Category Theory and Logic

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Category Theory (CT) is a branch of mathematics regarded by its proponents either as an alternative to or as a profoundly revised and generalized version of the Set theory in its role of unifying conceptual framework for mathematics.

Relationships between CT and logic are twofold. On the one hand, CT may be used as a powerful algebraic tool for combining logical systems and studying relations between logical systems. From this point of view applications of CT in Universal Logic are natural and straightforward. On the other hand, CT allows for an "internal" reconstruction of basic logical notions (truth-values, connectives, quantifiers) through a category-theoretic construction of topos. Since topos may be regarded as a generalised domain of discourse as well as a generalised geometrical space, this latter approach reveals new deep links between logic and geometry (topology).

The two approaches in the categorical logic are not formally incompatible but prima facie they assume different attitudes toward the followingphilosophical dilemma: Whether any mathematical theory needs certain logical foundations (to be made explicit through some formal logical system) or the formal logic itself is nothing but a particular application of mathematics, so one should build logic on a mathematical background rather than the other way round? CT sheds a new light on this old controversy, and allows for a substantial revision of its terms.

In this introductory tutorial we will give the basic notions of CT, briefly sketch both approaches in categorical logic, and consider the issue of Universal Logic from a category-theoretic point of view.

References

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