatization [of Euclidean geometry], say, but rather with Frege's *Begriffsschrift*. It is not a substantive doctrine, but a form of rational representation: a form of rational argument and inference.

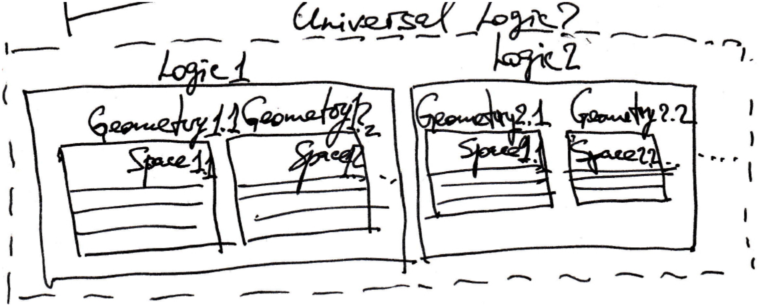
Modern Picture (Hilbert)



many geometries

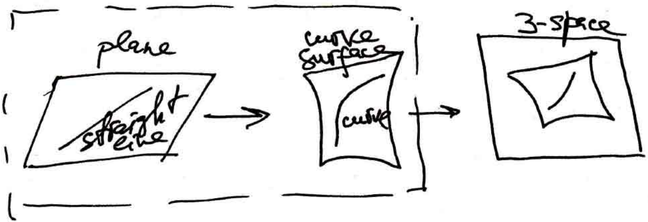
but one logic

Logical pluralism



many logics and many mathematics

The idea of intrinsic geometry (Gauss-Riemann-Klein)



Objects are maps!

Motivating example (classical). The expression "Euclidean plane" is ambiguous.
In one sense it means a geometrical space studied in Planimetry where live circles, triangles, etc (EPLANE);
In a different sense it means an object living in the Euclidean 3-space (ESPACE)(eplane):

$$EPLANE \xrightarrow{eplane} ESPACE$$

Objects are maps!

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- There are many different eplanes living in ESPACE;
- Circles, etc. in ESPACE factor through EPLANE:

$$\begin{tikzcd} CIRCLE \arrow[r, "circle_1"] \arrow[d, "circle_2"'] & EPLANE \arrow[dl, "eplane"] \\ ESPACE & \end{tikzcd}$$

Objects are maps!

General situation:

$$TYPE \xrightarrow{object} SPACE$$

<u>Remarks</u>:
Being a type and being a space are relational properties. Being an object is non-relational property.
Each object is of particular type and lives in a particular space.

Objects are maps!

Non-classical examples:

$$HPLANE \xrightarrow{pseudosphere} ESPACE$$

(Beltramy)

$$EPLANE \xrightarrow{horisphere} HSPACE$$

(Lobachevsky)
Remark: Pseudosphere and horisphere are not types/spaces but objects (without ambiguity).

Objects are maps!

Objects of the same type look differently in different spaces:

$$\begin{tikzcd}
 & HSPACE \\
EPLANE \arrow[ur, "horisphere"] \arrow[dr, "eplane"'] & \\
 & ESPACE
\end{tikzcd}$$

Objects of different types in the same space look always differently.

Different geometrical spaces are unified into a single whole through mutual mappings, i.e., through their shared objects. They typically form a *category*.
Examples: category of Riemanian manifolds and differentiable maps, category of topological spaces and continuous maps, etc.

Where is logic in this unification?
If the obtained category has appropriate properties (and, in particular "has enough objects") it supports an *internal logic* (cf. the notion of intrinsic geometry).
Examples:

Non-example: category of Rimanian manifolds

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Examples:

- category of sets and functions (Classical logic);
- Grothendieck toposes (Intuitionistic logic of many specific sorts);
- Model categories = categories of generalized topological spaces allowing for homotopy theory (Constructive Type theories)

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- Compare the conceptual shift from Gauss' theory of curve surfaces to Riemann's general theory of (differentiable) manifolds: intrinsic construction of manifolds; no fixed ambient space is needed.

Analogy with geometry

Epistemically intrinsic and extrinsic properties of a given manifold are to be treated on equal footing. In the language of arrows the *intrinsic* properties are expressed by <u>incoming</u> morphisms while the extrinsic properties are expressed by <u>outgoing</u> morphisms (in particular, by embeddings into outer spaces). A given type/space is characterized by morphisms of both sorts.

Analogy with geometry

However there is a sense in which any given space can be fully characterized intrinsically (in this case "full" means "intrinsic"). In that sense the Euclidean Planimetry fully describes EPLANE as a space. Extrinsic properties of EPLANE reveal themselves when the EPLANE embeds into ESPACE, HSPACE, etc.
Traditional essentialism requires to fix intrinsic properties first and study extrinsic (relational) properties afterwards. I do *not* share this view.

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Meta-logic

Is an external (meta-) logic really needed for axiomatic building? The answer depends on what one wants to get:

- It is not needed when one aims at building the target theory in the bottom-up way on the basis of raw facts. In this case the internal logic of the corresponding category is sufficient.
- It is needed when one wants to represent the target theory in some other theory. This method does not apply when the target theory is built independently from any other theory.

Epistemic independence

The usual epistemological meaning of being a *foundation* implies epistemic independence (in the narrow sense of the word explained above).

Claim: such independence is an epistemic value.

Object-building

Object-building

- (Geometrical) object-building turns (again) into an essential part of the axiomatic theory-building (cf. Euclid and Frege's *Begriffsschrift* and Hilbert&Ackermann axiomatization of logic). Thus NAM is object-oriented. With NAM the logical structure of a given theory is (partly) determined bottom-up, not (only) top-down.

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- The idea of internal logic of a given field of study is relatively old: cf. Reichenbach's Quantum Logic. However only the category-theoretic framework allowed for a precise mathematical formulation of this idea.

Claim

Lawere's axiomatization of Topos theory and Voevodsky's axiomatization of Higher Homotopy apply NAM rather than FAM.

Curry-Howard: Simply typed lambda calculus

Variable: $\dfrac{}{\Gamma, x : T \vdash x : T}$

Product: $\dfrac{\Gamma \vdash t : T \quad \Gamma \vdash u : U}{\Gamma \vdash \langle t, u \rangle : T \times U}$

$\dfrac{\Gamma \vdash v : T \times U}{\Gamma \vdash \pi_1 v : T}$ $\dfrac{\Gamma \vdash v : T \times U}{\Gamma \vdash \pi_2 v : U}$

Function: $\dfrac{\Gamma, x : U \vdash t : T}{\Gamma \vdash \lambda x.t : U \rightarrow T}$

$\dfrac{\Gamma \vdash t : U \rightarrow T \quad \Gamma \vdash u : U}{\Gamma \vdash tu : T}$

Curry-Howard: Natural deduction

Identity: $\dfrac{}{\Gamma, A \vdash A}$ (Id)

Conjunction: $\dfrac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \& B}$(& - intro)

$\dfrac{\Gamma \vdash A \& B}{\Gamma \vdash A}$ (& - elim1); $\dfrac{\Gamma \vdash A \& B}{\Gamma \vdash B}$ (& - elim2)

Implication: $\dfrac{\Gamma, A \vdash B}{\Gamma \vdash A \supset B}$ ($\supset$-intro)

$\dfrac{\Gamma \vdash A \supset B \quad \Gamma \vdash A}{\Gamma \vdash B}$ ($\supset$-elim aka *modus ponens*)

Curry-Howard Isomorphism

$$\& \equiv \times$$

$$\supset \equiv \rightarrow$$

Brouwer-Heyting-Kolmogorov (BHK interpretation)

- proof of $A \supset B$ is a procedure that transforms each proof of A into a proof of B;
- proof of $A \& B$ is a pair consisting of a proof of A and a proof of B

Historical remark

Foundational consideration played a crucial role in this story from the outset (Schönfinkel, Curry, Church, Kolmogorov, Lawvere, Lambek). The expression "Curry-Howard isomorphism", which suggests that we have here an unexplained/surprising formal coincidence, is due to Howard 1969. The *true* history (and the true meaning) still waits to be explored.

Lawvere and Lambek 1969

The structure behind the Curry-Howard isomorphism is precisely captured by the notion of *Cartesian closed category* (CCC), which is an (abstract) category with the terminal object, products and exponentials.

Examples: Sets, Boolean algebras

Simply typed lambda-calculus / natural deduction is the *internal language* of CCC.

- Objects: types / propositions
- Morphisms: terms / proofs

Lawvere's philosophical motivation

- objective invariant structures vs. its subjective syntactical presentations
- objective logic vs. subjective logic (Hegel)

From FAM to NAM

The concept of CCC was discovered by Lawvere in 1969 (as a general setting for diagonal arguments) 5 years after he first axiomatized Set theory as a (first-order) theory of the category of sets (ETCS in 1964). These 5 years mark Lawvere's shift from FAM to NAM: instead of "using" the external (classical) FOL as logical foundation he now aims at building FOL internally as a part of his target axiomatic theory!

Higher-order generalization: Hyperdoctrines (Lawvere)

- Quantifiers as adjoints to substitution; hyperdoctrines (1969)
- Toposes (1970)
- *Locally* Cartesian closed categories (LCCC) (1972)

Lawvere on logic and geometry

The unity of opposites in the title is essentially that between logic and geometry, and there are compelling reasons for maintaining that geometry is the leading aspect. At the same time, in the present joint work with Myles Tierney there are important influences in the other direction: a Grothendieck "topology" appears most naturally as a modal operator, of the nature "it is locally the case that", the usual logical operators, such as $\forall$, $\exists$, $\Rightarrow$ have natural analogues which apply to families of geometrical objects rather than to propositional functions, and an important technique is to lift constructions first understood for "the" category $\underline{S}$ of abstract sets to an arbitrary topos .

Lawvere on logic and geometry (continued)

We first sum up the principle contradictions of the Grothendieck-Giraud-Verdier theory of topos in terms of four or five adjoint functors [..] enabling one to claim that in a sense logic is a special case of geometry. (Lawvere 1970)

Lawvere's axioms for topos

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(Elementary) topos is a category which

- has finite limits
- is CCC
- has a subobject classifier

From NAM back to FAM

McLarty rebuilds Lawvere's axiomatic Topos theory by FAM standard. He notices that most of his axiomatic construction can be done internally in any topos (except specific constructions in *Set*). McLarty introduces the notion of internal language of topos (ch. 14) and then describes how a given topos "looks from inside", i.e., can be described in terms of its own internal language (ch. 16 titled "From the Internal Language to the Topos"). However unlike Lawvere McLarty does not try to use this internal description for the axiomatic development of topos theory. I claim that in this respect McLarty's version of the axiomatic Topos theory is not adequate to Lawvere's.

MLTT (Martin-Löf 1980): key features

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- double interpretation of types: "sets" and propositions
- double interpretation of terms: elements of sets and proofs of propositions
- higher orders: dependent types (sums and products of families of sets)
- MLTT is the internal language of LCCC (Seely 1983)

MLTT: two identities

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- Definitional identity of terms (of the same type) and of types: $x = y : A$; $A = B : type$ (substitutivity)

MLTT: two identities

- Definitional identity of terms (of the same type) and of types: $x = y : A$; $A = B : type$ (substitutivity)
- Propositional identity of terms x, y of (definitionally) the same type A:
  $Id_A(x, y) : type$;
  Remark: propositional identity is a (dependent) type on its own.

MLTT: Higher Identity Types

- $x', y' : Id_A(x, y)$
- $Id_{Id_A}(x', y') : type$
- and so on

Fundamental group

Fundamental group G_T^0 of a topological space T:

- a base point P;
- loops through P (loops are circular paths $l : I \rightarrow T$);
- composition of the loops (up to homotopy only! - see below);
- identification of homotopic loops;
- independence of the choice of the base point.

Fundamental (1-) groupoid

G^1_T:

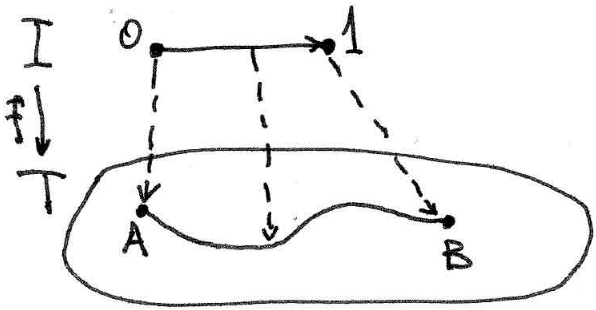
- all points of T (no arbitrary choice);
- paths between the points (embeddings $s : I \rightarrow T$);
- composition of the *consecutive* paths (up to homotopy only! - see below);
- identification of homotopic paths;

Since not all paths are consecutive G^1_T contains more information about T than G^0_T!

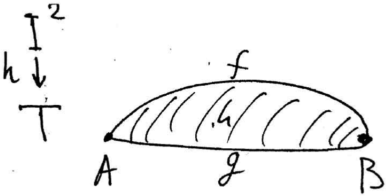
Path Homotopy and Higher Homotopies

$s : I \longrightarrow T$, $p : I \longrightarrow T$ where $I = [0, 1]$: paths in T
$h : I \times I \longrightarrow T$: homotopy of paths s, t if $h(0 \times I) = s$, $h(1 \times I) = t$
$h^n : I \times I^{n-1} \longrightarrow T$: n-homotopy of $n-1$-homotopies h_0^{n-1}, h_1^{n-1} if $h^n(0 \times I^{n-1}) = h_0^{n-1}$, $h^n(1 \times I^{n-1}) = h_1^{n-1}$;
Remark: Paths are zero-homotopies

Path Homotopy and Higher Homotopies



Homotopy categorically and Categories homotopically



$$\begin{tikzcd} A \arrow[r, bend left, "f"] \arrow[r, bend right, "g"'] \arrow[r, phantom, "\Downarrow"] & B \end{tikzcd}$$

Higher Groupoids and Omega-Groupoids (Grothendieck 1983)

- all points of T (no arbitrary choice);
- paths between the points ;
- homotopies of paths
- homotopies of homotopies (2-homotopies)
- higher homotopies up to n-homotopies
- higher homotopies ad infinitum

G^n_T contains more information about T than G^{n-1}_T!

Composition of Paths

Concatenation of paths produces a map of the form $2I \rightarrow T$ but not of the form $I \rightarrow T$, i.e., not a path. We have the whole space of paths $I \rightarrow 2I$ to play with! But all those paths are homotopical. Similarly for higher homotopies (but beware that n-homotopies are composed in n different ways!)
On each level when we say that $a \oplus b = c$ the sign $=$ hides an infinite-dimensional topological structure!

Grothendieck Conjecture:

G^{ω}_{T} contains all relevant information about T; an omega-groupoid is a complete algebraic presentation of a topological space.

Homotopy Type theory

- Groupoid model of MLTT: basic types are groupoids, terms are their elements, dependent types are fibrations of groupoids (families of groupoids indexed by groupoids - rather than families of sets indexed by sets). Extensionality one dimension up. (Streicher 1993).
- Higher (homotopical) groupoids model higher identity types. Intensionality all way up (Voevodsky circa 2008).

Voevodsky on Univalent Foundations

The broad motivation behind univalent foundations is a desire to have a system in which mathematics can be formalized in a manner which is as natural as possible. Whilst it is possible to encode all of mathematics into Zermelo-Fraenkel set theory, the manner in which this is done is frequently ugly; worse, when one does so, there remain many statements of ZF which are mathematically meaningless. This problem becomes particularly pressing in attempting a computer formalization of mathematics; in the standard foundations, to write down in full even the most basic definitions - of isomorphism between sets, or of group structure on a set - requires many pages of symbols.

Voevodsky on Univalent Foundations (continued)

Univalent foundations seeks to improve on this situation by providing a system, based on Martin-Löf's dependent type theory whose syntax is tightly wedded to the intended semantical interpretation in the world of everyday mathematics. In particular, it allows the direct formalization of the world of homotopy types; indeed, these are the basic entities dealt with by the system. (Voevodsky 2011)

h-levels

- (i) Given space is called A *contractible* (aka space of h-level 0) when there is point $x : A$ connected by a path with each point $y : A$ in such a way that all these paths are homotopic.
- (ii) We say that A is a space of h-level $n + 1$ if for all its points x, y path spaces $paths_A(x, y)$ are of h-level n.

h-universe

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- Level 0: up to homotopy equivalence there is just one contractible space that we call "point" and denote pt;

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- Level 0: up to homotopy equivalence there is just one contractible space that we call "point" and denote pt;
- Level 1: up to homotopy equivalence there are two spaces here: the empty space $\emptyset$ and the point pt. (For $\emptyset$ condition (ii) is satisfied vacuously; for pt (ii) is satisfied because in pt there exists only one path, which consists of this very point.) We call $\emptyset, pt$ *truth values*; we also refer to types of this level as *properties* and *propositions*. Notice that h-level n corresponds to the logical level $n-1$: the propositional logic (i.e., the propositional segment of our type theory) lives at h-level 1.

h-universe

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- Level 2: Types of this level are characterized by the following property: their path spaces are either empty or contractible. So such types are disjoint unions of contractible components (points), or in other words *sets* of points. This will be our working notion of set available in this framework.

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- Level 3: Types of this level are characterized by the following property: their path spaces are sets (up to homotopy equivalence). These are obviously (ordinary flat) *groupoids* (with path spaces hom-sets).

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- Level 3: Types of this level are characterized by the following property: their path spaces are sets (up to homotopy equivalence). These are obviously (ordinary flat) *groupoids* (with path spaces hom-sets).
- Level 4: 2-groupoids

h-universe

- ..
- Level n+2: n-groupoids
- ..
- ω-groupoids
- ω-groupoids $(\omega + 1 = \omega)$

How it works

Let *iscontr*(A) and *isaprop*(A) be formally constructed types " A is contractible" and "A is a proposition" (for formal definitions see Voevodsky:2011, p. 8). Then one formally deduces (= further constructs according to the same general rules) types *isaprop*(*iscontr*(A)) and *isaprop*(*isaprop*(A)), which are non-empty and thus "hold true" for each type A; informally these latter types tell us that for all A "A is contractible" is a proposition and "A is a proposition" is again a proposition.

How it works

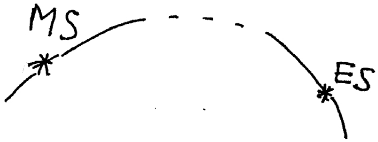
With the same technique one defines in this setting type $weq(A, B)$ of *weak equivalences* (i.e., homotopy equivalences) of given types A, B (as a type of maps $e : A \to B$ of appropriate sort) and formally proves its expected properties. These formal proves involve a *different* type $isweq(A, B)$ of h-level 2, which is a proposition saying that A, B are homotopy equivalent, i.e., that type $weq(A, B)$ is inhabited.)

Axiom of Univalence

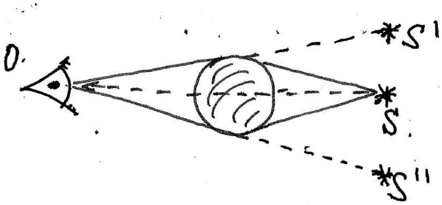
Homotopically equivalent types are (propositionally) identical. This means that the universe *TYPE* of homotopy types is construed like a homotopy type (and also modeled by ω-groupoid).
Axiom of Univalence is the <u>only</u> axiom of Univalent Foundations on the top of MLTT.

Object-building with NAM and HoTT

Identity through time

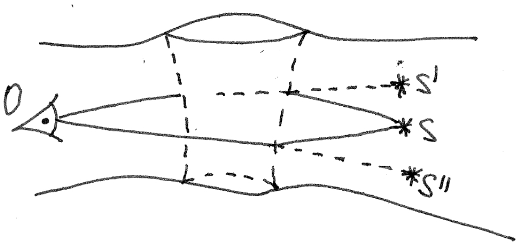


Object-building with NAM and HoTT



Gravitational lensing

Object-building with NAM and HoTT



Wormhole lensing

Conclusion

The New Axiomatic Method is the Good Old Genetic Axiomatic Method of Euclid, Newton and Clausius.

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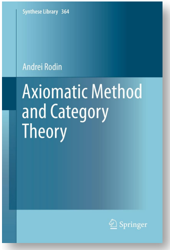

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- **Provides a deep textual analysis of Euclid, Hilbert, and Lawvere that describes how their ideas are different and how their ideas progressed over time**
- **Presents a hypothetical New Axiomatic Method, which establishes closer relationships between mathematics and physics**

This volume explores the many different meanings of the notion of the axiomatic method offering an insightful historical and philosophical discussion about how these notions changed over the millennia.
The author, a well-known philosopher and historian of mathematics, first examines Euclid who is considered the father of the axiomatic method, before moving onto Hilbert and Lawvere. He then presents a deep textual analysis of each writer and describes how their ideas are different and even how their ideas progressed over time. Next, the book explore category theory and details how it has revolutionized the notion of the axiomatic method. It considers the question of identity/equality in mathematics as well as examines the

http://arxiv.org/abs/1210.1478

THANK YOU!