Philosophical Issues of the Category Theory

General Outline

The Category Theory is a relatively new branch of mathematics which was started in the late forties by MacLane and Eilenberg. Firstly the Category Theory was considered as a very general algebraic theory which reached a higher level of abstraction than previously existing theories and thus gave powerful new tools to deal with problems in algebra, algebraic topology and functional analysis. It was recognized very soon however that the Category Theory not only made a step toward higher abstraction in algebra, but also gave a reason to rethink the foundations of algebra and mathematics in general. The reason was that while the Set Theory in spite of its known difficulties was widely considered as a proper foundation of mathematics, the language of categories seemed to be a generalization of or even an alternative to that of sets. This meant that Category Theory could not be placed in the set theoretic «Cantor's paradise» as other algebraic theories (no matter how abstract) but rather had to redefine for itself such basic mathematical concepts as «element», «set», «part» (i.e. a «subset» in set-theoretical language), «point», «variable and its domain», «equality», «number», «function» and others. The new perspective, of course, could not leave logical issues untouched: attempts to redefine standard logical notions in categorial terms gave very interesting new logical concepts, particularly that of «internal logic» and «local truth value».

Since the Category Theory redefines very basic mathematical and logical concepts philosophers should not be indifferent to it. Since these concepts (e.g. «element», «part», «point») are significant not only for mathematics but also for general metaphysics, the philosophical importance of the Category Theory is not exclusive to the Philosophy of Mathematics. An especially important metaphysical problem which relates directly to Category Theory is that of Change and Time. Another related problem is that of the relationships between locality and universality in thinking. (The reason being that categorial language can be distinguished from set-theoretic as «dynamic» and «local» as opposed to «static» and «absolute».)

Until recently it seemed that even for a beginner in Category Theory a fairly strong mathematical background is a must. In fact, however, the situation with Category Theory is not different from that of any other mathematical theory of general significance (such as set theory, for example): while special questions require a suitable background, basic concepts can be explained to anyone who whishes to understand. In any case the most important concepts of contemporary mathematics, e.g. those of algebraic group and topological space, can be easier explained through Category Theory rather than the other way round.

Requirements and Organization

Although familiarity with mathematics is an advantage, no specific background is required, except a willingness to work at a certain level of abstraction and rigour.

The course consists of three types of activity - explanation of basic theoretical concepts, exercises and discussions. The requirements are a mid-term short paper, on a topic to be planned in advance (approximately 8-12 pages), and a final examination.

Readings

The principle reading is

F.W. Lawvere, S.H. Schanuel, Conceptual Mathematics: A first introduction to categories, Cambridge 1997 (Below referred to as CM)

In addition, I shall use material from the following texts:

J.L. Bell, From Absolute to Local Mathematics // Synthese, Vol. 69, N3, Dec.1986, pp. 409-26

J.L. Bell, Category Theory and the Foundation of Mathematics // British Journal for the Philosophy of Science 32 (1986), p. 349 ff.

R. Goldblatt, Topoi: the Categorial Analysis of Logic, Amsterdam, NY, Oxford 1979

Hajnal, A. Set theory, Cambridge, NY, 1999.

F.W. Lawvere Continuously variable sets: Algebraic Geometry = Geometric Logic // Logic Colloquium '73, ed. H.E.Rose, Shepherdson 1975, pp. 135-156

A.G. Kurosh, The Theory of Groups, NY 1960

Z.P Mamuzic, Introduction to general topology, Groningen, 1963.

Newton I. Mathematical Works

Plato, Republic

Ross S.D. Locality and Practical Judgment NY 1994

B.A.W. Russell, The Principles of Mathematics

Provisional Schedule

1. Introduction: Can mathematical objects change? Variables and functions. *Newton I. Mathematical Works, B.A.W. Russell, The Principles of Mathematics, ch.7*
2. Sets, maps and compositions. External and internal diagrams.*CM, Part 1, Session 2, Hajnal, A. Set theory, ch.1 (part 1)*
3. The category of sets. Arrows and urelements.*CM, Part 1, Session 1, , R. Goldblatt, Topoi: the Categorial Analysis of Logic, ch.1; J.L. Bell, Category Theory and the Foundation of Mathematics*
4. The algebra of compositions: isomorphisms and automorphisms; monoids and groups. *CM, Part 2 , Sessions 4, 7, Part 3, Session 1; A.G. Kurosh, The Theory of Groups, NY 1960*
5. The algebra of compositions: retractions and sections, naming and sorting, monomorphisms and epimorphisms.*CM, Part 2, Sessions 5, 6; Hajnal, A. Set theory, ch.1 (part 2)*
6. The algebra of compositions: samples and kinds, idempotents and Plato's Theory of Forms. *CM, Part 2, Session 9, Plato, Republic, ch.7*
7. Continuous maps and Brouwer's theorem: objectification and «mapification» of concepts.*CM, Part 2, Session 10*
8. Categories of structured sets: groups and topological spaces; continuity and the idea of topology; Erlangen Program for foundation of geometry.*CM, Part 3, introductory notes, R. Goldblatt, Topoi: the Categorial Analysis of Logic, ch.2*
9. Set-theoretic definition of topological structure and continuous maps. *Z.P Mamuzic, Introduction to general topology*
10. Categories of structured sets: graphs and dynamical systems.*CM, Part 3, Session 12*
11. Natural numbers.*CM, Part 3, Session 15, R. Goldblatt, Topoi: the Categorial Analysis of Logic, ch.2*
12. Universal mapping properties: Terminal and Initial objects; duality, Movement and Rest in the categorial language; points of an object; Zeno's paradoxes. *CM, Part 4, Session 19, Goldblatt, Topoi: the Categorial Analysis of Logic, ch.3*
13. Universal mapping properties: sums and products; exponentiation *CM, Part 4, Session 21, Goldblatt, Topoi: the Categorial Analysis of Logic, ch.3*

14. Local truth: external and internal logic; local and absolute; locality and openness. *Goldblatt, Topoi: the Categorial Analysis of Logic, ch.7, 14; J.L. Bell, From Absolute to Local Mathematics*
15. Topoi: parts of an object; locality and temporality.*CM, Part 5, Session 32; Lawvere F.W. Continuously variable sets*
16. Concluding discussion: How can local and temporal thinking be universally significant? *Ross S.D. Locality and Practical Judgment, ch.1*

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