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Axiomatic Method and Category Theory

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Preface

I first learned about category theory about 20 years ago from Yuri I. Manin's course on algebraic geometry (Manin 1970) when I was preparing my dissertation on Euclid's *Elements* and was focused on studying Greek mathematics and classical Greek philosophy. Then I convinced myself that the mathematical category theory is philosophically relevant not only because of its name but also because of its content and because of its special role in the contemporary mathematics, which I privately compared to the role of the notion of *figure* in Euclid's geometry. Today I have more to say about these matters. The broad historical and philosophical context, in which I studied category theory, is made explicit throughout the present book. My interest to the Axiomatic Method stems from my work on Euclid and extends through Hilbert and axiomatic set theories to Lawvere's axiomatic topos theory to the Univalent Foundations of mathematics recently proposed by Vladimir Voevodsky. This explains what the two subjects appearing in the title of this book share in common.

The next crucial biographical episode took place in 1999 when I was a young scholar visiting Columbia University on the Fulbright grant working on ontology of events under the supervision of Achille Varzi. As a part of my Fulbright program I had to make a presentation in a different American university, and I decided to use this opportunity for talking about the philosophical significance of category theory (I cannot now remember how exactly I married then this subject with the event ontology). Achille Varzi kindly arranged for me the invitation from Barry Smith to give a talk at his seminar on formal ontology in the SUNY in Buffalo. When I sent to Barry Smith my abstract he replied that nobody except probably Bill Lawvere will be able to understand my paper, and suggested to make the paper more accessible to the general audience. By that time I had already read some of Lawvere's papers but was wholly unaware about the fact that Lawvere worked in the same university and could attend my planned talk. So I took Smith's words for a joke. When I realized that this was not a joke I was very excited and, as it turned out, not without a reason

because my meeting with Lawvere during this visit indeed determined the direction of my research for many years to come. This book is a summary of what I have achieved so far working in this direction.

Saint-Petersburg, Russia

Andrei Rodin

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