

Objectivity, Objecthood and Genetic Axiomatic Methods in Modern Categorical Mathematics

In 1934 Hilbert and Bernays distinguished between their novel notion of formal axiomatic method, which later became standard, and a more traditional notion of axiomatic method that they describe as "constructive" or "genetic". While the traditional genetic method requires building of theoretical objects from given primitive objects according to certain construction rules (think of points and constructions by ruler and compass in the traditional geometry), the modern formal axiomatic method uses what Hilbert and Bernays describe as "existential form", i.e., an assumption that any given consistent theory has certain models which, generally, are simply posited rather than constructed. At the same time the formal method allows for constructive (genetic) procedures applied (not to theoretical objects proper but) to formulas expressing properties of (and relations between) the theoretical objects. Thus the formal method does not exclude the genetic method altogether but delimits its application to (informal) meta-mathematics.

In spite of the fact that Hilbert-style axiomatic setting still provides an "official" picture of how a well-founded mathematical theory should look like, its application in the mathematical practice remains very limited. I argue that certain important developments in the axiomatic thinking of 20th and 21st centuries including Lawvere's axiomatic Topos theory and Voevodsky's axiomatic Homotopy Theory (developed along with his project of building new Univalent Foundations of mathematics), can be adequately described as a revival of the traditional genetic approach in the new contexts. I show that these developments help us to pave the existing gap between foundations of mathematics and foundations of physics and thus give us some sensible strategies of applying modern mathematics in natural sciences. On the basis of my analysis of the new genetic methods in mathematics I suggest an original account of mathematical objectivity and of mathematical objecthood.