

Models of HoTT and the Semantic View of Theories (2nd version)

P. Suppes famously argued that a typical scientific theory should be identified not with any particular class of statements (formal or contentual) but rather with a certain class of models. On this basis Suppes and his followers designed a Bourbaki-style format of formal presentation where a scientific theory is presented through an appropriate class of its set-theoretic models. Even if such a Bourbaki-style presentation can be useful for purposes of logical and structural analysis, it appears to be useless as a practical tool, which may help working scientists to formulate and develop their theories in an axiomatic way. In practice, the existing non-statement approach to axiomatizing scientific theories demonstrates the same limitations as its syntactically oriented rival.

HoTT provides a novel notion – as well as an example – of theory, which does not reduce to a class of propositions but has a further higher-order non-propositional structure. The axiomatic basis of this theory consists of a system of rules, which apply both at the propositional and non-propositional levels. At the semantic level these rules apply as rules for constructing further semantic constructions from primitive semantic elements assigned to primitive syntactic types. The model theory of HoTT does not fit into the standard Tarskian model-theoretic framework and requires a re-definition of the key concept of satisfaction.

The new constructive notion of theory motivated by HoTT and its model theory provides a precise sense in which a theory, generally, does not reduce to a class of its propositions. Thus it supplements the semantic aka non-statement view of theories in its usual form with a new axiomatic technique, which has a potential of applications in the theoretical physics, engineering and some other disciplines.