

What is a Constructive Theory?

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The received notion of (axiomatic) theory, which stems from Hilbert [1], is that of set T of propositions (either contentful or non-interpreted aka propositional forms) with subset $A \subset T$ of axioms provided with a notion of consequence, which generates T from A in the obvious way. I argue that this standard notion is too narrow for being an adequate theoretical model of many mathematical theories; the class of such conterexamples is apparently very large and ranges from the geometrical theory of Euclid's *Elements*, Book 1 to the recent Homotopy Type theory [2]. In order to fix this problem I introduce a more general notion of theory, which involves typing and a generalized notion of consequence relation applicable also to objects of other types than propositions. I call such a theory *constructive* and explain how this notion of theory generalizes upon the notions of axiomatic and *genetic* theories discussed in the earlier literature [3]. The corresponding notion of being constructive is based on Kolmogorov's [4] and Smirnov's ideas while its formal type-theoretic representation is largely due to Martin-Löf [5]. Putting these ends together I show how the proposed general constructive axiomatic framework allows for distinguishing a number of more specific notions of constructivity, which are useful in special contexts. Finally I provide an epistemological argument intended to show that the proposed notion of constructive axiomatic theory is more apt to be useful in natural sciences and other empirical contexts than the standard notion. An extended version of this argument is given in [6].

Bibliography

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