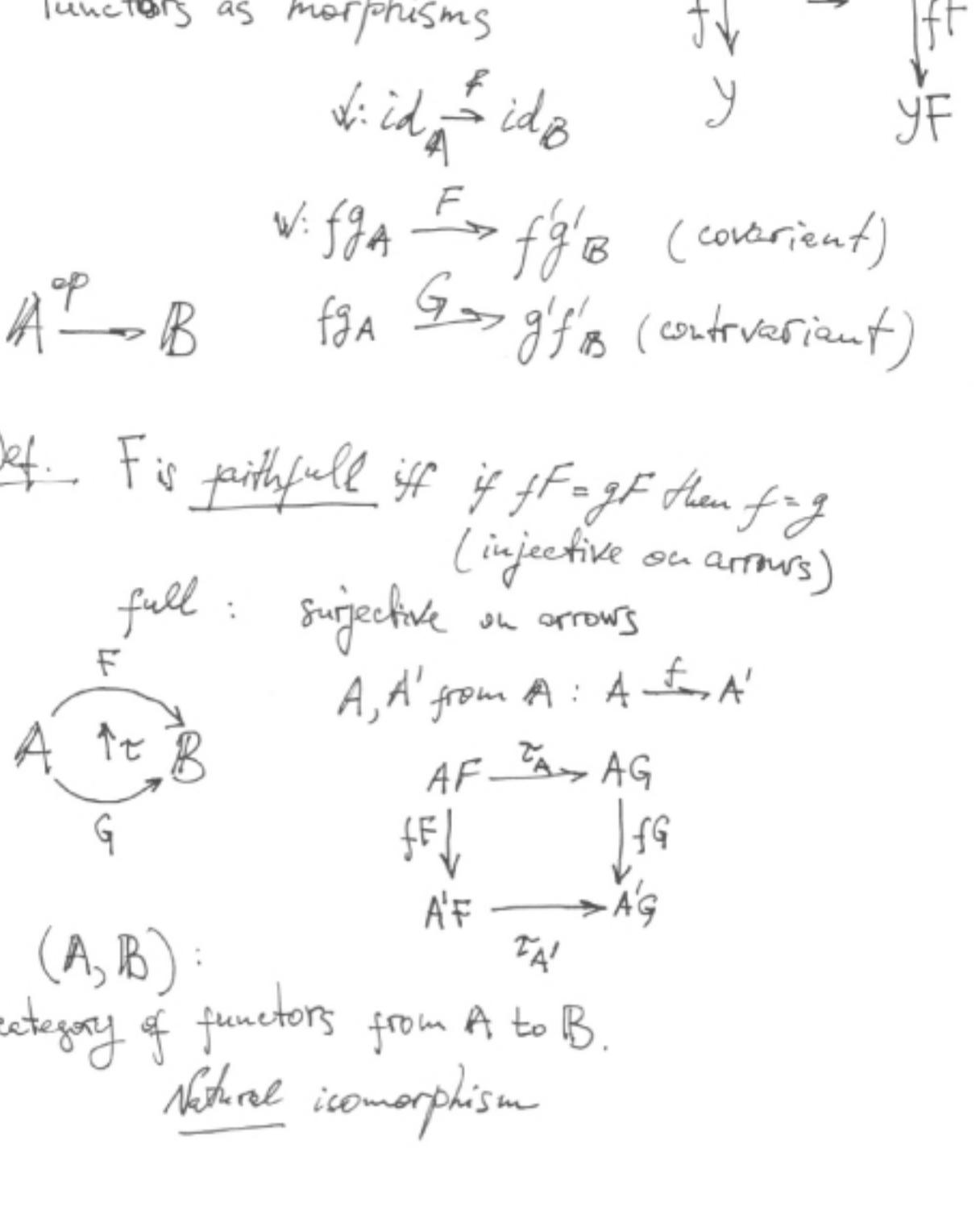


2. [Categories of Categories]

(1)



Def. F is faithful iff if $fF = gF$ then $f = g$
(injective on arrows)

full: surjective on arrows

$$A, A' \text{ from } A : A \xrightarrow{f} A'$$

$$\begin{array}{ccc} AF & \xrightarrow{\cong_A} & AG \\ fF \downarrow & & \downarrow fG \\ A'F & \xrightarrow{\cong_{A'}} & A'G \end{array}$$

(A, B) :

the category of functors from A to B .

Natural isomorphism

Categories from categories

$$S \xrightarrow{i} A \quad \text{monic}$$

Sub-category; full sub-category

$$\begin{array}{ccc} \text{Arrow category} & A^2 & A \xrightarrow{s} A' \\ (\text{of } A) & & \downarrow f \qquad \downarrow g \quad \text{arrow in} \\ & & B \xrightarrow{t} B' \quad A^2 \\ \text{Slice (comma) category} & A/X & \end{array}$$

$$\begin{array}{ccc} \text{Sheaf} & & A \xrightarrow{f} X \\ \text{Pre-Sheaf} & & f \downarrow \qquad g \\ & & A \xrightarrow{\text{op}} \text{Sets} \end{array}$$

Top/X (local homeo)

Ex1: 1 - category with one object 1

$$G: A \rightarrow 1 \quad F \dashv G \quad \begin{array}{c} 1 \xrightarrow{f} b \\ \downarrow f \xrightarrow{=} b \\ 1 \xrightarrow{f} b \end{array}$$

$$1 \xrightarrow{f} b \quad : b \text{ is initial}$$

$$H \dashv G : a \text{ is terminal}$$

$$\Delta: A \rightarrow AA: \text{ diagonal}$$

$$\begin{array}{c} a \xrightarrow{f} b \\ aa \xrightarrow{ff} bb \end{array}$$

$$G \dashv \Delta$$

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