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★**Mathematics of the transcendental.**
Edited, translated and with an introduction by A. J. Bartlett and Alex Ling.
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Mathematics of the transcendental by Alain Badiou is a collection of two different texts written at different times. Realizing in his own singular way the idea of *mathematical philosophy*—which in a different form is also popular in the *analytic* philosophical circles where Badiou definitely does not belong—the author provides in this book a philosophically laden introduction to elementary category theory and topos theory. Badiou's general method of combining mathematics with philosophy is technically the following: he takes some relevant pieces of mathematics in a ready-made form using standard mathematical textbooks and then renders the mathematical contents into a philosophical discourse using an original philosophical vocabulary and an original system of concepts. This establishes fairly intimate relationships between mathematical and philosophical ways of reasoning, which Badiou explains in detail and justifies in his recent book [*Éloge des mathématiques*, Café Voltaire, Flammarion, 2015; MR3497423].

Badiou's principal choice of philosophically relevant mathematics is as expected: like most of other philosophers of the past and the present he discusses foundations of the discipline. His *Being and event* [Continuum, London, 2005; first French edition, *L'être et l'événment*, Seuil, Paris, 1988] is a systematic philosophical treatment of set theory; his *Logics of worlds: being and event, 2* [Continuum, London, 2009; first French edition, *Logiques des mondes*, Seuil, Paris, 2006] is a treatment of category theory and topos theory. The book under review treats the same mathematical subjects as the *Logics of worlds* but gives more space to mathematical proofs and constructions. The first part of the book represents results of Badiou's personal effort to learn basic concepts of category theory and topos theory and to think them through in philosophical terms; here the author exposes elementary category and topos theory and evaluates potentials of these theories as rivals of set theory in foundational and ontological matters.

The aim of the second part, which was written two decades later, is to reconcile Badiou's view on set theory as the theory of ontology in a strong philosophical sense—i.e., *the* theory of what there is—as presented in his *Being and event* with his later interpretation of topos theory developed in the *Logics of worlds*. The proposed solution, which in a less technical form is also found in the *Logics of worlds*, is to think of topos structure as a *transcendental* rather than an ontological structure. The word "transcendental" in the title and elsewhere in the book refers not to things that somehow transcend human understanding but rather to basic structures and setups of this understanding itself. In a nutshell, the mature Badiou's view on the relationships between set theory and topos theory is this: while set theory describes what there is, topos theory describes how one can think of and understand what there is. This is a somewhat conservative view on the topos concept, which can be described as a version of "set-theoretic orthodoxy". It obviously leaves aside certain developments and ideas related to topos theory. However, a reader of Badiou should keep in mind that mathematical generality is not the author's goal in the present book or elsewhere, so the above argument is rather at cross-purposes and thus hardly constitutes an objection.

Indeed, the choice of exposed mathematical material in the second part of this book is very specific and strongly determined by the author's philosophical project. Badiou's main mathematical source here, which he acknowledges in the introductory part, is R. I. Goldblatt's book [*Topoi*, second edition, Stud. Logic Found. Math., 98, North-Holland, Amsterdam, 1984; MR0766560]. However, instead of following Goldblatt systematically, Badiou picks up from Goldblatt's book certain specific constructions which best serve his philosophical purposes. He uses essentially the concept of existence predicate due to Fourman and Scott [R. I. Goldblatt, op. cit. (p. 267)], introduces Heyting-valued sets [R. I. Goldblatt, op. cit. (p. 274)] and then builds from these elements a Grothendieck topos in a bottom-up way. Goldblatt describes the existence predicate as "a measure of the extent to which an individual is defined (exists)" [R. I. Goldblatt, op. cit. (Preface, p. xii)] and makes throughout his book many other remarks which sound exciting for a philosopher. Badiou's philosophical account of set theory and topos theory can be seen as a systematic elaboration of such hints found in Goldblatt's book and in some other contemporary mathematical literature. It goes without saying that Badiou uses these hints very selectively, and indeed leaves aside those which lead to alternative philosophical interpretations of the same mathematical contents.

The idea that specific mathematical constructions and a mathematical reasoning with these constructions can adequately represent a reasoning about ultimate philosophical matters such as being, existence and understanding, in my own view, is rather objectionable, but it belongs to the core of Badiou's philosophical thinking. Many proponents of *mathematical philosophy* in the analytic camp use essentially the same method even if their writing style is very different. This way of reasoning has been playing a major role in philosophy since at least the beginning of the 20th century, and it certainly deserves serious consideration independently of writing styles.

The two parts of the book are closely related thematically. But they were written independently and unfortunately no attempt has been made by the author to provide the two pieces published under the same cover with some cross-references. Such cross-references would be particularly useful in the second part of the book where the concept of topos appears at a certain point without a proper explanation or reference.

Whether or not Badiou's method of developing a philosophical discourse using mathematics is justified on theoretical grounds, it creates a specific terminological difficulty for a reader. A part of Badiou's game is to give to standard mathematical concepts non-standard philosophical names. For example, he calls a *transcendental* what is usually called in mathematics a complete Heyting algebra, calls a category (topos) a *situation*, and a subobject a *territory*. Such terminological puzzles are easy to resolve when the author also provides the standard terminology; in some other cases the corresponding standard terms can be easily found by the reader in the usual external sources. But there are also difficult cases when the philosophical and the mathematical terminology conflate severely, so the risk of conceptual confusion becomes very high. In particular, in the second part of the book Badiou chooses to call general morphisms—which are actually functions since only set-based structures are considered—by the name of *relations*. The author makes it clear that he has in mind a general philosophical concept of relation, which he specifies mathematically in this particular way. But since the name of *relation* is also commonly used in mathematics and mathematical logic in a related but still different technical sense, confusion becomes very likely. The responsibility for this terminological disorder is partly shared by the translators of Badiou's texts into English, who should have taken greater care of using the standard logical and mathematical English vocabulary.

Having said that, I cannot detect in this book any essential conceptual lack of clarity behind the terminological one. This is why I believe that the terminological problem is solvable. One may hope that a devoted reader might someday make some sort of glossary for translating Badiou's original terms into common mathematical parlance.

This book may be interesting and useful both for a philosophical reader who wants to learn some basics elements of category and topos theory, and for a mathematical reader interested in philosophical interpretations of these theories. Of course, philosophical readers have other—and arguably better—options for learning these pieces of mathematics. Badiou's writings may seem like a very strange place to start learning category theory and topos theory, but I personally know of several young people who first got interested in studying these theories—and even modern mathematics more generally—by reading Badiou, and who later proceeded with standard mathematical textbooks. So the pedagogical value of Badiou's popular writings should not be underestimated, even though this influential thinker is pictured by certain academics as a corruptor of youth.

Reciprocally, a mathematical reader interested in today's philosophy of mathematics and mathematical philosophy will find in this book an exciting example of systematic philosophical thinking about mathematics and with mathematics. Once again, such a reader is advised to evaluate Badiou's contribution against the existing concurring philosophical approaches to the same mathematical matters. Such literature is not extensive, but it exists. A mathematical reader of Badiou also needs to bear in mind that philosophers, unlike mathematicians, as a matter of brute sociological fact, don't form today a global community with universally agreed standards of the profession, even if significant efforts towards this goal have been taken in academia and elsewhere. Badiou's unique philosophical style and his singular place among writers and researchers who describe themselves as philosophers perfectly demonstrates this point.

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