Logical Forms versus Translational Categories

Logic is usually described as a discipline concerned with codification, systematisation of and theorising on general forms of reasoning. This is, of course, a very imprecise description but it is sufficient for my present purpose. For I am going to show that in there is a sense in which the given description and corresponding traditional notion of logic are too restrictive and then suggest an extension.

The concept of logical form stems from the fundamental observation that reasonings (however specified) like many other linguistic, social and natural phenomena come in patterns, that is, are in some sense repeatable. Given a sufficiently large set of individual organisms biologists find appropriate equivalence relations between them, and so classify them into species, types, etc. Then talking about features of, say, goldfishes a biologist at certain point may forget about any individual goldfish and pretend that the referred features belong to an abstract individual. Similarly a logician may talk about propositions, inferences and proofs without mentioning concrete examples (although they are often helpful).

We can see that the notion of form is not specific for logic. Why then logic (together with mathematics) is often described as formal science (as distinguished from empirical sciences like biology)? In my view this is a matter of degree but not of principle. One can reasonably talk about goldfishes without mentioning any individual goldfish but one can hardly say anything reasonable about an abstract animal forgetting that there are different kinds of animals. In logic, on the contrary, abstract form-concepts are supposed to work in higher levels of generality. For example Aristotle's "perfect syllogism" in Aristotle's own view was supposed to work through all semantic contexts allowing for a responsible scientific treatment. But how Aristotle or anybody else could possibly check that syllogisms or other putative logical forms work in this way indeed?

One may argue that unlike the case of biology such a check cannot be made empirically. For logic unlike biology purports not to describe how people do reason but to teach them how they should reason. (I am agree with this but I think that this is again rather a matter of degree than of principle. Think about bio-technology.) Perhaps the following argument in favour of the traditional idea of logic is crucial: unless certain context-independent rules of reasoning are assumed and respected no rational discussion, which is not restricted to a particular subject-matter would be possible. In other words, unless a speaking community agrees on a certain system of formal logic it cannot be a rational community, and so cannot develop anything like classical science. Notice the unifying function of logic here: no specialised esoteric knowledge requires logic by itself; logic is needed for making knowledge public and for providing different kinds of knowledge with a common space.

As far as there is only one system of logic on the market the above argument can sound persuasive and the available system of logic can play its intended role better or worse. However the development in logic started in 20th century completely changed the situation, and nowadays systems of logic are massively produced like any other mathematical structures. Although only few of these new symbolic logical calculi may and do pretend to replace the logic in the old good sense any choice of one of them as "basic" or "universal" looks at least problematic and at most wholly arbitrary. On the other hand, a sheer logical pluralism simply avoids the problem rather then resolves it: as far as logic ceases to be universal (or at least to pretend to be universal) it ceases to be logic in the traditional sense; in particular it cannot be any longer a backbone of rationality. So one may reasonably doubt whether various symbolic calculus selling itself as systems of logic really deserve this name and then look for "true logic" somewhere else.

A way of meeting the challenge is by trying to find a weakest structure shared by known logical calculi and on this ground stipulate this weakest logic as universal. Perhaps such a choice can be also made on a different and more complicated grounds but I think that it will remain disputable anyway. I propose a different solution. I claim that a shared logic - by which I now mean a formal logic in the traditional Aristotelian sense - is not in fact necessary for supporting a rational discussion. Imagine a community of speakers where each speaker reasons according to his or her own private logic. By private logic I mean a system of rules applied by a single person with respect to his or her own reasoning. It might seem that in this situation no rational discussion is ever possible. However this is not the case, at least if one is not too dogmatic about what rationality amounts to. What can allow for a rational discussion in the given situation is a well-organised network of translations between speaker's private logics. The common practice of translation between natural languages suggests that this might work. (The fact that such translations are, generally, non-reversible, can be easily shown by linguistic examples. Although translation between natural languages is not quite the same thing as translation between systems of logic the analogy seems me appealing.)

One can go further and suggest that what I have just called a translation network may replace logic (in the usual sense) also on the individual level, so after all we don't need the doubtful notion of private logic here. In other words even a particular reasoning can be viewed as a coherent translation between arguments. One more way to put this is by saying that logic can vary at the private scale as well as at the scale of community. However this latter description is rather unfortunate because it only says that there is no longer any fixed logic but doesn't make it clear what is suggested instead. As far as the term "logic" is concerned I don't think that this would be a good idea to qualify my present suggestion as going beyond logic tout court; what I'm trying to do is not to replace logic by something else but rather enlarge the traditional notion of logic (and the corresponding notion of rationality).

The above suggestion might looks extremely vague but I have a piece of mathematics in my pocket, which will help me to make it more precise. This is the Category theory. The mathematical notion of Category captures well pre-theoretical notions of translation, transformation, mapping and the like. Let's provisionally assume that reasonings and translations between reasonings make categories. Similarly, assume that reasonings themselves are categories. (This latter assumption is important in order to stress that my present proposal doesn't reduce to the idea of translation between different systems of formal logic.)

Remark that the notion of category is a generalisation (rather than specification) of the notion of form (or structure). For the notion of form (structure) is an abstraction made on the basis of certain equivalence relation (cf. Fregean abstraction). An equivalence relation can be described as existence of isomorphism of a certain kind. The notion of category is more general than that of form (structure) for the simple reason that categorical morphisms are, generally speaking, not isomorphisms (they are generally not reversible). Generally objects of a category don't share anything like common form (unless this category is a groupoid).

At this point we don't need to assume much about reasonings, in particular I suggest to avoid any straightforward distinction between form and content. One can imagine a lot of very different ways of how one reasoning can be "translated" into another - one may even think about Freudian free association in these terms. The question is now the following: What is a logical morphism between reasonings? or in other words: What kind of category can be reasonably called logical ? These questions replace traditional questions about logical form, and the former questions don't reduce to the latter.

One reasonable answer is that logical categories should involve truth values and that logical morphisms should preserve them. Such categories are known in mathematics as toposes. Remarkably toposes allow not only for logical morphisms but also for geometrical ones. In my talk I shall revise Topos theory from the point of view explained above and try to answer the question whether or not this theory can provide a universal traslational framework for reasoning.