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Endurance, Perdurance and Quantum Duality.

Abstract:

In this paper I suggest an account of mechanical movement which uses formal means of Category Theory. This account of movement implies an account of space and time which resolves the Endurance vs. Perdurance controversy as follows: an «enduring object» (say, a moving body) and a corresponding «perduring object» (a trajectory of this body) are dual descriptions of one and the same thing, where the sense of «being dual» is defined as in Category Theory. Then I show that the Quantum duality between waves and particles can be described in the same way. This allows to consider movement uniformly in the Quantum and classical cases.

#1. ABC of Categories

Definition: A **category** comprises:

1) objects A,B, ...
2) arrows f,g,... between objects; if there is an arrow f: $A\rightarrow B$, then it is possible that A=B, i.e. that A and B are identical.

Axiom 1: For every two arrows f:$A\rightarrow B$ and g:$B\rightarrow C$ there exist a **composition** arrow gf:$A\rightarrow C$; the composition of arrows is associative, i.e. if there exists a composition g(fh) then g(fh) = (gf)h = gfh.

Axiom 2: For every object A there exists its **identity** arrow 1_A: $A\rightarrow A$ such as for every incoming arrow f: $\rightarrow A$ $1_A f = f$ and for every outgoing arrow g:$A\rightarrow$ $g1_A = g$.

Example: The Category of Sets (objects are sets, arrows are mappings of sets)

Definition: An arrow f is called **monic** iff it is left-cancelable, i.e. iff for any arrows g,h fg=fh ⇒ g=h.
Corollary: in the category of sets monic arrows are injective mappings; the category-theoretic notion of monic arrow is an analogue of the set-theoretic notion of injective mapping.

Definition: A monic arrow into an object A is called **subobject** of A
Corollary: the intuitive meaning of a subobject of A is a part of A (cf. subsets).

Definition: An object T of a category such as for any other object A of this category there is one and only one arrow A→T is called **terminal** object of this category.
Fact 1: If a category has a terminal object then it is unique up to isomorphism.

Definition: An arrow T→A from **the** terminal object of a category to any other object of this category is called a **point** of A.
Fact 2: A point is **monic** arrow (for proof see the diagram below where T is the terminal object).

$B \rightrightarrows T \to A$

Definition: Two diagrams of a category are called **dual** with regard to each other iff one can be converted into the other by changing of directions of all its arrows. A diagram of a category is **self-dual** iff it is converted by changing of directions of all its arrows into itself.
Example: Diagrams A→B and A←B are dual; diagram $A \rightleftarrows B$ is self-dual.

#2. Movement

Let us call objects of a category *things* and arrows of the category *placements.*

Interpretation: A→B is read as A **is located at** B.

Definition: A **rests with regard to** B iff there is only one placement of A at B;
A **moves with regard to** B iff there are at least two placements of A at B:

A→B: A rests with regard to B

$A \rightrightarrows B$: A moves with regard to B

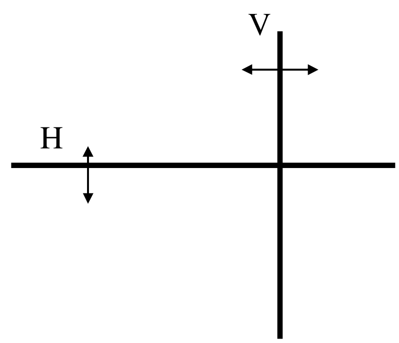
What is unusual in this definition? The fact that *A moves* (*rests*) *with regard to B* does **not imply** *B moves* (*rests*) *with regard to A.*

Examples: (1) $A \to B$, (2) $A \rightrightarrows B$, (3) $A \;\substack{\longrightarrow \\ \longleftarrow \\ \longrightarrow}\; B$

(1) A rests with regard to B but the movement relation of B with regard to A is not defined;
(2) A moves with regard to B but the movement relation of B with regard to A is not defined;
(3) A moves with regard to B but B rests with regard to A.

Explanation: ultra-relational (local) account of movement: (move3.gif)

$H \rightrightarrows V$, then $H \leftleftarrows V$;



a man walks across the Earth: $M \rightrightarrows E$ (the man moves but the Earth does not)

Corollary: Suppose a diagram shows that A moves with respect to B. Then the dual diagram shows that B moves with respect to A «in the same way»:

$A \rightrightarrows B$; $A \leftleftarrows B$

#3. Space

Definition: A thing A is **extended** iff A has two or more incoming arrows.

Corollary: If A moves with regard to B then B is extended.

Definition: A monic arrow coming into A is called a **part** of A.

Motivation: See the Definitions of monic arrow and subobject from **#1**.

Definition: **Space** is a thing S of a given category such as any other thing A of the category is located at S, i.e. there is an arrow $A \to S$.

Motivation: Space is where **all things** are located.

Corollary 1: A category can have a lot of spaces.

Corollary 2: Though the above account of movement is (ultra-)relationalist it gives a substantialist concept of space.

Definition: A space S of a category is called **static** if every thing A of this category rests at S, i.e. there is only one arrow A→B.

Corollary: A static space of a category is **the** terminal object of this category; if a category has a static space it is unique in this category (see Fact 1 from **#1** above).

We will see that what is usually considered as **the** space is the **static** space of a given category provided the following interpretation: everything rests in the space **in a given moment of time**.

#4. Time

Consider a thing A moving with respect to T: $A \rightrightarrows T$. When we might call T a **trajectory** of A? When nothing (whenever) is located at T except A and its parts (or A and things located at A?).

Definition: T is called a (**proper**) **trajectory** of A iff (1) A moves with respect to T and (2) if B is located at T then B is located at A (and is a part of A):

$$\begin{tikzcd} A \arrow[r, shift left] \arrow[r, shift right] & T \\ B \arrow[u] \arrow[ur] & \end{tikzcd}$$

Definition: **Time** (T) is **a** trajectory of **the** static space (S) of the category.

Corollary 1: If S moves with respect to a thing A then A is a trajectory of S, i.e. time. (Static space cannot move with regard to anything but time).

Corollary 2: If a category has a time then every object of this category is located in this time (by composition):

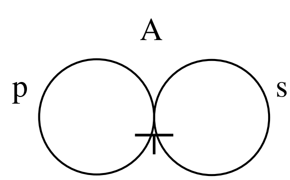
$$
\begin{tikzcd}
 & S \arrow[dr] & \\
A \arrow[ur] \arrow[rr] & & T
\end{tikzcd}
$$

Hence, time is a non-static space.

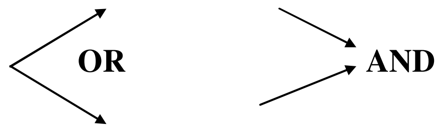
Corollary 3: This account of time does not presuppose a special notion of «temporal extension» different from «spatial extension». The crucial difference is not between two types of extension but between **extension** and **movement** (with respect to an extended thing). The two are **dual** in the following sense: when a diagram $A \rightrightarrows B$ shows that A moves and B is extended, the dual diagram $A \leftleftarrows B$ shows otherwise that A is extended and B moves.

Definition: A placement $S \to T$ of the static space at a time is called a **moment** of time.
Corollary 1: A moment of time T is a point of T (as defined in **#1**).
Corollary 2: A moment of time T is a **part** of T(see Fact 2 from **#1**).
Corollary 3: The static space can be viewed of as the class of all the «momentary photos» of all the things of the given category.

#5. Endurance vs. Perdurance ? (for the case of mechanical movement)
What is real - a moving particle (an enduring object) or its trajectory (a perduring event)? Consider the following diagram:



This diagram shows that A moves with regard to itself (this can be thought of as A's rotation). In this case **A is its own trajectory**: check that the condition mentioned in the definition of trajectory is trivially satisfied. The diagram is self-dual: this allows to say that *A is a moving particle* and *A is a trajectory* are two **dual** descriptions of A. By Reichenbach's word (*Philosophical Foundations of QM*) the former description is that in ***or-terms*** while the letter description is that in ***and-terms***:



Apparently none of the two descriptions suggests an ontological priority.

<u>Note</u>: A category with only one object is called **monoid**. If all arrows of a monoid are invertable, i.e. are isomorphisms, then the monoid is a (algebraic) **group**. Hence for the case of **invertable** movements we should restrict monoids to groups.

The only case when taking into consideration only one thing A one can reasonably ask whether A is a moving particle or a trajectory is the above case when A moves with regard to itself. For generally the predicates *to be a moving particle* and *to be a trajectory* are relational: A can move with regard to its trajectory B being itself a trajectory of another thing C, while B in its turn moves with regard to its trajectory D. Thus the only possible answer to the question is: A is both *trajectory* and *particle*; the two are A's dual descriptions.

#6. Space, Time and Duality in QM

The above account of movement, space and time does NOT presuppose that

1) time has a special structure, particularly the structure of linear or partial order (though such a structure can be introduced); correspondingly, a moving particle's trajectory is not necessarily a line (one-dimensional manifold).
2) a moving thing (particle) has an extension or is a point (though a moving particle **may** have an extension as at the diagram below)

$$
\begin{tikzcd}
{} \arrow[dr] & & \\
& A \arrow[r, shift left] \arrow[r, shift right] & B \\
{} \arrow[ur] & &
\end{tikzcd}
$$

However this account DO presuppose that any trajectory has an extension.

This features apparently allow to apply this account to QM case as well as to the classical case. Particularly, OR-AND (particle-trajectory) duality in the QM case is as above provided that a particle's trajectory is its wave-function (thus in the QM case the particle-trajectory duality is the particle-wave duality). This allows to think of a QM object as a moving particle along the same lines as of a classical object provided that, generally speaking, the only extension which can be prescribed to the particle is the extension of its trajectory.

<u>Hypothesis</u>: the difference between the classical and the QM cases is that categories of QM objects have no static spaces and hence times (as defined above, i.e. «universal» times of a given category; this does not prevent QM objects to move).

<u>Problems</u>:

1) The above account allows a situation when a moving thing has no trajectory:
   $A \rightrightarrows B \leftleftarrows C$. Does it make sense?
2) How to combine this account with mathematical apparatuses of classical and Quantum mechanics? Particularly, what happens when a wave-function is not static?