$g: X \to B$ which satisfies (1) for all $x \in X$. Let us call such subsets *determined*. Then we can prove

THEOREM. *There exists a unique subset U of A such that (a) U is R-closed, i.e., $\forall x \in U, x^R \subseteq U$; (b) U is determined and all R-closed subsets of U are determined; (c) U is the largest subset of A satisfying (a) and (b). This theorem ensures, for any relation R, the existence and uniqueness of a function $g: U \to B$ which satisfies (1) on its domain and is defined on a domain U which extends the wellfounded part W of R.*

[1] R. MONTAGUE, *Well-founded relations: Generalizations of principles of induction and recursion*. ***Bulletin American Mathematical Society***, vol. 61, p. 442.

▸ GEMMA ROBLES, FRANCISCO SALTO, AND JOSÉ M. BLANCO, *Routley–Meyer semantics for natural implicative expansions of Kleene's strong three-valued matrix*.

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Routley–Meyer semantics, originally introduced for interpreting relevance logic, is a highly malleable semantics capable of modelling families of nonclassical logics very different from each other. Let us now understand the notion of a "natural implication" following [2]. Then, there are exactly six natural implicative expansions of Kleene's strong three-valued matrix with 1 as the sole designated value.

The aim of this article is to endow each one of the logics characterized by these six expansions with a Routley–Meyer type ternary relational semantics. There are well-known logics among those determined by these six expansions. Łukasiewicz three-valued logic Ł3 is an example.

Acknowledgments. Work supported by research project FFI2014-53919-P, financed by the Spanish Ministry of Economy and Competitiveness.

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[2] N. TOMOVA, *A lattice of implicative extensions of regular Kleene's logics*. ***Reports on Mathematical Logic***, vol. 47 (2012), pp. 173–182.

▸ ANDREI RODIN, *Two "styles" of axiomatization: Rules versus axioms. A modern perspective*.

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In a Hilbert-style nonlogical axiomatic theory the semantics of logical symbols is rigidly fixed, while the interpretation of nonlogical symbols usually varies giving rise to different models of the given theory. All nonlogical content of such a theory is comprised in its nonlogical axioms (e.g., axioms of ZF) while rules, which generate from these axioms new theorems, belong to the logical part of the theory (aka underlying logic).

An alternative approach to axiomatization due to Gentzen amounts to a presentation of formal calculi in the form of systems of rules without axioms. Gentzen did not try to extend his approach to nonlogical theories by considering specific nonlogical rules as a replacement for nonlogical axioms. However the more recent work in Univalent Foundations of Mathematics [2] suggests that the Gentzen-style rule-based approach to formal presentation of theories may have important applications also outside the pure logic.

A reason why one may prefer a rule-based formal representation is that it is more computer-friendly. This, in particular, motivates the recent work on the constructive justification of the Univalence Axiom via the introduction of new operations on types and contexts [1]. However this pragmatic argument does not meet the related epistemological worries. What kind of knowledge may represent a theory having the form of a bare system of rules? Is such a form of a theory appropriate for representing a knowledge of objective human-independent reality? How exactly truth features in rule-based nonlogical theories?

Using HoTT as a motivating example I provide some answers to these questions and show that the Gentzen-style rule-based approach provides a viable alternative to the standard axiomatic approach not only in logic but also in science more generally.

Acknowledgments. The work is supported by Russian Foundation for Basic Research, research grant 16-03-00364.

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► ALEKSANDRA SAMONEK, *Relation algebras, representability, and relevant logics*.
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This talk is an introduction to the problems concerning certain relevant logics and relation algebras.

[4] has shown how to obtain sound and complete semantics for RM, *i.e.*, the implicational fragment $R \rightarrow$ of R with the axiom mingle $A \rightarrow (A \rightarrow A)$. He also demonstrated how one can obtain a sound but not complete interpretation of R by replacing sets with commuting dense binary relations. But RM does not have a variable-sharing property (VSP) which R has. A modal restriction of RM in case of which the VSP is preserved was given in [5] together with the argument that from an intuitive semantical point of view, this modal restriction of RM is an alternative to Anderson and Belnap's logic of entailment E ([1]).

[6] has studied a version of positive minimal relevant logic B and [2] demonstrated that B is fully interpretable in the variety of weakly associative relation algebras which are not representable. [3] went on to show that if representability is dropped, one can obtain a complete interpretation of certain relevant logics in the language of relation algebras.

We will examine the mentioned results in order to clarify the connection between certain relation algebras and relevant logics like R and RM and see (i) whether such connection entails full interpretability of relevant logics in terms of relation algebras and (ii) what are the consequences of achieving this interpretability for representability and completeness.

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