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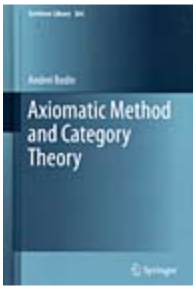
Axiomatic Method and Category Theory



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[Reviewed by Felipe Zaldivar, on 12/14/2013]

It is a debated issue whether mathematics should be thought and justified as a (language of) natural science or as independent science driven by its own dynamics, with important applications to other sciences. The author of the book under review takes as his goal to justify an extreme version of the former viewpoint. It is expressed in an epigraph that quotes the late V. I. Arnold's intellectually provocative statement that "mathematics is a part of physics […] where experiments are cheap". As the author remarks, a book containing a full argument for this thesis would have to contain as much physics as mathematics, so instead he limits his goals, focusing on an aspect of mathematical thought that seems very distant from the natural sciences: the *axiomatic method*.

The first part of the book gives a brief survey of the axiomatic method from Euclid's formalization of geometry and Hilbert's program to Lawvere's categorical logic. The occasional reader may well wonder what the underlying thread that runs from Euclid to Lawvere's closed Cartesian categories might be, but the book under review is not particularly clear about this. Save for some paragraphs, on pages 137 and 138, where in a rather sketchy manner Grothendieck's pioneering ideas are mentioned, there is no emphasis or even mention of the geometry underlying this whole set of ideas.

Most of the time I would not mind this, since after all this is a book devoted to the philosophy of mathematics with a particular focus on categorical logic. The author stated goals include, however, to discuss the intuitive nature of Euclid's axioms, contrasted with Hilbert's axioms in his *Foundations of Geometry* (translated into English, Open Court, 1950), and to offer some discussion of Bourbaki's structural use of the axiomatic method. In that context, it would have been more than appropriate to discuss the evolution of the notion of geometrical objects from Euclid to Grothendieck, for example of the notion of point, thus making clearer the notion of pointless topology, mentioned by the author in a footnote on page 137.

Nevertheless, the main focus of the book is on the categorical foundations of mathematics, and for this the last chapter of Part 1 introduces and discusses Lawvere's contributions, and in Part 2 continues the discussion of the categorification of mathematics up to Voevodsky's univalent foundations.

The final part of the book puts forward the author's main ideas on the relations and contrasts between intuitions and structures and outlines his ideas on an axiomatic method, in which intuition would be a bridge joining pure mathematics and natural science. To illustrate his ideas, the author introduces examples of the changing notion of intuition, starting with the usual overview of non-Euclidean geometries but adding a section on the evolution of ideas that led to the development of modern algebra at the turn of the 19^{th} century, namely Dedekind's notion of ideal. I don't buy the statement, in page 225, that Kummer called the numbers in the ring of integers of a cyclotomic extension of the rationals *ideal* numbers because he could not justify their *existence*. And, being a little nitpicky, the footnote 6 on page 226 describing the cyclotomic integers is plainly wrong. In this same vein, the explanation of the covariance or contravariance of functors given as commutative diagrams in page 254 is, to say the least, confusing. Perhaps I am being too demanding, but in a text where precision of ideas is being discussed, it would have been better that the reader should not have any misunderstandings due to small errors in the formulation of these ideas.

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