Rules versus Axioms: a Constructive View of Theories. Andrei Rodin

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In a Hilbert-style non-logical axiomatic theory the semantics of logical symbols is rigidly fixed, while the interpretation of non-logical symbols usually varies giving rise to different models of the given theory. All non-logical content of such a theory is comprised in its non-logical axioms (e.g. axioms of ZF) while rules, which generate from these axioms new theorems, belong to the logical part of the theory (aka "underlying logic"). This Hilbertian notion of axiomatic theory and its model has been used by Patrick Suppes and his many followers in their attempts to develop a general formal framework for representing scientific theories belonging to a wide range of disciplines [3].

An alternative approach to axiomatization due to Gentzen amounts to a presentation of formal calculi in the form of systems of rules without axioms. Gentzen did not try to extend his approach to non-logical theories by considering specific non-logical rules as a replacement for non-logical axioms. However the more recent work in Univalent Foundations of Mathematics [2] suggests that the Gentzen-style rule-based approach to formal presentation of theories may have important applications also outside the pure logic.

A reason why one may prefer a rule-based formal representation is that it is more computer-friendly. This, in particular, motivates the recent work on the "constructive justification" of the Univalence Axiom via the introduction of new operations on types and contexts [1]. Another reason is that such form of the representation allows one to represent formally various extra-logical methods, which play an important role in the justification of scientific theories but are left aside in the standard axiomatic representations.

Using HoTT and the Univalent Foundations as a motivating example I argue that the Gentzen-style rule-based approach provides a viable alternative to the standard axiomatic approach not only in logic but also in science more generally.

This work is supported by the Russian Foundation for Basic Research, research grant number 13-03-00384

References

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