Category Theory Foundation For Knowledge Representation

(how to make the axiomatic method work for engineering and science)

Henson Graves

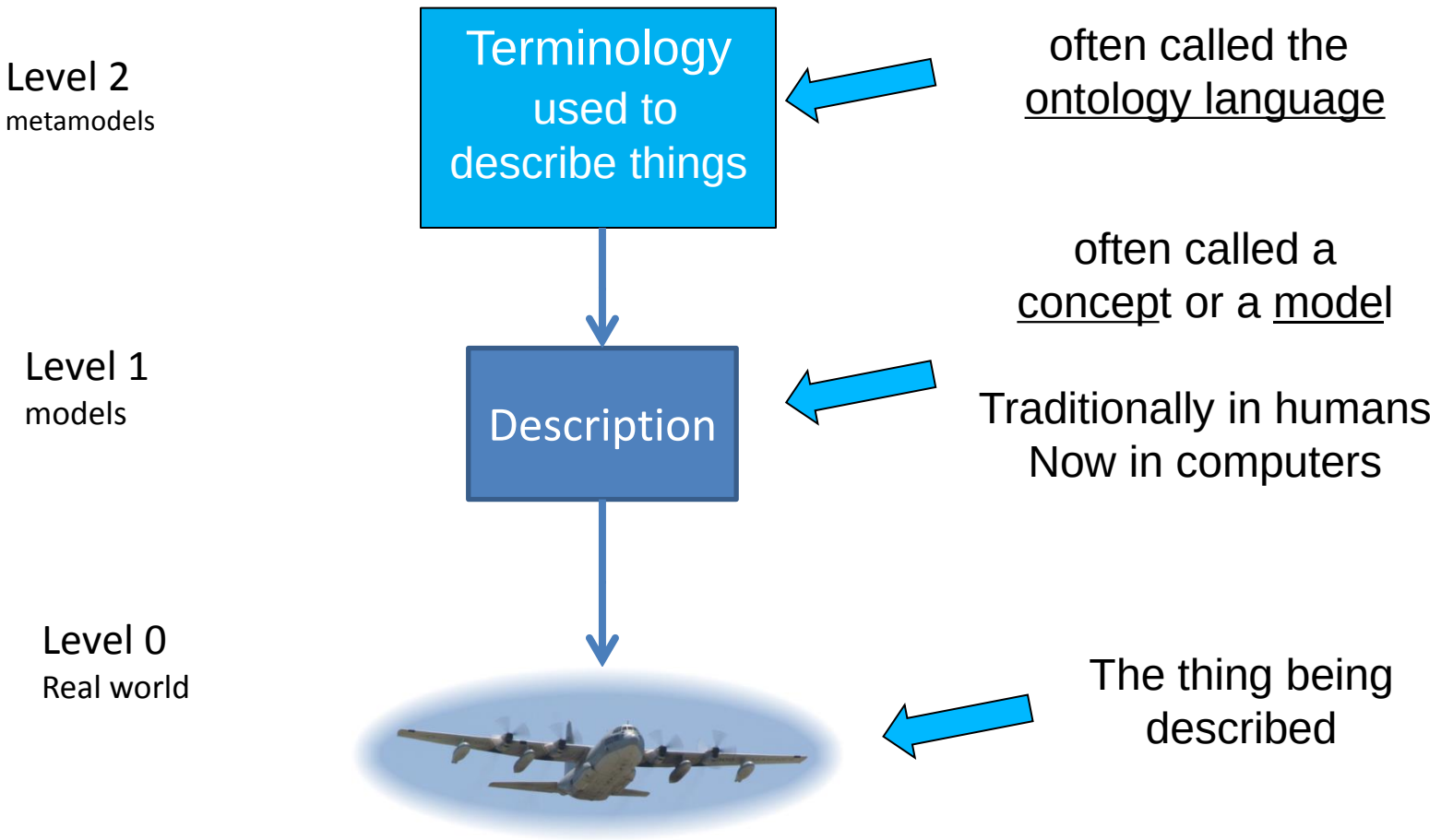
Andrei Rodin Seminar

Moscow, September 2014

Topics

- **Knowledge Representation (KR) by computer**
- **KR as an application of Axiomatic Method**
- **Why Category Theory is fundamental for KR**
- **What kind of Categories**
- **Kind of logic needed to represent a category**
- **Representing multiple categories**

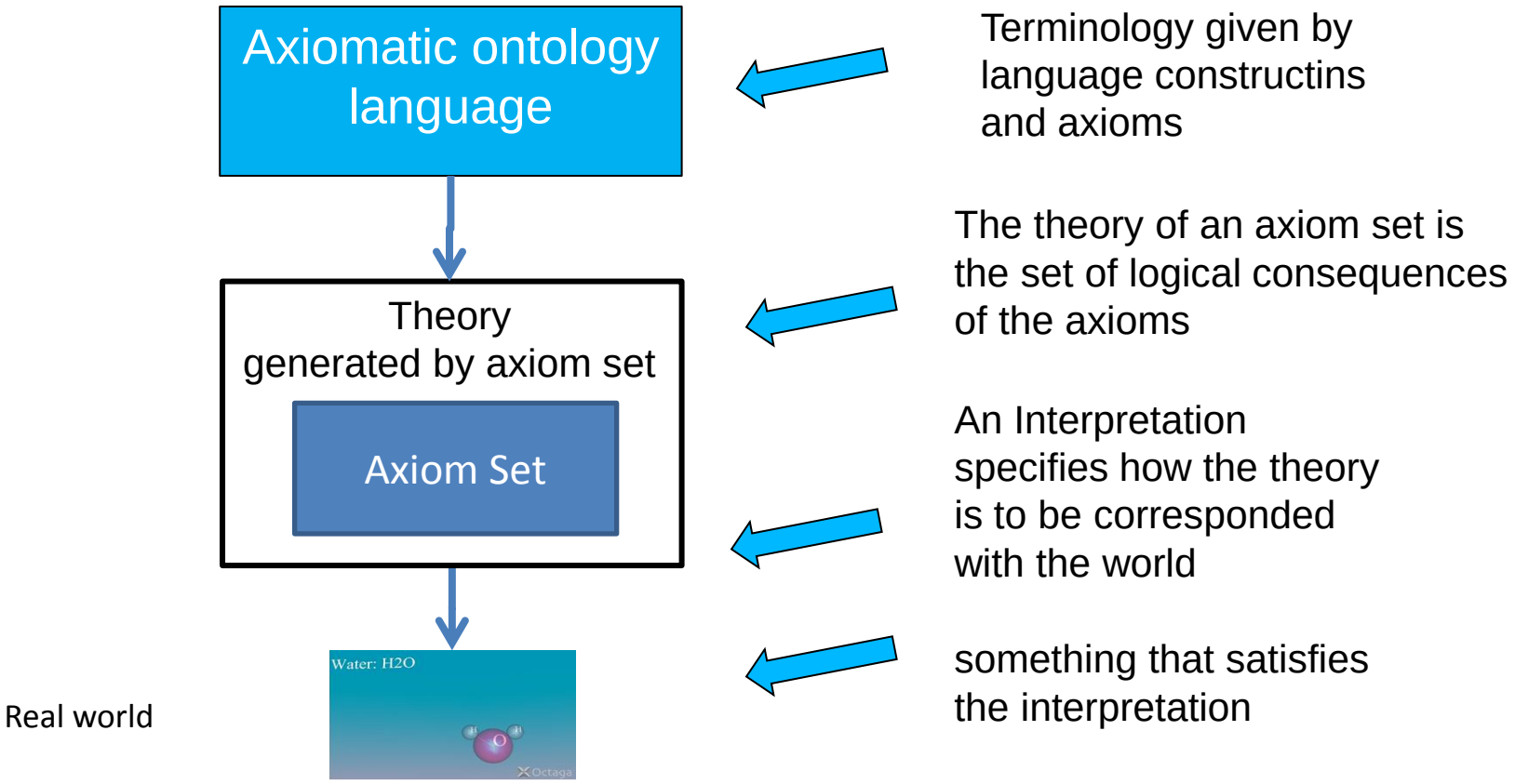
Knowledge Representation Generally Distinguishes, Things Described, Descriptions, and Terminology



... this taxonomy works in many philosophical approaches, it doesn't presuppose realism

The Axiomatic Method Provides a Solid Foundation For Knowledge Representation

...this viewpoint was originally controversial in Artificial Intelligence, but less so now



...the only real controversy is that it has been perceived as too hard to do

The Axiomatic Method Provides

- **Basis for knowledge reuse**
- **Criteria for logically correct reasoning**
- **Equivalence of application questions with logical questions**

...the thesis that axiomatic method was the way to do Artificial Intelligence originally spilt AI into pro and con camps

Engineers, Mathematicians, Philosophers Use Axiomatic Method In Different Ways

..., but it provides a unified way of looking at KR

Philosophers

Tend to look for some way, possibly unique, to describe reality

Mathematicians

Give axioms, and try to work out what all possible interpretations are. If they can do that they are happy as they have characterized the concept

Engineers

Look for axioms that can be used to describe things precisely, i.e., rule out non-intended interpretations, so automated reasoning can be used

The Application Of The Axiomatic Method For Knowledge Representation Has Been Limited

practically, it has been a hard problem to find a suitable axiomatic ontology, and it is difficult for humans to produce axiom sets for applications

- **Axiom sets in many logical formalisms are unable to characterize intended representations**
- **Logic of axioms needs to be restricted for automated theorem proving**
- **Practically people cannot build good axiom sets**

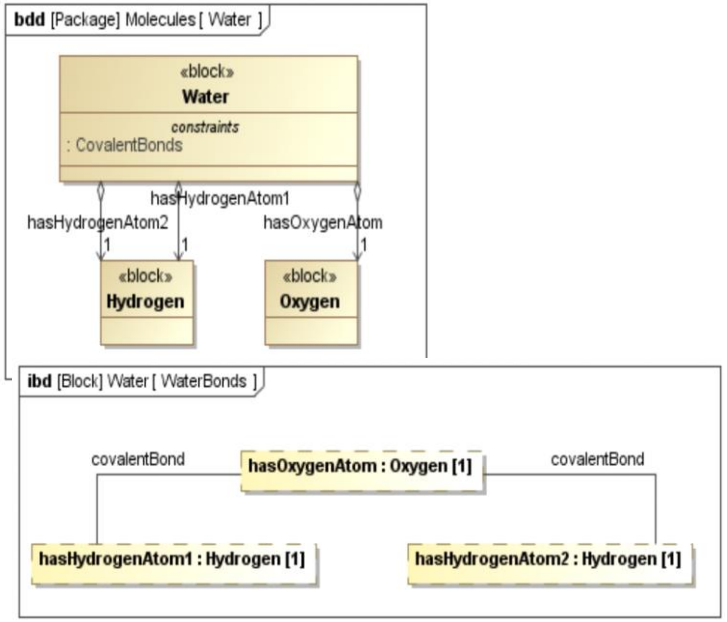
...yet there are game changers:

Graphical syntax and the ability to embed diagrams into an expressive axiomatic ontology

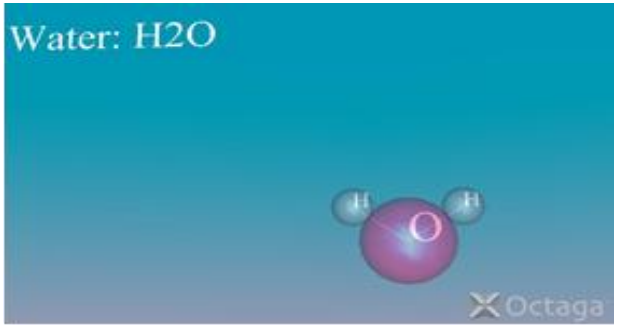
Engineering Diagrammatic Models Can Be Embedded In An Axiomatic Ontology

... This means that humans do not have to write axioms using logical axioms, however axioms beyond the diagram have to be added

model of H2O



One Interpretation



- SysML model of H2O, not all information shown
- 3D simulation generated from model
- Interpretation may have one or more molecules (class model)

- **Graves, Henson. "Structural Modeling in Biomedical and Product Engineering." INCOSE 2013,** www.omgwiki.org/MBSE/lib/exe/fetch.php?..

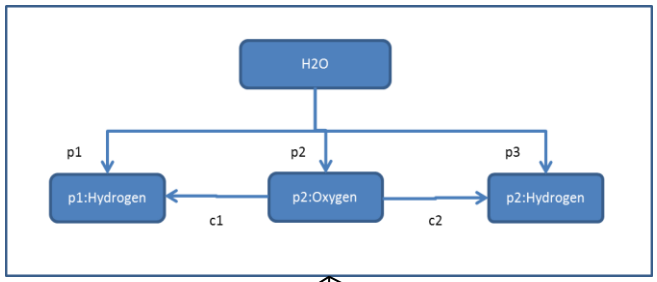
Application Of Axiomatic Method Has To Ensure That

- **Reasoning with axioms yields correct physical results**
- **Restated: Embedded into logic so that**
  valid interpretations = intended interpretations
- **This problem does not have the recognition it deserves**

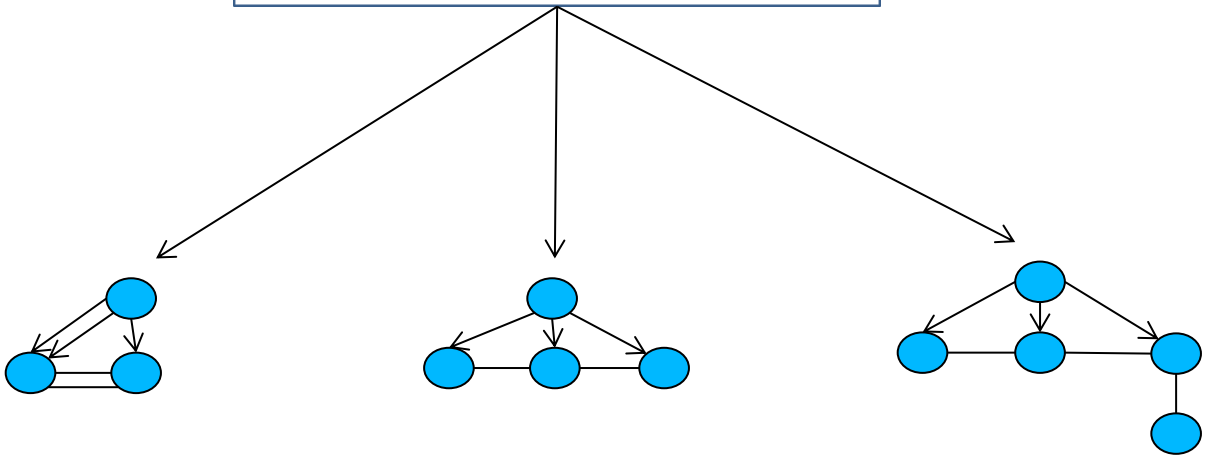
For The Valid Interpretations To Be The Intended Ones

...depends on the ontology language and form of axioms (metalogic)

model



Interpretation(s)



...DL axioms for H20 cannot rule out unintended interpretations

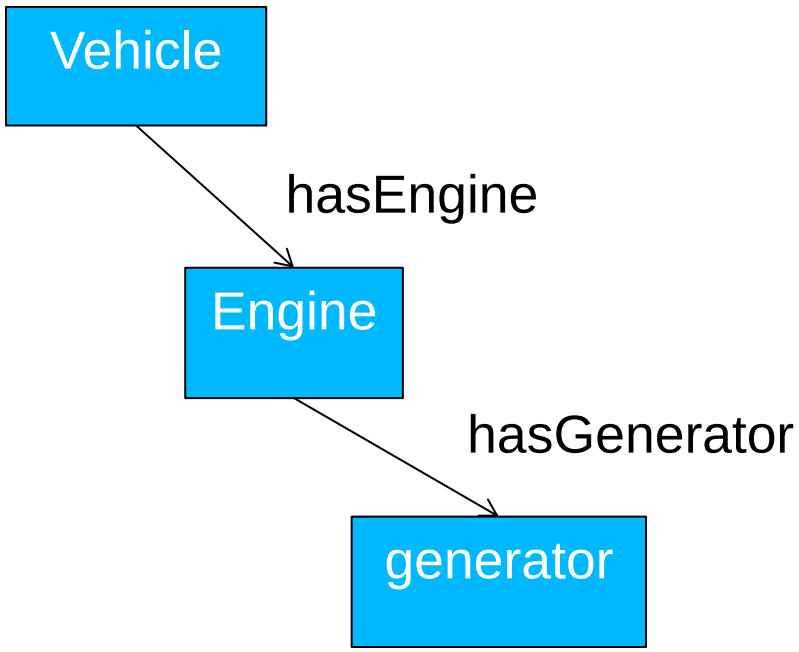
Magka, Despoina, Boris Motik, and Ian Horrocks. "Modelling structured domains using description graphs and logic programming." *The Semantic Web: Research and Applications*. Springer Berlin Heidelberg, 2012. 330-344.

Graves, Henson. "Structure Diagrams in Type Theory." *TURING* (2012): 63.

What kind of Ontology Language Is Required?

- **Strong argument that first order logic without function symbols is insufficient**
  - implies Description Logic is limited
- **Requirements for ontology language**
  - the expressiveness of Set Theory
  - Computationally tractable logic
  - A computationally engineered version of topos theory works nicely for many applications

First Order Category Theories Make Good Axiomatic Ontologies For Engineering & Science



A category is a generalization of a directed graph

Nodes:

Vehicle, Engine, Generator

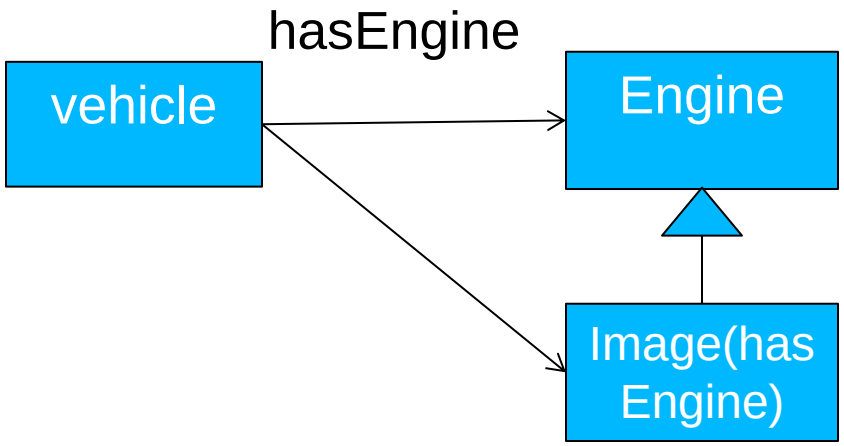
Arrows:

hasEngine:Vehicle -> Engine

An interpretation requires ability to recognize that a specific engine is associated with a specific vehicle

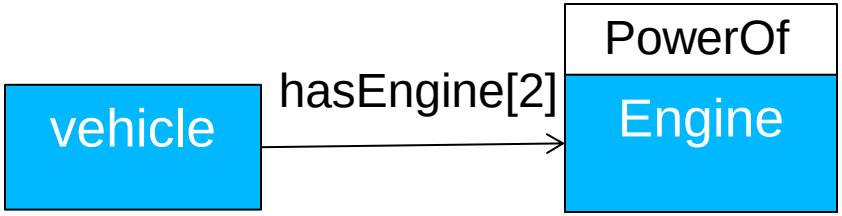
easy to interpret language constructions and axioms in physical world (functorial semantics)

Lots Of Useful Language Constructions such as subobject and power objects



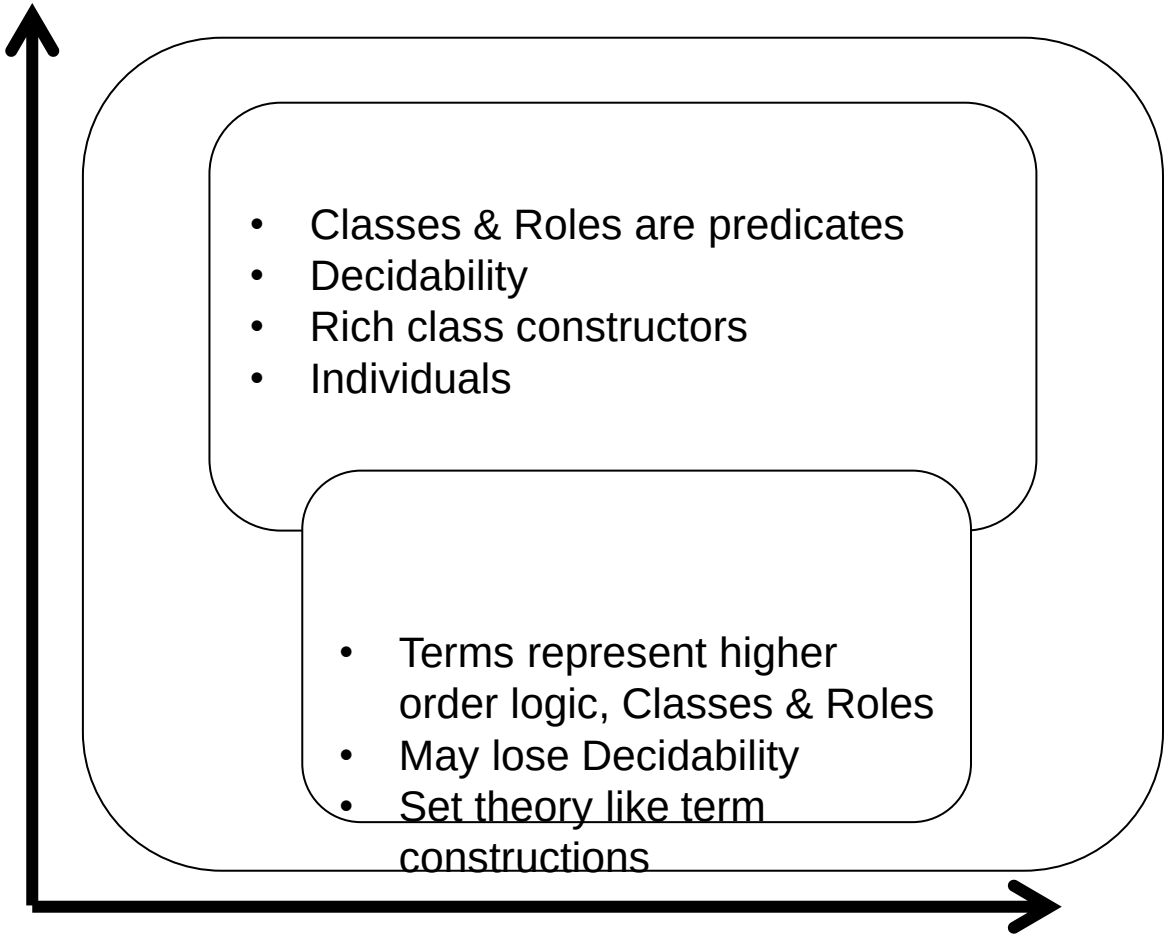
hasEngine =

$$\text{hasEngine}^{1}.\text{incl}_{\text{Image(hasEngine)}}$$



category theory can be presented very algebraically

Restricted First Order Logic Is Needed For Tractable Reasoning



Examples of First Type Theories

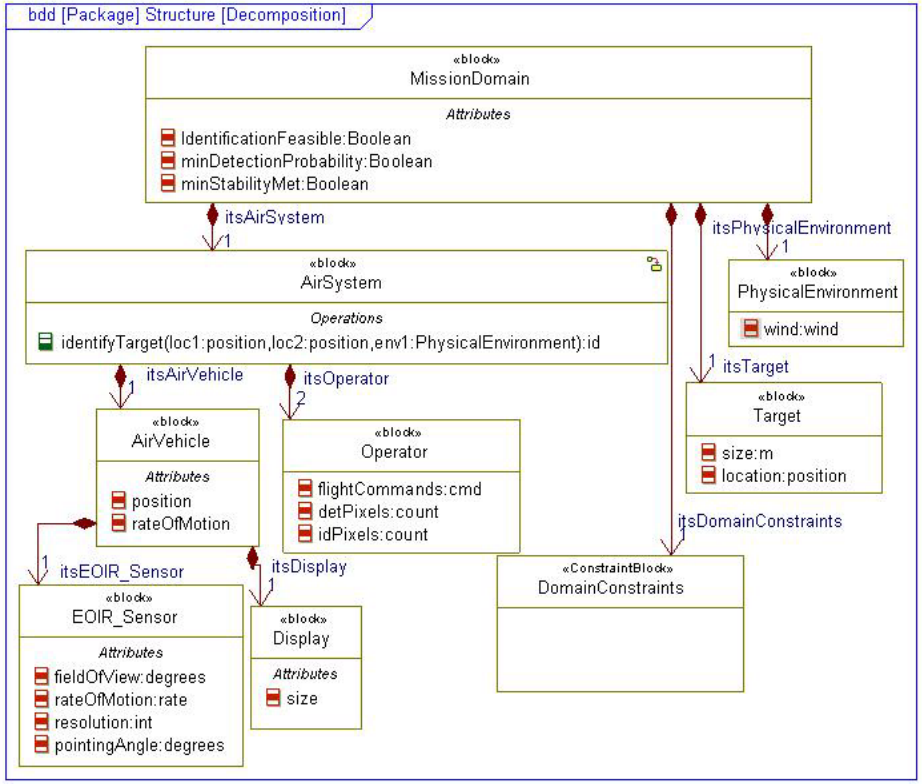
- Traditional type theories
- HOL in Rule Logics
- Rule Based topos theory

... Topos axioms can be given in fol rule form…

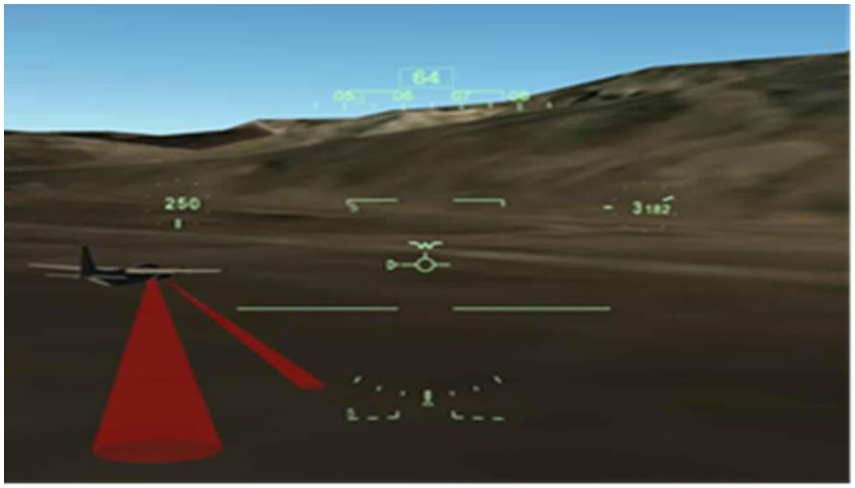
Engineering Example Of Computer Based KR
Products are designed and analyzed using models

... and these models can be embedded in a topos…

Engineering Model of Aircraft
& Operating Environment



Aircraft & Operating Environment

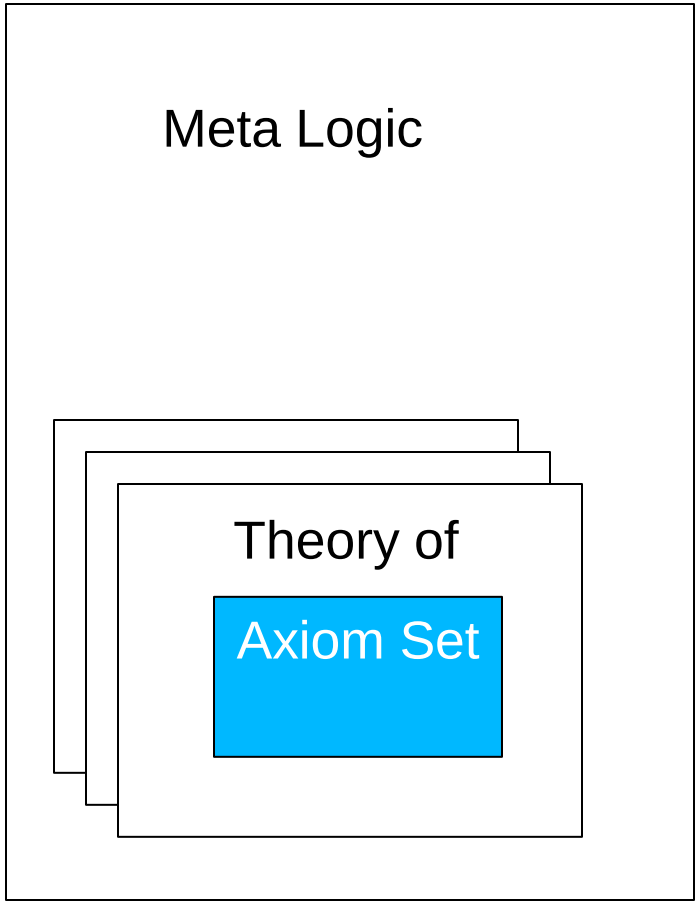


- Models for design and analysis
- Use of physics models
- Data analysis of simulations
- Model-based reasoning

Graves, Henson, and Yvonne Bijan. "Using formal methods with SysML in aerospace design and engineering." *Annals of Mathematics and Artificial Intelligence* 63.1 (2011): 53-102

KR Requires Combining and Modifying Multiple Axiom Sets

a 2-category is a good ontology language for this



Meta logic
- first order logic
- sorts become meta-types
- 2-category whose objects are axiom sets

Axiom sets
- in multisorted first order logic
- decidable well-formed conditions
- signature includes topos language constructions
- axioms are Horn rules

This was the topic of my phML2014 paper

Summary

- **Engineering models can be embedded as axiom sets in logic**
- **Many engineering problems translate to logic problems**
- **An elementary topos is a good axiomatic ontology language for many applications**
- **Operations on multiple axioms sets is a doctrine (2-category)**
  - provides a good development framework for model (axiom set) development