Kant’s Philosophy of Mathematics and Variable

The distinction between general laws and particulars which comply with the general laws is in the core of the Kant’s system. In the Preface to 2nd edition of «Critique of Pure Reason» Kant writes: «Before objects are given to me, that is a priory, I must presuppose in myself laws of understanding which are expressed in conceptions a priori.» The task of the «Critique» then is to explicate those general laws leaving the study of particular objects to scientists. More specifically the distinction works in Kant’s account of mathematical reasoning. Accordingly to Kant this comprises three things: general «conceptions» of the understanding, particular «constructions» of the intuition and «schemata» of the faculty of judgement (i.e. the faculty to subsume a particular construction under a general conception) which «mediates» between the former two giving a method of how to construct a particular of a kind determined by a given conception.

The basic examples which support the Kant’s account are geometrical: think about the general notion of triangle given by its definition, an imaginary «particular» triangle which may by identified, say, as ABC and a principle which allows to say that «ABC is triangle». So far so good. But think about the number 5. Is it particular object or general concept? Or possibly we loosely apply the name «five» both to a particular and to a concept just as we may say «this book» referring either to a concrete material thing, that is to a book-token, or to a certain text, for example, to «Anna Karenina», that is to a book-type? But unlike the case of book prima facie there is no reason to suspect ourselves in making such a confusion; it seems that «the number five» is a thing which is not «particular» in the sense in which a particular triangle ABC is nor «general» in the sense in which the general concept of triangle is.

We speak about *the* number 5 but we also can sum up *two* fives: 5+5. This might make us believe that the Kant’s theory works and the mentioned confusion actually happens. For the fives making the sum 5+5 look like two tokens of the same type just as a triangle ABC is a token of the type «triangle in general». Kant himself supposes that when we cognate numbers we draw imaginary pictures as well as in the case of geometrical figures, something like ••••• for the number 5. According to Kant such a picture is thought of as a picture of a particular number as well as a picture of triangle is thought of as a picture of a particular triangle. But think about the product 5x5 (five times five). While one five of the couple can be thought of as a row of particular units like ••••• the other should be thought of as a row of bigger units each consisting of five units mentioned above:

•••••   •••••   •••••   •••••   •••••

If you still think that Kant’s theory complies with this case think about 5^5.

Considering algebra Kant suggests that algebraic symbols as well as images of geometrical figures stand for certain imaginary particulars. But think about a function $f(x) = x^2$. Prima facie a function is not a particular but a general rule, in the given example this is the rule saying that any number which x stands for is to be squared. On the other hand in certain cases a function *is* considered as a particular, namely when they construct a «function between functions», i.e. a functional. Consider a simpler example. If x is a name of variable it is *not* a proper name of particular like the name ABC of a triangle. We may fix a domain of x and operate with x as a particular. But we should not forget that x stands for *any* number of its domain but not for a concrete number. If the domain of x is, say, all real numbers then in a sense x stands for real numbers in general.

The above examples are cases of the type/token relativity. They show that the case of a geometrical figures such as triangles which allows to distinguish between general «conceptions» and particulars absolutely is specific. It is important to understand what in this case makes the absolute distinction possible. Suppose that you are said about a distinction between two triangles, triangle X and triangle Y, but you do not know whether X and Y are names of triangle-types (like «isosceles» and «equilateral») or names of triangle-tokens (like «ABC» and «DEF» in usual notation). Then you may ask about sides of X and Y and be said that sides of X are K,L,M while sides of Y are N,O,P. Again you have two options: or those sides are particular segments (as «AB» in usual notation) or those are some classes of segments, for example classes of congruent and/or parallel segments. But if you are said that vertexes of X are points A,B,C while vertexes of Y are points D,E,F you have the only option: A,B,C,D,E,F are particular points and hence X,Y are particular triangles. The reason is that unlike any other geometrical figure points in no «natural» way may be classified into classes: all points are exactly «similar» and moreover so are relations between any two given points. Thus it is the specific concept of point which makes possible the absolute distinction between general «concepts» and particular intuitive «constructions» in the case of geometrical figures.

The notion of point as informally described above may be formalized by means of mathematical category theory. This apparatus shows that in many cases when points are usually presupposed, particularly in algebraic topology, they are actually unnecessary. The formal account of type/token relativity may be also given by means of this theory.

Although the situation when types and tokens are distinguishable absolutely appears to be specific in mathematics, it still might be argued after Kant that this situation is of general metaphysical significance because the type/token relativity never takes place among real (sensible) objects. This argument does not apply to (real) events however. Take a rain for example. Seemingly we may distinguish between rain as a kind of events and a particular rain the same way as we distinguish between apple as a kind of objects and a particular apple. To distinguish a particular rain we have to specify its location in space and time, for example «Upper Manhattan, around noon, April 23, 1999» seems to be suitable for the purpose. The problem is that unlike objects events may not be located in space and time pointwise: it is impossible to point out exact moments of time when a rain begins and ends and to distinguish sharply between points of space occupied and not occupied by a rain. It may be shown that those difficulties are principle and that to locate events in space and time we must reconstruct space and time without points. As far as space and time have no points, a spatio-temporal location of an event does not particularizes it absolutely because of the reasons mentioned above.

The following is an explicit example of type/token relativity of real events. Consider a conversation in English, that is a particular of the kind of English conversations. «English» in this context is a property of the conversation. On the other hand in historical perspective English language is nothing but a broad conversation itself which started where and when English language was born (however to mark this place and time), which is going on in our times worldwide but which very probably will stop one day. In this perspective English may be properly considered as a particular event, one event of the kind of language events among others. Similarly the very existing of human languages may be considered as an episode, i.e. a particular event of the history of the Universe.

I believe that the understanding of type/token relativity, i.e. a relativity of the distinction between particulars and universals makes necessary a fundamental revision of the Kantian doctrine from its very beginning. On the other hand the Kant's analysis of the special case when the type/token distinction can be made absolutely (regarding to certain frameworks of consideration) may help us to elaborate a better theory.