Foundations of physics and phenomenological reduction.

Pechenkin A.

My understanding of foundations of physics presupposes the idea of criticism. To reach the fundamental concepts and principles of the theory, the concept and principles representing reality, we need to take this theory under criticism, we need to follow the logical connections within the theory. We don't mean the criticism resulted from a competition of the rival scientific theories, for example, matrix and wave formulations of quantum mechanics. We mean the soft criticism which would improve the structure of the theory. However this criticism presupposes the fixation of stereotypes which beset our understanding of the foundations of the theory. Within a scope of the stereotype the current formulation of the theory looks like natural, productive, familiar… By fixing the stereotypes, we come to the problems of the structure of a theory (we see the logical breaks, the vague formulations) We see that we should restructure the theory to obtain true fundamental principles and concepts.

This criticism is close to what Edmund Husserl calls the **phenomenological reduction**. The phenomenological reduction means bracketing (or clothing in inverted commas) of current beliefs or notions. This means that we suspend these beliefs and notions to anyze and clarify them. We temporally postpone to apply these beliefs and notions to instead focus on analysis of them and on clarification of them.

Let us concentrate on the mathematical foundations of quantum mechanics. Could we treat the development of mathematical foundations of quantum mechanics as a unidirectional linear increase in rigor? Or there are alternative concepts of mathematical foundation?

To illustrate our version of scientific realism we provide a kind of the rational reconstruction of the history of mathematical foundations of quantum mechanics: from Dirack's "Principles" to Neumann-Birkhoff's logic of quantum mechanics.

As is known, quantum mechanics appeared in the form of two theories: the matrix and wave. Sometimes say that E.Schrödinger establish the equivalence of the two theories. However, as correctly noted N.R.Hanson, Shrödinger found only mutual translatability of these two formulations of quantum theory (2). Equivalence has been established by Dirac, who suspend discussing the dignities of matrix and wave schemes. This means that he fixed the stereotype of visibility and the stereotype of practical productivity.

True, the story is more complicated. Dirac originally supported the matrix mechanics. However, as early as 1925 he distinguished two levels of quantum language: he distinguished between the abstract q numbers (which are defined by their algebraic properties alone) and ordinary c numbers. In his 1930 classic he distinguished the symbolic method which is concerned with fundamental quantities of the theory (invariants and quasi-invariants of the transformations) and the method of representatives. For the sake of generality and logical consistency "I have chosen the

symbolic method, introducing the representatives later merely as an aid to practical calculations. This meant a complete break from the historical line of development".

J. von Neumann improved Dirac's presentation of quantum mechanics. He pointed to the tacit dependence of Dirac's theory from the ideology of matrix mechanics. In other words, he fixed the stereotype of the discreetness. Von Neumann emphasized that Dirac kept mathematical rigorous only for the theory of discrete spectrum. To describe the continuous spectrum, von Neumann writes, Dirac "insincerely" admitted improper functions – δ-function. For such functions there was no rigorous theory then.

Von Neumann consistently presented quantum mechanics as a theory of self-adjoined operators in Hilbert space. Von Neumann wrote "what does non belong to Hilbert space, does not exist for us".

In 1936 Birkhoff and von Neumann published an article aimed to describe the logical structures which are present in physical theories which, like quantum mechanics, don't satisfy classical logic. The basic idea of quantum logic is to replace the Boolean lattice appropriate to the phase space of classical physics by the projection lattice of Hilbert space. However, in the context of the present project it is important namely Birkhoff-von Neumann's version of quantum logic.

"To see why von Neumann insisted on the modularity of quantum logic, one has to understand that he wanted quantum logic to be not only propositional calculus of a quantum mechanical system, but also wanted it to serve as the event structure in the sense of probability theory. In other wards, what von Neumann aimed at was establishing the quantum analogy of the classical situ can be interpreted both as the Tarski-Lindenbaum algebra of a classical propositional logic and as the algebraic structure , where a Boolean algebra representing the random events of a classical probability theory, with probability being an additive normalized measure on the Boolean algebra" (3, P.157).

Birkhoff- von Neumann's quantum logic can be taken as a result of self-criticism of von Neumann's treatment of quantum probability in his preceding writings including "Mathematical foundations" (1932). Von Neumann proceeded from von Mises' empirical interpretation of probability (as relative frequency) in 1932. This interpretation has conceptual difficulties which led to the violation of mathematical rigidity proper to his "Mathematical foundations".

"Von Neumann does indeed abandon the frequency interpretation from 1936. But for von Neumann this option was out of question in the years 1927-1932. In his paper von Neumann speaks of the frequency interpretation of probability as the (i.e. unique) theory of probability. In the twentieth and early thirties his view of probability was clearly shaped under the influence of von Mises' relative frequency interpretation". (3, P.161).

In a ward, in 1936 von Neumann came to the second step of his struggle for foundations of quantum theory and by bracketing the achieved foundations took under consideration the stereotype of empiricism in the interpretation of probability.

Warning. So, we draw a straight line connecting the writings in mathematical foundations of quantum mechanics: Dirac, von Neumann, and again von Neumann

(together with Birkhoff). However, have we really a straight line? Have we really a continuous struggle for rigidity rather than criticism arising from the competition of scientific theories?

There are historical facts which evidence in favor of the latter proposition. Von Neumann's criticism of Dirack's foundations of quantum mechanics was above outlined. However, there is another criticism: in Bogoliubov-Logunov-Todorov's book on the foundations of quantum field theory there is another attack on Dirack's δ-function. In contrast to von Neumann, these three authors kept this mathematical structure in their book. However, they wrote (1, footnote on p.39). "In physics literature there are definitions like the following: "δ function equals to zero except the point x=0 and what means " . L.Schwartz. Théorie des distributions v.1-2 Paris , 1957 shows that this is anachronism which can be explained by the force of habit".

By drawing the straight line Dirack – von Neumann – von Neumann (together with Birkhoff) we have passed by some historical details. We did not emphasize that Dirack was ideologically connected with matrix mechanics, and von Neumann was connected with wave mechanics. To show this one needs to return to eigenvalues problem in both matrix mechanics and wave theory and follow the further interpretation of this problem by Dirack and correspondingly by von Neumann.

Literature

1. N.N.Bogoliubov, A.A.Logunov, I.T.Todorov. Foundations of the axiomatic approach to quantum field theory. Moscow.1969. P.39 (in Russian).
2. Hanson N.R. The concept of the positron. A philos. Analysis. Cambridge. 1963.
3. Redei M. Soft Axiomatization: John von Neumann on method and von Neumann's method in the physical sciences// Intuition and Axiomatic Method. Springer, 2006. P.