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Lawvere and Hegel
Throughout his works in Categorical logic Lawvere refers to Hegels Dialectical logic as a major source of his intellectual inspiration. At several instances Lawvere also suggests to consider his newly invented Categorical logic as a version of Dialectical logic put into a precise mathematical form. The aim of this talk is to explore the impact of Hegel's philosophical logic onto Lawvere's pioneering works in Categorical logic.
Among several Hegelian themes referred to in Lawvere's writings the most frequent and, in my view, the most important is Hegel's distinction between the *subjective* and the *objective* logic. Hegel describes these two kinds of logic not as two parts of the same static whole but rather as two stages of the same dialectical reasoning, which begins with the most general ontological categories of Being, Non-Being and Becoming, then proceeds through some further categories like Thisness, Quality, Finiteness, Infinity, One, Many, Quantity, Measure, and some other, and only at a late stage involves the notion of subjectivity. Hegel's criticizes the traditional logic (descending from Aristotle) as well as Kant's Transcendental logic as merely subjective (and thus having no objective significance) by pointing the fact that these types of logic assume some notion of thinking subject from the outset without grounding this notion in the objective dialectics of categories.
Lawvere turns this Hegelian critique of logical subjectivism against the received notion of formal (symbolic) calculus by arguing that conventional symbolic presentations of formal logical calculi are chosen arbitrarily and fail to capture the objective invariant algebraic structures behind these calculi. As a remedy Lawvere puts forward his notion of Cartesian Closed Category (CCC), which captures a large class of relevant structures; then Lawvere upgrades the concept of CCC to that of *hyperdoctrine*, which captures the invariant structure of logical quantification with the notion of adjoint functor.
On a parallel development Lawvere discovers what he describes as the "unity of opposites [..] between logic and geometry" in the then-new field of topos theory pioneered by Grothendieck and his collaborators. Lawvere's basic observation is that all main features of a given topos have not only geometrical but also logical significance. This allows Lawvere to conceive of Topos logic as the *objective* logic in Hegel's sense, where usual (*subjective*) logical operation are grounded in spatial and other fundamental categories, which are commonly called non-logical.
Lawvere's work in Categorical logic paves the gap between between Hegel's Dialectical logic and the modern mathematical logic and thus makes it necessary to take Hegel's views on logic seriously in today's debates.
References:

(1) F.W. Lawvere, "Equality in hyperdoctrines and comprehension schema as an adjoint functor", *Applications of Categorical Algebra*, p. 1- 14, 1970

(2) F.W. Lawvere, "Quantifiers and sheaves", *M. Berger, J. Dieudonne et al. (eds.), Actes du congres international des mathématiciens, Nice*, p. 329 - 334, 1970