

What is Constructive Axiomatic Method?

The received notion of axiomatic theory, which stems from Hilbert, is that of set T of propositions (either contentful or non-interpreted aka propositional forms) with subset $A \subseteq T$ of *axioms* provided with a notion of consequence, which generates T from A in the obvious way. I argue that this standard notion is too narrow for being an adequate theoretical model of many mathematical theories; the class of such counter-examples is apparently very large and it includes such different theories as the geometrical theory of Euclid's Elements, Book 1, and the more recent Homotopy type theory. In order to fix this problem I introduce a more general notion of theory, which uses typing and a generalized notion of consequence applicable also to objects of other types than propositions. I call such a theory *constructive* axiomatic theory and show that this particular conception of being constructive indeed captures many important ideas concerning the mathematical constructivity found in the earlier literature from Hilbert to Kolmogorov to Martin-Lof. Finally I provide an epistemological argument intended to show that the notion of constructive axiomatic theory is more apt to be useful in natural sciences and other empirical contexts than the standard notion. *Disclaimer:* The notion of constructive axiomatic theory is not my invention. The idea and its technical implementation are found in Martin-Lof's constructive type theory if not already in Euclid. My aim is to make this notion explicit and introduce it into the continuing discussions on axiomatic method and mathematical and logical constructivity.