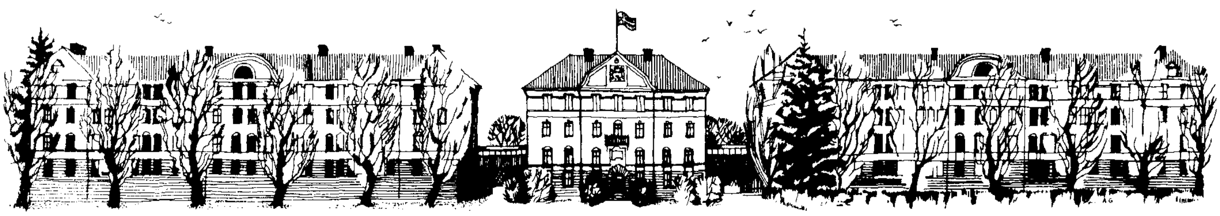
# CATEGORIFICATION
# in ALGEBRA and TOPOLOGY

List of Participants, Schedule and Abstracts of Talks



Department of Mathematics
Uppsala University

# CATEGORIFICATION in ALGEBRA and TOPOLOGY

Department of Mathematics
Uppsala University
Uppsala, Sweden
September 6-11, 2006

Supported by

- The Swedish Research Council

## Organizers:

- **Volodymyr Mazorchuk**, Department of Mathematics, Uppsala University;
- **Oleg Viro**, Department of Mathematics, Uppsala University.

# List of Participants:

1. **Ole Andersson**, Uppsala University, Sweden
2. **Benjamin Audoux**, Universite Toulouse, France
3. **Dror Bar-Natan**, University of Toronto, Canada
4. **Anna Beliakova**, Universitt Zrich, Switzerland
5. **Johan Bjorklund**, Uppsala University, Sweden
6. **Christian Blanchet**, Universite Bretagne-Sud, Vannes, France
7. **Jonathan Brundan**, University of Oregon, USA
8. **Emily Burgunder**, Universite de Montpellier II, France
9. **Paolo Casati**, Universita di Milano, Italy
10. **Sergei Chmutov**, Ohio State University, USA
11. **Alexandre Costa-Leite**, University of Neuchatel, Switzerland
12. **Ernst Dieterich**, Uppsala University, Sweden
13. **Roger Fenn**, Sussex University, UK
14. **Peter Fiebig**, Freiburg University, Germany
15. **Karl-Heinz Fieseler**, Uppsala University, Sweden
16. **Anders Frisk**, Uppsala University, Sweden
17. **Charles Frohman**, University of Iowa, USA
18. **Juergen Fuchs**, Karlstads University, Sweden
19. **Patrick Gilmer**, Louisiana State University, USA
20. **Nikolaj Glazunov**, Institute of Cybernetics, Kyiv, Ukraine
21. **Ian Grojnowski**, Cambridge University, UK
22. **Jonas Hartwig**, Chalmers University of Technology, Gothenburg, Sweden

23. **Magnus Hellgren**, Uppsala University, Sweden
24. **Martin Herschend**, Uppsala University, Sweden
25. **Magnus Jacobsson**, INdAM, Rome, Italy
26. **Zhongli Jiang**, BGP Science, P.R. China
27. **Louis Kauffman**, University of Illinois at Chicago, USA
28. **Johan Kåhrström**, Uppsala University, Sweden
29. **Sefi Ladkani**, The Hebrew University of Jerusalem, Israel
30. **Sergei Lanzat**, Haifa Technion, Israel
31. **Bernard Leclerc**, Universite de Caen, France
32. **Nicolas Libedinsky**, ENS-Paris, France
33. **Georges Maltsiniotis**, CNRS, Universite Paris 7, France
34. **Ciprian Manolescu**, Columbia University and CMI, USA
35. **Vassily Manturov**, Moscow State Regional University, Russia
36. **Gregor Masbaum**, Universite Paris 7, France
37. **Volodymyr Mazorchuk**, Uppsala University, Sweden
38. **Scott Morrison**, UC Berkeley, USA
39. **Gad Naot**, University of Toronto, Canada
40. **Serge Ovsienko**, Kyiv University, Ukraine
41. **Jerome Petit**, Institute of Mathematics, Montpellier, France
42. **Hendryk Pfeiffer**, MPI for Gravitational Physics, Germany
43. **Andrei Prasolov**, Universitetet i Tromsoe, Norway
44. **Josef Przytycki**, George Washington University, USA
45. **Jacob Rasmussen**, Princeton University, USA
46. **Andrei Rodin**, ENS, France
47. **Yongwu Rong**, George Washington University, USA
48. **Raphaël Rouquier**, University of Leeds, UK

49. **Ryszard Rubinsztein**, Uppsala University, Sweden
50. **Ingo Runkel**, King's College London, UK
51. **Bjrn Selander**, Uppsala University, Sweden
52. **Alexander Shumakovitch**, George Washington University, Washington, USA
53. **Wolfgang Soergel**, Freiburh University, FRG
54. **Catharina Stroppel**, University of Glasgow, UK
55. **Joshua Sussan**, Yale University, USA
56. **David Treumann**, Princeton University, USA
57. **Oleg Viro**, Uppsala University, Sweden
58. **Ben Webster**, UC Berkeley, USA
59. **Yasuyoshi Yonezawa**, Nagoya University, Japan

# Schedule of Talks

## Thursday, September 7th, 2006.

| | | |
|---|---|---|
| 08.30-09.20 | Louis H. Kaufmann | An Extended Bracket Polynomial for Virtual Knots and Links |
| 09.30-09.55 | Gad Naot | The universal Khovanov link homology theory |
| 10.00-10.25 | Hendryk Pfeiffer | Tangle homology from 2-dimensional extended Topological Quantum Field Theories |
| 10.30-11.00 | | Coffee-break |
| 11.00-11.25 | Magnus Jacobsson | $sl(3)$-homology for knotted webs and their cobordisms |
| 11.30-11.55 | Vassily Manturov | Khovanov homology for virtual knots with arbitrary coefficients |
| 12.00-14.00 | | Lunch |
| 14.00-14.50 | Scott Morrison | Disoriented and confused: Fixing the functoriality of Khovanov homology |
| 15.00-15.25 | Oleg Viro | Khovanov homology of signed chord diagrams and vice orientations |
| 15.30-15.55 | Benjamin Audoux | Surfaces with pulleys and categorification of refined Jones polynomials |
| 16.00-16.30 | | Coffee-break |
| 16.30-16.55 | Charles Frohman | TBA |
| 17.00-17.50 | Anna Beliakova | Categorification of the colored Jones polynomial |
| 18.30-21.00 | | Welecome Party - Eklundshof (Markan) |

## Friday, September 8th, 2006.

| | | |
|---|---|---|
| 08.30-09.20 | Wolfgang Soergel | Graded version of tensoring with finite dimensional representations |
| 09.30-09.55 | Peter Fiebig | Categories associated to Bruhat graphs and Kazhdan-Lusztig theory |
| 10.00-10.25 | Joshua Sussan | Category $\mathcal{O}$ and $sl_k$ Link Invariants |
| 10.30-11.00 | | Coffee-break |
| 11.00-11.25 | Volodymyr Mazorchuk | Category $\mathcal{O}$ as a source for categorification |
| 11.30-11.55 | Andrei V. Prasolov | Coherent homotopy commutative diagrams in algebraic topology |
| 12.00-14.00 | | Lunch |
| 14.00-14.50 | Jonathan Brundan | Symmetric functions, parabolic category $\mathcal{O}$ and the Springer fiber |
| 15.00-15.25 | Roger Fenn | Linear Biracks and Virtual Link Invariants |
| 15.30-15.55 | Ryszard Rubinsztein | Topological quandles: invariants of links and quandle spaces |
| 16.00-16.30 | | Coffee-break |
| 16.30-16.55 | Ingo Runkel | Categorification and correlation functions in conformal field theory |
| 17.00-17.50 | Catharina Stroppel | Khovanov cohomology as an offspring of representation theory and geometry |

## Saturday, September 8th, 2006.

| | | |
|---|---|---|
| 08.30-09.20 | Jacob Rasmussen | Differentials on Khovanov-Rozansky homology |
| 09.30-09.55 | Józef H. Przytycki | Hochschild homology as Khovanov homology of torus links |
| 10.00-10.25 | Alexander Shumakovitch | Behavior of knot invariants under genus 2 mutation |
| 10.30-11.00 | | Coffee-break |
| 11.00-11.25 | Sergei Chmutov | Thistlethwaite's theorem for virtual links |
| 11.30-11.55 | Christian Blanchet | Link homology from trivalent TQFT |
| 12.00-14.00 | | Lunch |
| 14.00-14.50 | Ciprian Manolescu | A combinatorial description of knot Floer homology |
| 15.00-15.25 | Yongwu Rong | Khovanov Type Homologies for Graphs |
| 15.30-15.55 | Patrick Gilmer | Integral lattices in TQFT |
| 16.00-16.30 | | Coffee-break |
| 16.30-16.55 | Gregor Masbaum | Integral TQFT and perturbative expansion |
| 17.00-17.50 | Dror Bar-Natan | Algebraic Knot Theory |

## Sunday, September 10th, 2006.

| | | |
|---|---|---|
| 09.00-09.50 | Bernard Leclerc | Preprojective algebras and categorification of cluster algebras |
| 10.00-10.25 | David Treumann | Constructible stacks |
| 10.30-11.00 | | Coffee-break |
| 11.00-11.25 | Zhongli Jiang | Lie powers of the natural module for finite general linear groups |
| 11.30-11.55 | Andrei Rodin | Categorification and Interpretation: Beyond the Formal |
| 12.00-14.00 | | Lunch |
| 14.00-14.50 | Raphaël Rouquier | Higher Representation Theory |
| 15.00-15.25 | Nikolaj Glazunov | Categorification of hard and soft geometry with applications to substantiation of conjectures |
| 15.30-15.55 | Alexandre Costa-Leite | Philosophical categorification |
| 16.00-16.30 | | Coffee-break |
| 16.30-17.20 | Ian Grojnowski | TBA |

# ABSTRACTS

## Surfaces with pulleys and categorification of refined Jones polynomials

**Benjamin Audoux**

Jones polynomials can be generalized to links in $I$–bundle. They can also be refined in order to be invariants of links up to some restricted notion of isotopies (e.g. braid–like and star–like isotopies). In all these constructions, one have to distinguish two kinds of circles among the connected componants of a Kauffman state. This distinction is at the root of the definition of categories of surfaces with pulleys. Thanks to them, we can define some invariants of "links" (by "links", we mean links in $I$–bundle, braid–like or star–like links). By applying a fonctor, we find again the usual categorifications of all these refined Jones polynomials.

Université Paul Sabatier, Toulouse, France

## Algebraic Knot Theory

**Dror Bar-Natan**

Wearing the hat of a topologist, I will argue that despite the (justified) great interest in categorification, the good old "Kontsevich Integral" is even more interesting and highly under-studied. The gist: the Kontsevich Integral behaves well under cool operations that make a lot of 3-dimensional sense.

A summary is, and handouts will be, at:
`http://www.math.toronto.edu/drorbn/Talks/Uppsala-0609/`

University of Toronto

# Categorification of the colored Jones polynomial

**Anna Beliakova** & Stephan Wehrli

Using the Bar-Natan approach, we modify the Khovanov's categorification of the colored Jones polynomial by introducing two parameters. Moreover, we extend the coefficients from $\mathbb{Z}/2\mathbb{Z}$ to $\mathbb{Z}$ or $\mathbb{Z}[1/2]$. We argue that for special choices of these parameters (not reproducing to the original Khovanov's construction), Khovanov and Lee homology theories are functorial with respect to colored framed link cobordisms (up to sign). Finally, we show that the Rasmussen invariant extended to links is a strong obstruction to sliceness.

Institut für Mathematik, Universität Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland

# Link homology via trivalent TQFT

**Christian Blanchet**

For each integer $N$, we construct a TQFT for planar trivalent graphs and trivalent surfaces. Following Khovanov construction for $sl(2)$ and $sl(3)$, we use this TQFT to define an homology of links which should be equivalent to Khovanov-Rozansky $sl(N)$ link homology.

University Bretagne-Sud, Vannes France

# Symmetric functions, parabolic category $O$ and the Springer fiber

**Jonathan Brundan**

I will explain how the direct sum of all partial coinvariant algebras (a.k.a. the cohomology algebras of all partial flag varieties in an $n$ dimensional vector space) can be made into a polynomial representation of the general linear group $GL(V)$, giving a simple algebraic construction of a naturally graded model for the tensor space $V^{\otimes n}$. Combined with Soergel's approach to the BGG category $O$ for the Lie algebra $gl_n(C)$, this allows me to give an explicit description of the center of an arbitrary block of an arbitrary parabolic category $O$ as a quotient of a partial coinvariant algebra. For regular blocks, the resulting presentation is the same as Tanisaki's presentation for the cohomology algebra of the corresponding Springer fiber, as was conjectured by Khovanov. Catharina Stroppel has found an independent proof of this conjecture.

University of Oregon

# Thistlethwaite's theorem for virtual links

**Sergei Chmutov**

The celebrated Thistlethwaite theorem relates the Jones polynomial of a link with the Tutte polynomial of a corresponding planar graph. We give a generalization of this theorem to virtual links. In this case the graph will be embedded into a (higher genus) surface. For such graphs we use a generalization of the Tutte polynomial discovered by B. Bollobás and O. Riordan.

The Ohio State University, Mansfield

# Philosophical categorification

**Alexandre Costa-Leite**

The purpose of this talk is to explain what is philosophical categorification. Logic is a powerful tool which philosophers utilise to better understand their problems and concepts. As it is well-known, the relationships between logic and category theory appear in different levels: 1) logical operators can be represented in categories, since objects are propositions and morphisms are proofs (for instance the works of Lambek and Goldblatt); 2) logics can also be assumed as objects of categories where morphisms are translations (for example, some articles of Carnielli and Coniglio); 3) methods for combining logics are universal constructions in some categories where objects are logics (the approach developed by Sernadas and Caleiro). Indeed, there are in the literature some other examples of how category-theoretic concepts can replace logical concepts. However, given that logic is a very important tool in order to understand concepts from philosophical areas such as epistemology and metaphysics, a natural conjecture is that category theory can also play an important role in philosophy. Category theory is a tool which can be used in philosophical theories and itself has an ontological status (some attempts of applying categories in philosophy are those of Badiou, Marquis, Rodin and Reyes). Philosophical categorification is the philosophical counterpart of categorification introduced by some mathematicians (Dolan and Baez), but replacing logical concepts for categorial concepts, and also set-theoretic notions by category-theoretic notions in order to investigate philosophical concepts. In this talk, some concrete examples of philosophical categorification and the philosophical foundations of categorification are investigated.

University of Neuchâtel - CH
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# Linear Biracks and Virtual Link Invariants

**Roger Fenn**

Biracks and biquandles are curious algebraic objects and in general little is known about them. However some things are known about linear biracks (=biquandles) and this is sufficient to give workable invariants for virtual and flat links.

Sussex

# Categories associated to Bruhat graphs and Kazhdan-Lusztig theory

**Peter Fiebig**

To each Coxeter system we associate an exact category and determine its projective objects by both a local (the *localization on the Bruhat graph*) and a global method (the *translation combinatorics*). Thus we get an identification of Soergel's bimodules and the Braden-MacPherson canonical sheaves. We derive new properties, and in particular we prove the multiplicity free case of the corresponding Kazhdan-Lusztig conjecture, and give an explicit description of the exceptional characteristics of the ground field.

Universität Freiburg

# Integral lattices in TQFT

**Patrick Gilmer**

We will describe a version of the SO(3)-TQFTs at roots of unity of odd prime order which is defined over a cyclotomic ring of integers and on the cobordism category of targeted morphisms. We will give explicit bases, found with Gregor Masbaum, for the modules associated to surfaces.

Louisiana State University, Baton Rouge

# Categorification of hard and soft geometry with applications to substantiation of conjectures

**Nikolaj Glazunov**

Many problems from arithmetic geometry, symplectic geometry are formulated and described as very rigid mathematical problems in terms of number theory, algebra and algebraic geometry.

Our aim is to make the link between hard and soft mathematics more precise. It will appear that many problems of hard mathematics can be parameterized and then investigated by soft mathematics. However, there are exceptions.

To illustrate the limits and the possibilities of investigating hard problems by soft methods, we will present a general category theory framework of hard and soft mathematics. We illustrate our considerations on the proof of Minkowski's conjecture about critical determinant of the region $|x|^p + |y|^p < 1,\ p > 1$, and on investigation of aspects of categorical mirror symmetry.

Glushkov Institute of Cybernetics National Academy of Sci. Prospect Glushkova 40, 03680, Kiev-187 GSP Ukraine. glanm@yahoo.com

# Lie powers of the natural module for finite general linear groups

**Zhongli Jiang**

Let $L$ be a free Lie algebra of finite rank $r$ over a field $F$. For each positive integer $n$, denote the degree $n$ homogeneous component of $L$ by $L_n$. The group of graded algebra automorphisms of $L$ may be identified with $GL(r, F)$ in such a way that $L_1$ becomes the natural module, and then the $L_n$ are referred to as the Lie powers of this module. When $F$ is of characteristic zero, the $GL(r, F)$-module structure of the $L_n$ was understood already fifty years ago. This talk will report current work on the case when $F$ is a finite prime field.

Beijing Guojing Pengrun Science and Trade Co., Ltd.

# An Extended Bracket Polynomial for Virtual Knots and Links

**Louis H. Kauffman**

This talk will discuss a new extended bracket polynomial for virtual knots and links. This polynomial, $E(K)$, takes values in Laurent polynomials in the variable $A$, and virtual equivalence classes of flat virtual diagrams associated to states obtained by smoothing the diagram. This polynomial is distinct from similar polynomials defined at the level of surface embeddings. It is defined in terms of the planar representation diagrams for the virtual knots, and can directly distinguish many diagrams such as the Kishino diagram and certain long knot diagrams whose closures are unknotted. We will compare this invariant with the original bracket polynomial and with the Miyazawa polynomial for virtuals. We will discusss the possibility for categorifications of the invariant $E(K)$.

University of Illinois at Chicago

# Preprojective algebras and categorification of cluster algebras

**Bernard Leclerc**

In a joint series of papers with Christof Geiss and Jan Schröer, we have obtained a categorification of the cluster algebra structure on the ring $\mathbb{C}[N]$ of polynomial functions on a maximal unipotent subgroup $N$ of a complex simple algebraic group $G$ of type A, D, E. More precisely, let $\Lambda$ denote the preprojective algebra attached to the Dynkin diagram of $G$. We proved that the clusters of $\mathbb{C}[N]$ are in natural bijection with maximal rigid modules over $\Lambda$, and we introduced a mutation operation on these modules lifting the Fomin-Zelevinsky mutation for clusters. This allowed us to prove that all cluster monomials of $\mathbb{C}[N]$ belong to the dual of Lusztig's semicanonical basis of the enveloping algebra of the Lie algebra of $N$.

The talk will review these results. If time allows, I will indicate how appropriate subcategories of mod$-\Lambda$ yield in a similar way new cluster algebra structures on the coordinate rings of the partial flag varieties $G/P$, where $P$ is an arbitrary parabolic subgroup.

Université de Caen, France

# A combinatorial description of knot Floer homology

**Ciprian Manolescu**

Knot Floer homology is an invariant of knots in the three-sphere, which detects the genus of the knot, and can be used to recover the Heegaard Floer homology of any surgery on that knot. The original definition, due to Ozsváth-Szabó and Rasmussen, involved counts of pseudoholomorphic disks in a symplectic manifold. In joint work with Peter Ozsváth and Sucharit Sarkar, we found a purely combinatorial description of this invariant. Starting with a grid presentation of the knot, one construct a special Heegaard diagram for the knot complement, in which the count of pseudolomorphic disks is elementary.

Columbia University and The Clay Mathematics Institute

# Khovanov homology for virtual knots with arbitrary coefficients

**Vassily Olegovich Manturov**

We construct the Khovanov homology theory for all virtual (and overtwisted virtual) knots with arbitrary coefficients, and several generalizations of it (Frobenius extensions etc). A key notion in this construction is an atom, a checkerboard 2-surface. We shall consider both algebraic (by using Hopf algebra) and topological counterpart (tanlges and cobordisms) part of the theory. As a partial case, this leads to Khovanov homology for knots in $RP^3$. This Khovanov homology admits a spanning tree which leads to some crossing number estimates (via atoms).

Moscow State Regional University, Moscow, Russia

# Integral TQFT and perturbative expansion

**Gregor Masbaum**

We study the mapping class group representations associated to the SO(3) TQFT at an odd prime. The main result of this talk is that when restricted to the Torelli group, these representations stabilize (in a sense to be explained) as the order of the root of unity goes to infinity. Here it is crucial to use the integral bases we found in previous joint work with Pat Gilmer. This gives in particular a skein-theoretical construction of Ohtsuki's power series invariant of homology spheres.

Insitut de Mathematiques de Jussieu, Paris, France

# Category $\mathcal{O}$ as a source for categorification

**Volodymyr Mazorchuk**

The action of translation functors on the (graded version of) principal block of the BGG category $\mathcal{O}$ for $\mathfrak{sl}_n$ gives rise to the regular representation of the symmetric group $S_n$ (the corresponding Hecke algebra) on the complexified Grothendieck group. Restricting to subcategories one gets categorification. For example a module induced from the sign module over a parabolic subgroup can be categorified via Rocha-Caridi's parabolic subcategory of $\mathcal{O}$; the Specht module can be categorified via the endomorphism algebra of the characteristic projective-injective module in this parabolic subcategory; the permutation module via some other parabolic subcategory of $\mathcal{O}$. Several modules even have many different categorifications, which naturally raises the question of comparing them and the information one can derive from these categorifications. In the talk I will try to describe these various categorifications, discuss questions related to them and present some explicit examples.

Uppsala University, Uppsala, Sweden, `mazor@math.uu.se`

# Disoriented and confused: Fixing the functoriality of Khovanov homology.

**Scott Morrison**

This is a joint work with Kevin Walker. I'll describe a modification of Bar-Natan's cobordism model for Khovanov homology, introducing "disorientations", and some rules for manipulating them. Using these, we discover that link cobordisms now induce honest well-defined maps between the complexes associated to links, not just up-to-sign maps. In addition to "disorientations", we can also add "confusions": places where a disorientation changes type. These live up to their name; they need a spin framing, and change sign when rotated. However, they fix a few defects of the "disoriented" model, allow nice proofs, and make contact with some familiar features of su(2)'s planar representation theory. There will be lots of pictures!

University of California, Berkeley

# The universal Khovanov link homology theory

**Gad Naot**

In this talk I will introduce the universal Khovanov link homology theory. This theory is developed using the full strength of the geometric formalism of Khovanov link homology theory and has many computational and theoretical advantages. The universal theory answers questions regarding the precise amount of information held within the complex associated to a link. It also answers questions regarding the extraction of this information by giving full control over the various TQFTs applied to the complex (along with control over other gadgets such as the various spectral sequences related to these TQFTs). After a brief overview and some reminders I will introduce the major tools and ideas used in developing the universal theory (such as surface classification, genus generating operators, complex isomorphisms and "promotions"). Then I will present some of the advantages of such a theory, time permitting (more on the topic can be found at arXiv:GT/0603347).

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# Tangle homology from 2-dimensional extended Topological Quantum Field Theories

**Hendryk Pfeiffer**

The definition of Khovanov's chain complex relies on a 2-dimensional Topological Quantum Field Theory (TQFT). Since the boundary of a 2-dimensional cobordism is a closed 1-manifold, this construction is automatically restricted to links as opposed to tangles. We use 2-dimensional extended TQFTs in which the cobordisms are replaced by manifolds with corners, in order to generalize Khovanov homology from links to tangles.

Our construction can be seen as a translation of Bar-Natan's picture world into algebra in which both the composition of tangles and the tensor product of tangles are described by algebraic operations. This is important if one tries to reverse-engineer Khovanov homology in order to uncover a braided monoidal 2-category.

Max-Planck-Institute for Gravitational Physics, Am Mühlenberg 1, 14476 Potsdam, Germany

# Coherent homotopy commutative diagrams in algebraic topology

**Andrei V. Prasolov**

We discuss three areas where coherent homotopy commutative diagrams (*CHCD*) arise:

- loop spaces and $\Gamma$-spaces;
- homotopy (co)limits of spaces and spectra;
- strong shape theory.

The methods used in all the three areas are similar, and can be formalized. One of the methods is to substitute a *CHCD* with an equivalent strict commutative diagram. We discuss the situations when such substitution is possible, and develop some important consequences. We consider also morphisms between *CHCD*s, and strict commutative substitutions for such morphisms.

Institutt for matematikk og statistikk Universitetet i Tromsø

# Hochschild homology as Khovanov homology of torus links

**Józef H. Przytycki**

We show that Khovanov homology and Hochschild homology theories share common structure. In fact they overlap: Khovanov homology of a $(2, n)$-torus link can be interpreted as a Hochschild homology of the algebra underlining the Khovanov homology. In the classical case of Khovanov homology we prove the concrete connection. In the general case of Khovanov-Rozansky, $sl(n)$, homology and their deformations and in the case of categorification of colored Jones polynomial we conjecture the connection. We will show, on several examples, how the knowledge of, Hochschild homology can be used to compute Khovanov type homology.

George Washington University

# Differentials on Khovanov-Rozansky homology

**Jacob Rasmussen**

For each $N > 0$, there is a spectral sequence relating Khovanov and Rozansky's HOMFLY homology to the $sl(N)$ homology. I'll describe these sequences and some of their applications to understanding the structure of KR homology.

Princeton University

# Categorification and Interpretation: Beyond the Formal

**Andrei Rodin**

Mathematics is often described as a "formal" science as opposed to sciences studying certain specific empirical "content" like physics, chemistry, biology or any of their subdivisions. The idea behind this description is that mathematical objects are forms (or more precisely "pure" or "idealized" forms) of things found in Nature. In this talk I analyze the relevant notion of form and claim the following: Categorification significantly widens the traditional notion of mathematics by extending it "beyond the formal".

The traditional notion of mathematical "form" is that of an invariant under certain reversible transformations (an invariant of a group). So this notion depends on the corresponding class of transformations. For example, in a metrical context the form of "circle" is defined up to motions and scale transformations, while in a purely topological context "circle" is defined up to homeomorphism. (This, of course, makes a difference: in a metrical context circular and oval forms are different but topologically they are the same.) This geometrical pattern applies to algebra thanks to reversibility of the substitution: so one may call formula x+y=y+x a "common algebraic form" of numerical expressions 1+2=2+1, 2+3=3+2, etc. . D. Hilbert in his "Grundlagen der Geometrie" applies this notion of form to a geometrical theory as a whole through his "axiomatic method": the idea is to describe a given theory only up to isomorphism leaving it to the user to choose his or her favorite model. Although this method, generally speaking, doesn't work - I mean the presence of so called "non-standard" models - Hilbert's formalist approach (in the wide sense just mentioned) still remains very influential in mathematics. Categorification changes this fundamental pattern and the associated intuition of "form" by the simple fact that it assumes the notion of non-reversible transformations (morphism) as primitive and defines reversible morphisms as a special case. Thus in the context of functorial semantics first proposed in Lawvere's thesis the usual worry of ruling out non-standard models makes no sense, and the very distinction between a theory and its model changes its usual meaning. Algebraic and geometric properties in a categorical context reveal profound links unnoticed before. In my talk I shall analyze these and some other consequences of "dropping the reversibility" via categorification.

ENS, France, rodin@ens.fr

# Khovanov Type Homologies for Graphs

**Yongwu Rong**

Following Khovanov, there have been a number of analogous homology theories for graphs. In this talk, we will explain several such theories which correspond to the chromatic polynomial (with Laure Helme-Guizon), or the Tutte polynomial (with E. Fanny Jasso-Hernandez), or the Bollobas-Riordan polynomial, or the Penrose polynomial (with M. Khovanov and Kerry Luse).

Department of Mathematics, The George Washington University, Washington, DC 20052, U.S.A.

# Higher Representation Theory

**Raphaël Rouquier**

Representation theory can be viewed as the study of vector spaces with symmetries. I would like to present some evidence that there is an interesting theory where vector spaces are replaced by abelian (or triangulated) categories. The main purpose is to study and understand certain module or sheaf categories from this viewpoint. I will discuss the joint work with Joe Chuang where the case of $sl_2$ is developed.

University of Leeds

# Topological quandles: invariants of links and quandle spaces.

**Ryszard Rubinsztein**

We discuss topological quandles and invariants of classical links derived from them. Topological quandles are quandles in the category of topological spaces. Given such a quandle $Q$ and an oriented link $L$ in $\mathbb{R}^3$ we construct an invariant $J_Q(L)$ which is a topological space well-defined up to homeomorphism. For a topological quandle $Q$ we also construct a quandle space $B_qQ$ playing a role of "classifying space" of $Q$ and similar to (but different from) the rack space $BQ$ of Fenn, Rourke and Sanderson.

Department of Mathematics, Uppsala University, Uppsala, Sweden.

# Categorification and correlation functions in conformal field theory

**Ingo Runkel**

Two-dimensional open/closed conformal field theory (CFT) can be defined through its correlation functions. They must satisfy certain consistency conditions which arise from the cutting of world sheets along circles or intervals. These conditions can be conveniently summarised by thinking of the correlators as a natural transformation between two functors from a geometric category to a category of vector spaces. Under suitable assumptions, specifying such a natural transformation is equivalent to giving a simple symmetric Frobenius algebra in an appropriate braided monoidal category, namely the representation category of the chiral symmetries of the CFT.

King's College London

# Behavior of knot invariants under genus 2 mutation.

Nathan M. Dunfield, Stavros Garoufalidis, **Alexander Shumakovitch,** and Morwen Thistlethwaite

We study behavior of various knot invariants under genus 2 mutation. We generalize results of Morton-Traczyk and Ruberman to show that the colored Jones polynomials (for all colors) and the hyperbolic volume are invariant under such mutation and, hence, under the Conway (that is, genus 0) mutation.

On the other hand, we show that the HOMFLY-PT polynomial and Khovanov homology may change under genus 2 mutation, although it is known that they are invariant under the Conway one. The question about the Kauffman polynomial is still open.

Finally, we present examples of knots with the same same colored Jones polynomials, HOMFLY-PT polynomial, Kauffman polynomial, signature, and hyperbolic volume, but different Khovanov homology.

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# Graded version of tensoring with finite dimensional representations

**Wolfgang Soergel**

Each finite dimensional representation defines a functor of category $\mathcal{O}$ to itself, namely the functor of tensoring with it. Such a functor admits one and even many different lifts to a functor on the graded version of category $\mathcal{O}$. We explain why one may simultaneously choose for each finite dimensional representation a graded lift of this functor such that the lift for a direct sum is isomorphic to the sum of the lifts and the lift for a tensor product is isomorphic to the composition of the lifts for its factors. However it is not possible to choose these isomorphisms compatibly and I would like to understand the obstruction.

Universität Freiburg

# Khovanov cohomology as an offspring of representation theory and geometry

**Catharina Stroppel**

We show how Khovanov cohomology emerges naturally from representation theory of the Lie algebras $sl(n)$ and from categories of perverse sheaves on Grassmannians. Finally we relate it to the geometry of Springer fibres. Parabolic generalisations of the Bernstein-Gelfand-Gelfand category $O$ can be used to categorify the Jones polynomial and to define functorial invariants of tangles and cobordisms. We want to illustrate that Khovanov's invariants are nothing else than certain restrictions of these functorial invariants. The algebras which are used to define Khovanov homology become endomorphism algebras of certain representations for $sl(n)$. (This was conjectured by Braden and Khovanov). We give an explicit equivalences between the categories of perverse sheaves on Grassmannians and the parabolic versions of category $O$ which are of interest. Afterwards we extend Khovanov's description to an easy diagrammatic description of these categories. It turns out that the centres of these categories are isomorphic to the cohomology ring of the associated Springer fibres (as was independently proved by Jonathan Brundan). Finally we want to illustrate how the intersection theory of the Springer fibres controls the dimensions of homomorphism spaces between projective objects in our categories.

University of Glasgow

# Category $\mathcal{O}$ and $\mathfrak{sl}_k$ Link Invariants

**Joshua Sussan**

We construct a functorial invariant of oriented tangles on certain singular blocks of category $\mathcal{O}$. Parabolic subcategories of these blocks categorify tensor products of fundamental $\mathfrak{sl}_k-$ representations. Projective functors restricted to these categories give rise to a functorial action of the Lie algebra. On the derived category, Zuckerman functors categorify $\mathfrak{sl}_k-$ homomorphisms. Cones of natural transformations between the identity functor and Zuckerman functors are assigned to crossings and these assignments satisfy the appropriate tangle relations.

Yale University

# Constructible stacks

**David Treumann**

For a Whitney stratification $S$ of a topological space $X$, we introduce the notion of an $S$-constructible stack of categories on $X$. The motivating example is the stack of $S$-constructible perverse sheaves. We introduce a 2-category $EP_{\leq 2,S}(X)$, called the exit-path 2-category, which is a natural stratified version of the fundamental 2-groupoid. Our main result is that the 2-category of $S$-constructible stacks on $X$ is equivalent to the 2-category of 2-functors $Funct(EP_{\leq 2,S}(X), Cat)$.

Princeton University

# Khovanov homology of signed chord diagrams and vice orientations

**Oleg Viro**

Khovanov homology of classical links is generalized to signed chord diagrams. This gives, in particular, generalizations to virtual links, twisted virtual links, oriented links in the 3-dimensional projective space and oriented links in oriented thickened surfaces. It is inspired by recent Manturov's construction for virtual links, and differs from it mainly by emphasizing of conceptual topological ingredients such as orientations and vice orientations of manifolds and chord diagrams, local coefficient systems, fermionic configurations of points, etc.

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