COMPUTATIONAL FRACTURE MECHANICS: TOWARDS MULTI-$*$ ANALYSIS

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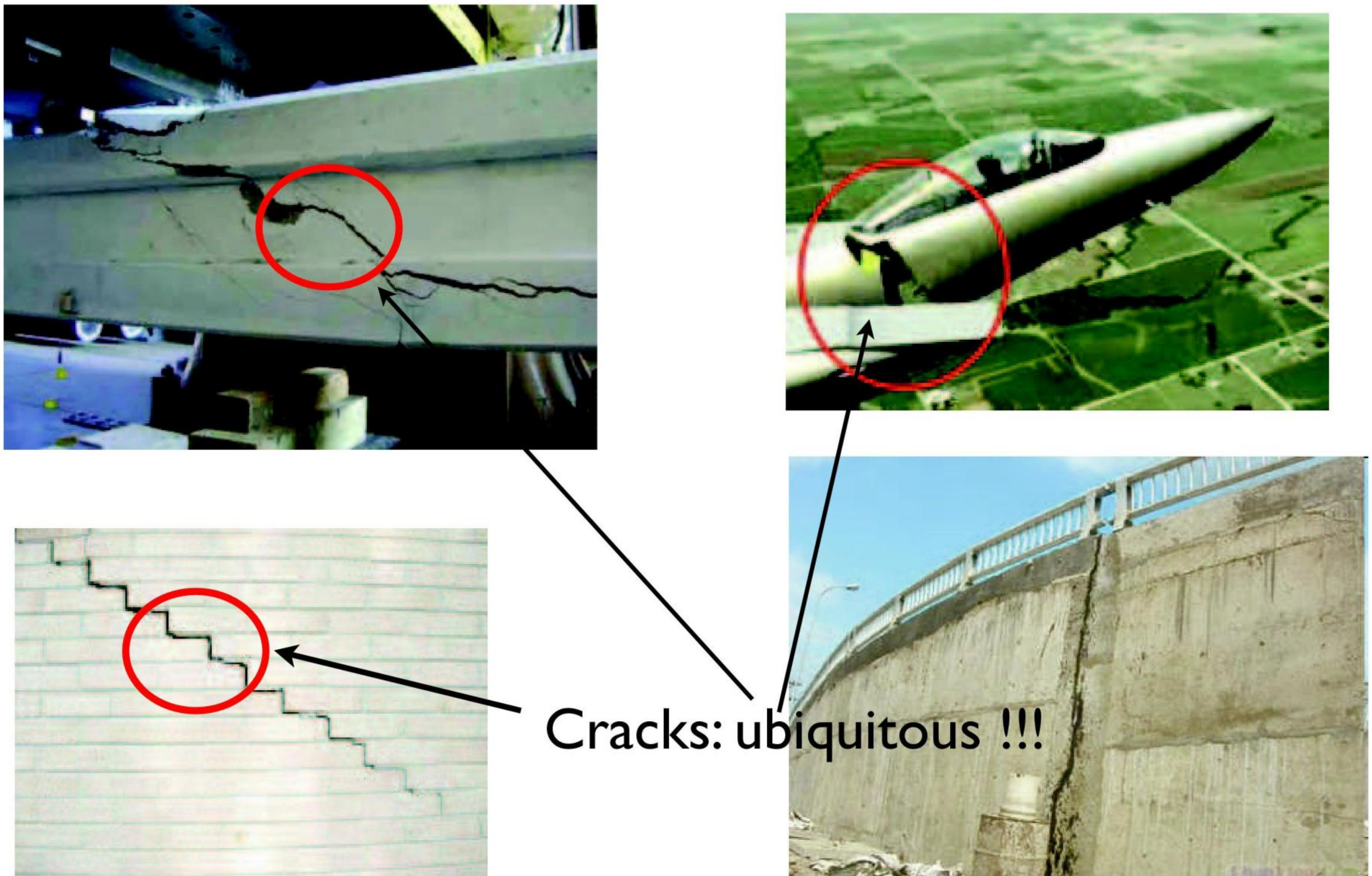
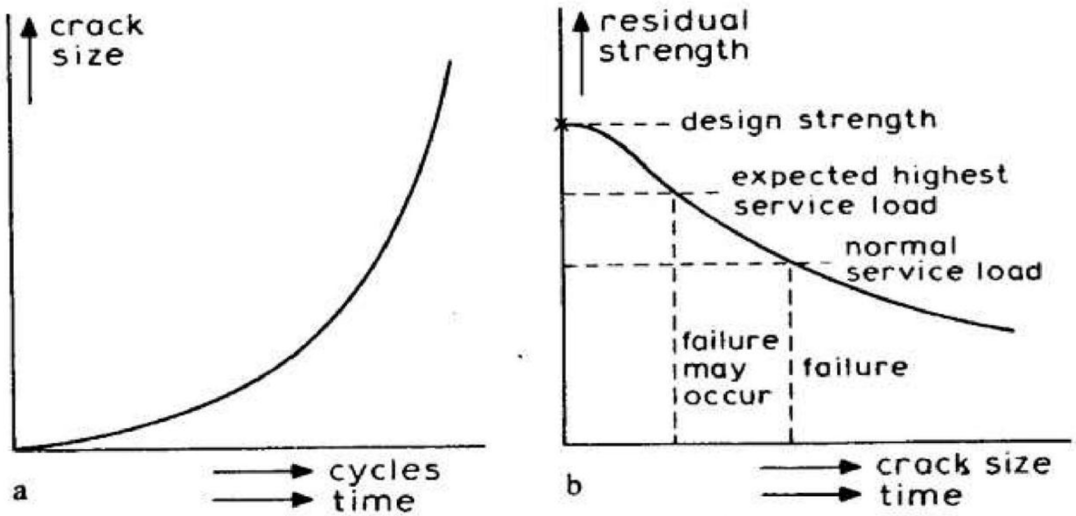
Problem Domain of Computational Fracture Mechanics (CFM)



[Phu N.V., 2012]

Basic Fracture Mechanics Problems



$$\sigma_{\mathrm{res}} = \frac{K_c}{f(a/W)\sqrt{\pi a}}$$

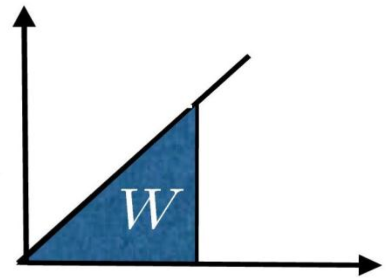
- What is the residual strength as a function of crack size?
- What is the critical crack size? $f(a_c/W)\sigma\sqrt{\pi a_c} = K_c$
- How long does it take for a crack to grow from a certain initial size to the critical size? $N = N_0 + \int_{a_0}^{a_c} \frac{da}{C(\Delta\sigma\sqrt{\pi a})^m}$

[Phu N.V., 2012]

Energetic Fracture Criteria

Consider a linear elastic bar of stiffness k, length L, area A, subjected to a force F, the work is

$$W = \int_0^u F du = \int_0^u k u du = \frac{1}{2} k u^2 = \frac{1}{2} F u$$

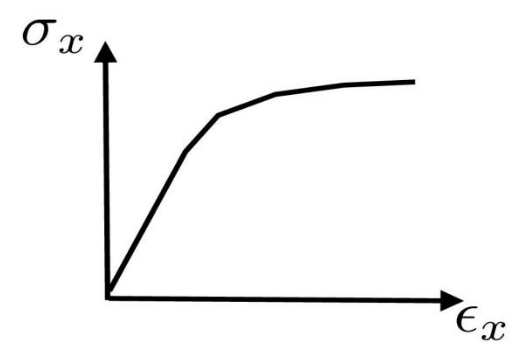


This work will be completely stored in the structure in the form of **strain energy**. Therefore, the external work and strain energy are equal to one another

$$\boxed{U = W = \frac{1}{2} F u}$$

In terms of stress/strain

$$\boxed{U = \frac{1}{2} F u = \frac{1}{2} \frac{F}{A} \frac{u}{L} A L}$$

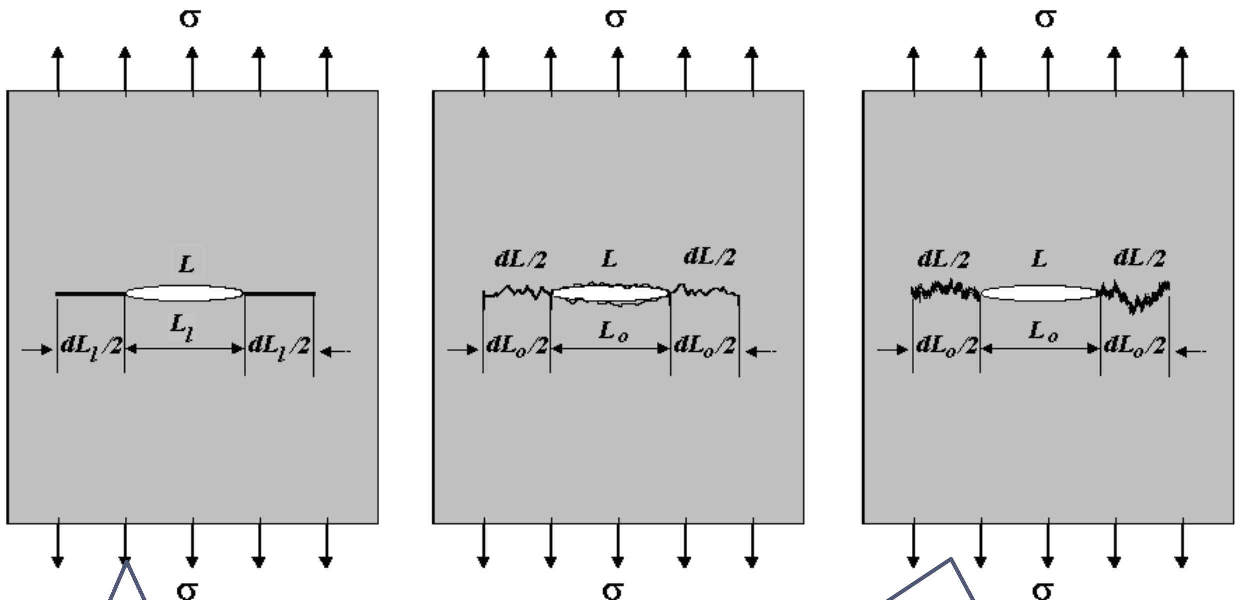


Strain energy density $[\mathrm{J/m^3}]$

$$u = \frac{1}{2} \sigma_x \epsilon_x \longrightarrow \boxed{u = \int \sigma_x d\epsilon_x}$$

[Phu N.V., 2012]

"Linear" vs Fractal Fracture Mechanics



$$G_0 = \frac{m^* \sigma_r^{\,2}}{E'} L_0$$

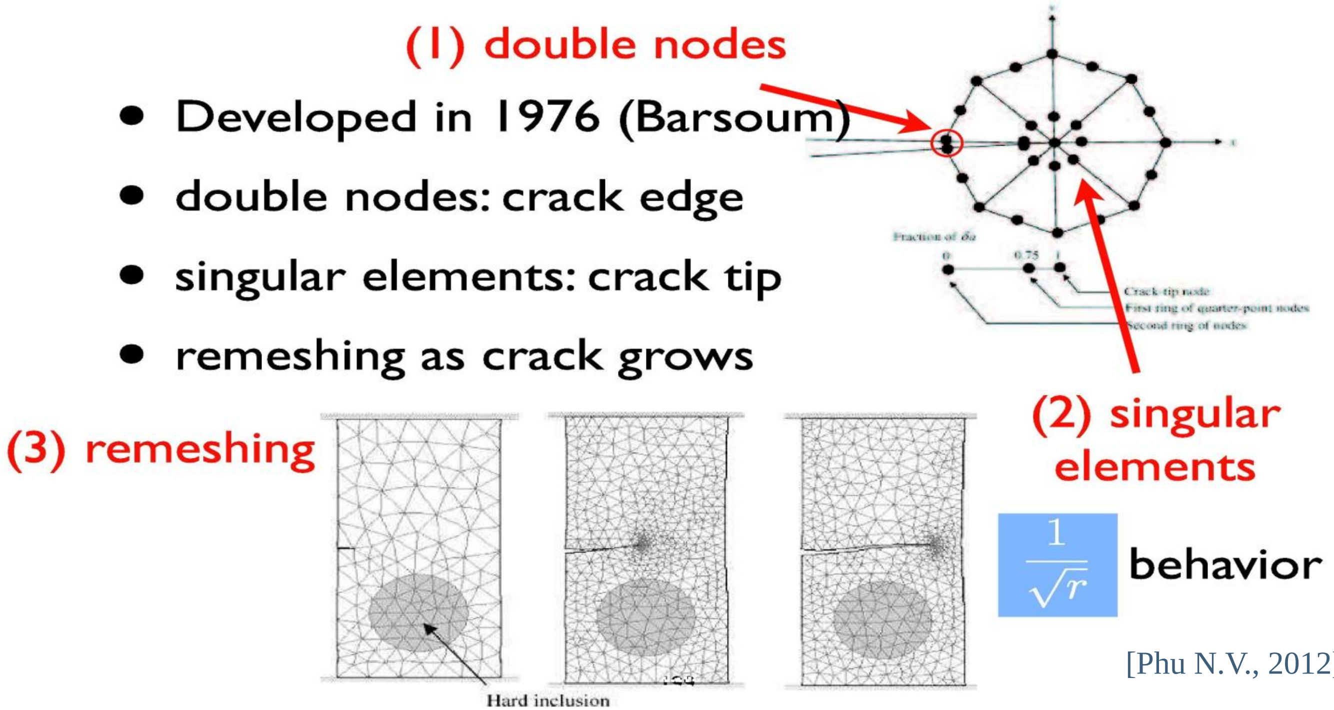
$$G_0 = \frac{m^* \sigma_r^{\,2}}{E'} L_0 \left[1 + (2-H)\left(\frac{H_0}{l_0}\right)^2 \left(\frac{l_0}{L_0}\right)^{2H-2} \right]$$

[Alves L.M., de Lacerda L.A., 2012]

Multi-$*$ CFM Problems

- **Multi**criteria
  - involving comparative analysis of different crack propagation criteria basing on different conceptual and physical models of fracture
- **Multi**scale
  - requiring coherent modeling of crack initiation and impact across all scales of the product
- **Multi**physical
  - examining wrecking effects jointly caused by mechanical forces, gravitation, heat, electromagnetic fields, and chemical reactions
- **Multi**material
  - being applied to products made from composite materials or highly heterogeneous structures produced by additive 3D-printing technologies

Finite-Element Method (FEM) to Solve CFM Problems



[Phu N.V., 2012]

- Issues associated with FEM
  - making a mesh with edges conforming to the crack geometry is time-consuming
  - different physical forces are best computed over different incompatible meshes
  - meshes usually don’t compose over an assembly structure, esp. if constructed by welding or the likes

Alternatives to FEM

- Strong Discontinuity Method
- Extended Finite Element Method
- R-adaptive methods, such as those based on Configurational Forces or Universal Meshes
- Meshfree methods, such as Scan&Solve™
- Methods based on Peridynamics
- Phase-field models in brittle fracture
- Discontinuous Galerkin and Polytopal Finite Element Methods
- Methods for Cohesive Fracture Models
- Methods based on Functional-Voxel geometrical models
- Movable Cellular Automatons Method

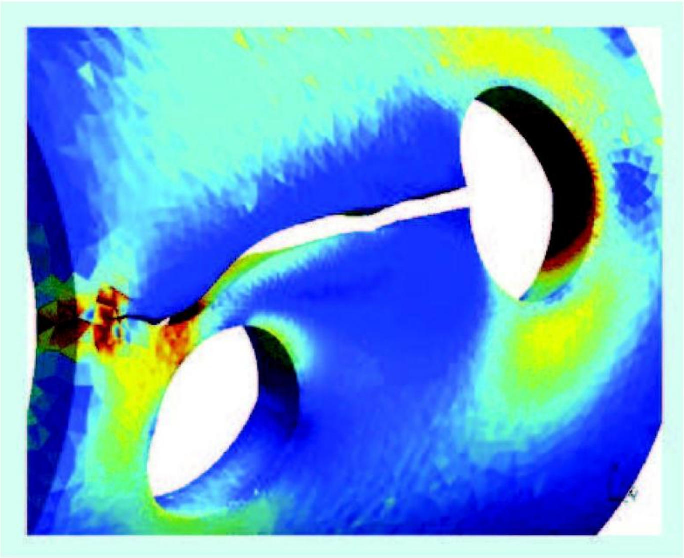
Extended FEM (XFEM)

Belytschko et al 1999

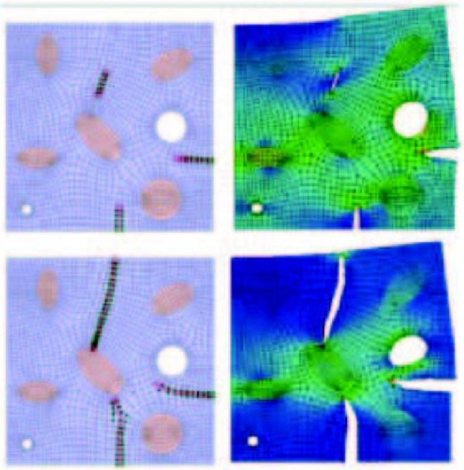
set of enriched nodes

$$\mathbf{u}^h(\mathbf{x}) = \boxed{\sum_{I \in \mathcal{S}} N_I(\mathbf{x})\mathbf{u}_I} + \boxed{\sum_{J \in \mathcal{S}^c} N_J(\mathbf{x})\Phi(\mathbf{x})\mathbf{a}_J}$$

standard part enrichment part

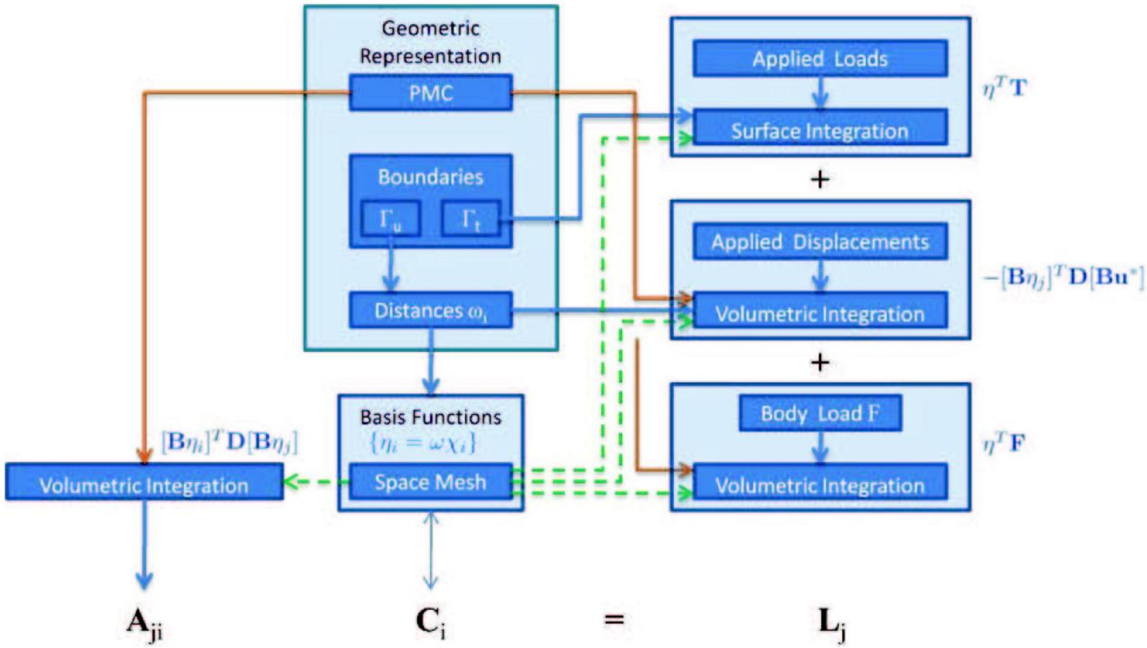


CENAERO, M. Duflot



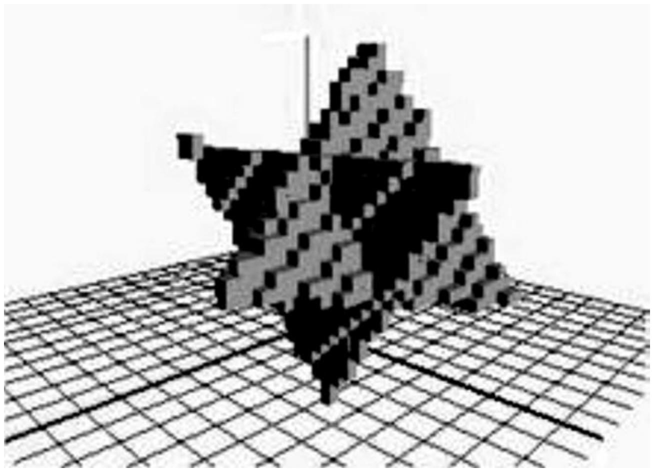
[Phu N.V., 2012]

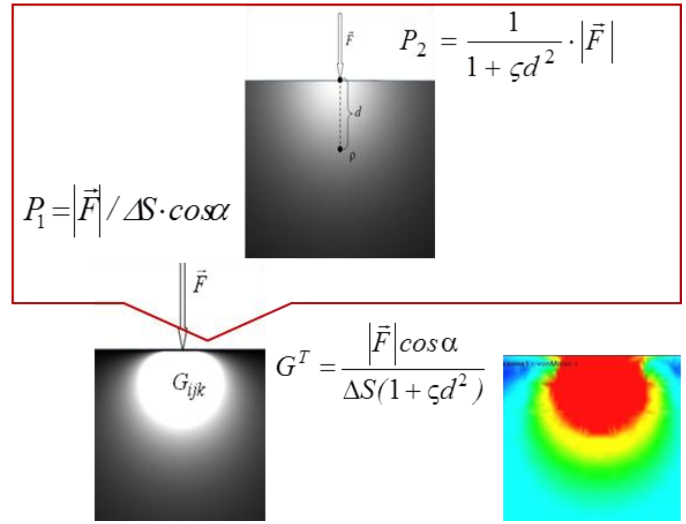
Meshfree Methods

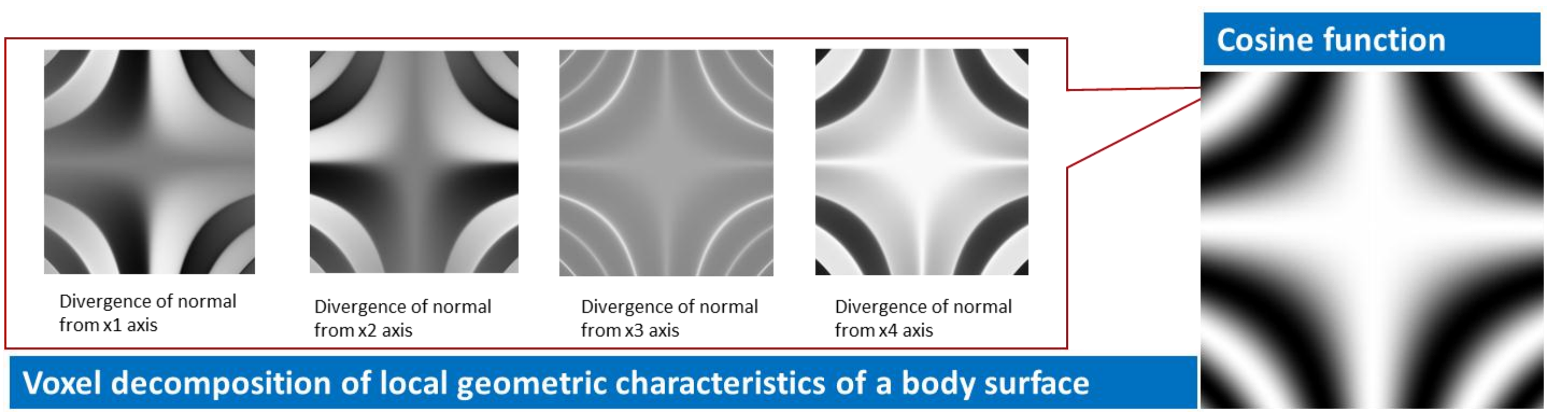


[Intact Solutions, 2009]

Functional-Voxel Approach

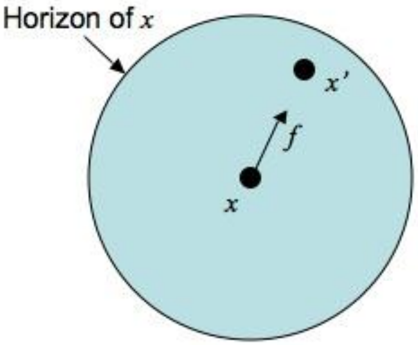


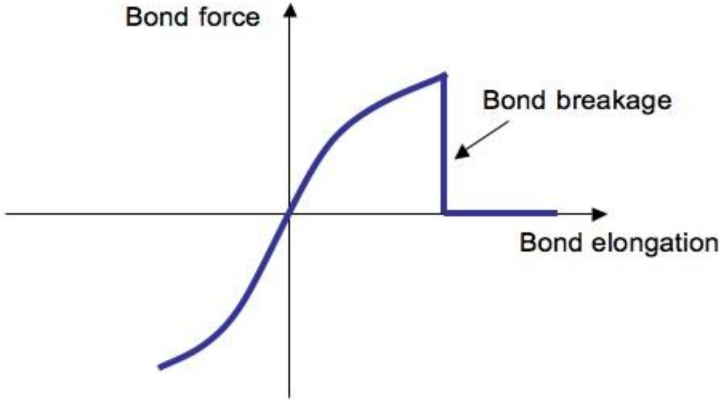


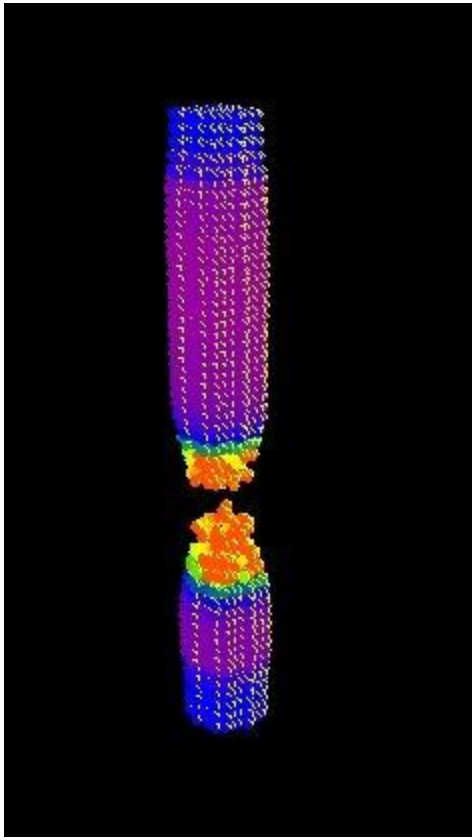


Peridynamics

$$\rho(x)\ddot{u}(x,t) = \int_R f(u(x',t) - u(x,t), x' - x, x)dV_{x'} + b(x,t)$$







Phase-field Models

$$L(\boldsymbol{u}, \dot{\boldsymbol{u}}, c) = \int_\Omega \left(\frac{1}{2}\rho \dot{\boldsymbol{u}} \cdot \dot{\boldsymbol{u}} - \frac{1}{2}(c^2 + \eta)\boldsymbol{\varepsilon}^T \boldsymbol{C} \boldsymbol{\varepsilon} \right) d\Omega - \int_\Omega \mathcal{G}_c \left(\frac{(1-c)^2}{4\epsilon} + \epsilon \boldsymbol{\nabla} c \right) d\Omega$$

The Euler-Lagrange equations of the problem can be used to arrive at the following strong form of the equations of motion

$$\boldsymbol{D}^T \boldsymbol{\sigma} + \boldsymbol{b} = \rho \ddot{\mathbf{u}}$$

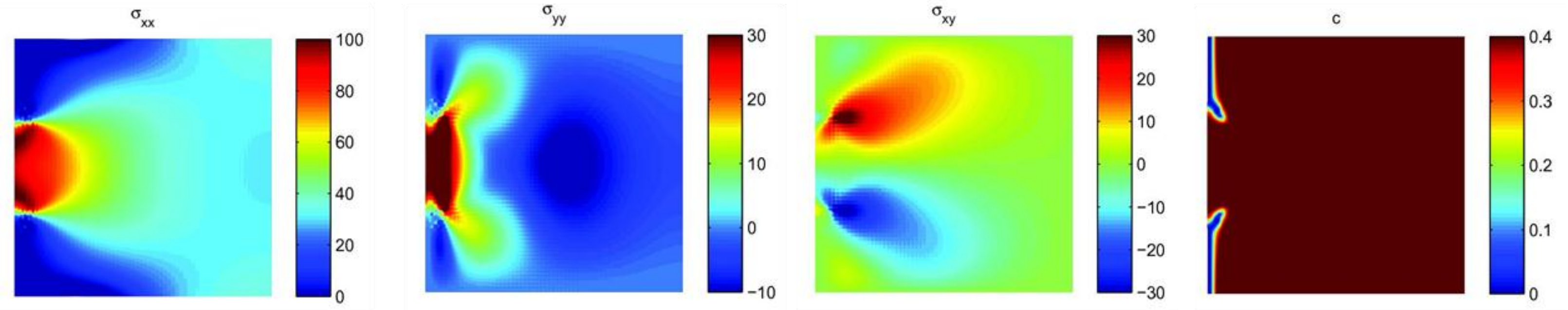
$$\dot{c} = -M \left(c \boldsymbol{\varepsilon}^T \boldsymbol{C} \boldsymbol{\varepsilon} - \mathcal{G}_c \left(2\epsilon \Delta c + \frac{1-c}{2\epsilon} \right) \right)$$

with

$$\boldsymbol{\sigma} = (c^2 + \eta) \boldsymbol{C} \boldsymbol{\varepsilon}$$

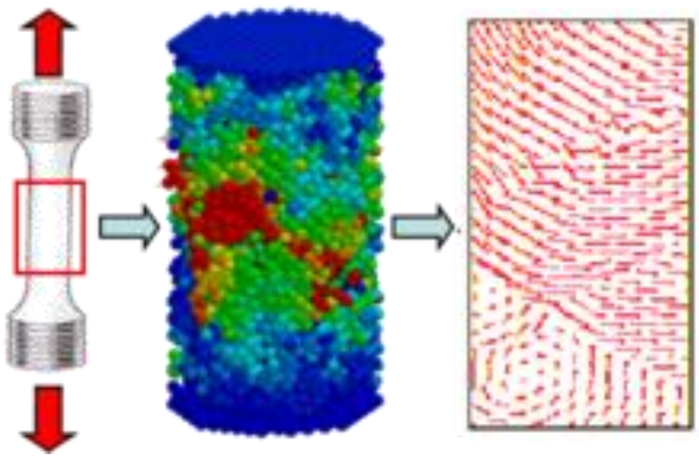
M is a mobility parameter and is assumed in this work to be constant and positive. As a result, the adopted phase-field evolution model represents the classical Ginzburg-Landau model.

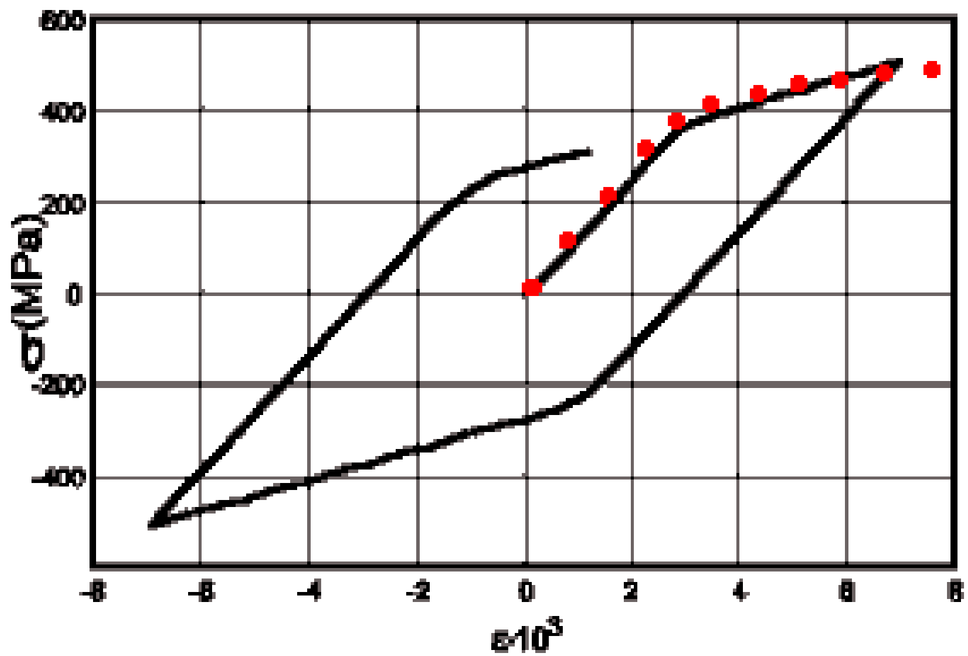
[Santos H., Silberschmidt V., 2014]



Movable Cellular Automatons Method

$$\frac{d^2 h^{ij}}{dt^2} = \left(\frac{1}{m^i} + \frac{1}{m^j}\right) p^{ij} + \sum_{k \neq j} C(ij, ik)\psi(\alpha_{ij,ik}) \frac{1}{m^i} p^{ik} + \sum_{l \neq i} C(ij, jl)\psi(\alpha_{ij,jl}) \frac{1}{m^j} p^{jl}$$





[Psakhie S.G. et al, 1995-2005]

Thank you for your attention