

**PROOFS AND FUNDAMENTALS:  
THEMES FOR THE FINAL EXAM**

- **Logic:**

- Statements; Arguments; Propositional Connectives, Truth-Tables, Tautologies
- Rules of Inference, Formal Derivations, Syntax and Semantics, Soundness and semantic Completeness
- First-Order Logic: Predicates, Quantifiers, Instantiation and Generalization
- Direct Proofs; Proofs by Contrapositive and by Contradiction.

- **Sets:**

- Notation, Russell Paradox, Axioms of Extensionality, Separation and Well-Foundedness;
- Cardinality of sets, Cantor Theorem;
- Boolean operation on sets, the Cartesian Product of sets.

- **Functions:**

Domain and Codomain, Composition of functions; Invertible functions, image and inverse image, injective, surjective, and bijective functions.

- **Relations:**

- Equivalence Relations; Equivalence Classes
- Partial Order Relations; Partially Ordered Sets (Posets); isomorphism and (homo)morphism of posets.

- **Algebraic Operations:**

Binary Operations; Groups of Permutations; Homomorphisms and Isomorphisms of Groups.

- **Arithmetic:**

- Natural numbers, Integers; Rational Numbers; Real Numbers (as Dedekind Cuts); Complex Numbers;
- Mathematical Induction