BOOK OF ABSTRACTS

New Mathematical Methods in Today's Physics: Logical, Epistemological and Computational Aspects

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Contents

1 Organizers:

Andrei Rodin (Institute of Philosophy RAS and Saint-Petersburg State University) and

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2 Aim and Scope:

Over the last few decades there were a number of important attempts to apply the new 20-th century mathematics in physics. Mathematics in its turn borrowed from physics many important ideas and motivations. During the planned Conference these developments has been discussed and scrutinized from various viewpoints including logical, epistemological and historical ones.

The Conference brings together physicists, mathematicians and philosophers working on mathematical foundations of physics. We aim, in particular, at developing a new theoretical and pragmatic perspective on the famous problem of *unreasonable effectiveness* of mathematics in the natural sciences stressed by E. Wigner back in 1960. This event explores in today's scientific context various conceptual ties between mathematics and logic, on the one hand, and physics and other natural sciences, on the other hand. The Conference has been designed as the concluding event of 3-year long research project "Epistemological Strategies of Application of Mathematics in Natural Sciences" funded by RFBR (grant N 13-06-00515), which allows its participants to share their results and exchange their ideas with a larger scientific community and establish a new horizon for their further researches.

3 Abstracts:

3.1 Jairo Jose da Silva (Uiversity Sao Paulo) Structuralism and the Applicability of Mathematics in Physics

We may not be able to read our destiny in the cards, but we can foretell future events by playing with mathematical symbols: mathematics has *predictive* powers vis-à-vis our experience of reality. There are good evidences that mathematics is also *heuristically* relevant in science; that is, that we can find out how the world works by means of mathematical manipulations, independently of *observing* how it works. For the most part, mathematics is created without much attention to how the world is; the world, on the other hand, is what it is independently of our mathematical creations. How, then, is it possible that mathematics has *anything* to say about the world, let alone disclosing its innermost secrets? Often, great mysteries are born out of great prejudices or *idées reçues* that go either unquestioned or unnoticed. This is a case in point. The belief, so ingrained in us so as to pass for established truth, that the empirical world exists "out there", in itself, *given* ready-made as an object of inquiry, and that mathematics, a creation of man, *just happens* to be our best instrument to investigate it must be called into question if the usefulness of mathematics as a tool to explore physical reality ceases to be a mystery or a gateway to the mystic. Since the usefulness of man-made mathematics in natural science is an unquestionable fact, one must consider with suspicion the belief that physical reality is something that we simply stumble upon ready-made. I propose an alternative view: nature, *as conceived by the empirical sciences*, is an *intentional construct*, a *mathematical* surrogate of perceptual reality devised for methodological purposes. This view, advanced most notably by the philosopher Edmund Husserl in his last published work *The Crisis of European Sciences and Transcendental Phenomenology*, but embraced by important physicists such as Hermann Weyl (who studied with Husserl), offers a natural, simple and historically well-founded naturalist solution for the problem of the applicability of mathematics in science (by "naturalist" I mean a solution that does not give man a privileged position in the natural scheme of things). Mathematics is applicable in science because the object of science is *not* reality as *perceived*, but reality as *conceived* - and only indirectly, via a relation of *approximation*, perceptual reality - , and our scientific *conception* of physical reality is mathematical through and through. From this perspective the mystery of the applicability of mathematics in science utterly vanishes, becoming nothing but an instance of the applicability of mathematics into mathematics itself, a much less momentous phenomenon (which however raises interesting logical questions).

3.2 Serge P. Kovalyov (Institute of Control Science RAS) Computational Fracture Mechanics: Towards a Multi-$\star$ Analysis

Methods of Computational Fracture Mechanics (CFM) are designed for numerical analysis and simulation of the propagation of cracks in solid products. CFM results are crucial in improving the mechanical performance of industrial products. Physical models for CFM are being steadily developed since the very end of XIX century, routinely based on energy balance analysis of crack propagation by means of simplified elasticity theory for homogeneous materials. However, contemporary approach to highly-automated digital industry challenges CFM problems to advance far beyond this paradigm, stipulating the following features:

- Multicriteria (involving comparative analysis of different crack propagation criteria basing on different conceptual and physical models of fracture);
- Multiscale (requiring coherent modeling of crack initiation and impact across all scales of the product ranging from each smallest mechanical part up to the large organizationaltechnical system that includes the product as a functional element);
- Multiphysical (examining wrecking effects jointly caused by mechanical forces, gravitation, heat, electromagnetic fields, and chemical reactions);
- Multimaterial (being applied to products made from composite materials or highly heterogeneous structures produced by additive 3D-printing technologies).

All these features are collectively addressed as "multi-$\star$ in the present report. Traditional Finite Element Method (FEM) used more than 40 years as a major computational physics device is known to fail to provide satisfactory simulation results for real-life multi-$\star$ CFM problems even when the most powerful supercomputers in the world are utilized. A number of alternative methods are being intensely developed recently, such as:

- Strong Discontinuity Method;

- Extended Finite Element Method;
- R-adaptive methods, such as those based on Configurational Forces or Universal Meshes;
- Meshfree methods, such as $Scan\&Solve^{TM}$, or methods based on Peridynamics;
- Phase-field models in brittle fracture;
- Discontinuous Galerkin and Polytopal Finite Element Methods;
- Methods for Cohesive Fracture Models;
- Methods based on Functional-Voxel geometrical models;
- Moving Cellular Automatons Method;
- Fractal Fracture Mechanics methods.

A number of these CFM methods are surveyed in the report.

3.3 Marc Lachièze-Rey (University Paris-Diderot) Which mathematics for quantum gravity ?

Since (at least) Plato, the world is described with mathematics and their "unreasonable effectiveness" appeared more and more clearly in the recent developments of physics. One of the challenge of present physics is to build a theory of quantum gravity. This search is very active but we do not know in which extent the mathematical tools used for gravity (mainly Riemanian Geometry), dynamics (symplectic geometry) and quantum physics (algebra, group theory) may remain relevant for this task. On the other hand, mathematics have exhibited correspondences, dualities and various links between these distinct branches, and physicists have proposed different reformulations of a given theory in different frameworks.

I review some of the various mathematical tools involved in these formulations of gravity, quantum physics and Loop quantum gravity. I insist on their relations and correspondences, and present the possible benefits that new mathematical approaches (in particular category theory) can bring.

3.4 Alexander D. Panov (Skobeltsyn Institute of Nuclear Physics) Bell's Theorem, Computability of Quantum Theory, and the Relativity of Local Realism

Using an explicit counterexample we show, in an apparent contradiction with the well-known theorems about the impossibility of hidden variables in quantum mechanics, that the Bell's inequalities can be violated in a system, which satisfies all the requirements of local realism. This local-realistic system is a classical computer that simulates quantum evolution of an Einstein-Podolsky-Rosen entangled pair. The possibility of exact simulation of quantum systems by classical machines follows from algorithmic computability of quantum theory. It is shown that the inaccuracy of the usual proof of theorems on the impossibility of hidden variables is a simplified interpretation of the concept of the local realism. The actual reality may incorporate many different "layers" with different kinds of realisms, but not just one as implicitly supposed in the theorems.

3.5 Alexander Pechenkin (Moscow State University) Foundations of physics and phenomenological reduction

My understanding of foundations of physics presupposes the idea of criticism. To reach the fundamental concepts and principles of the theory, i.e., the concepts and the theoretical principles representing reality, we need to put this theory under a criticism, i.e., follow the logical connections within the theory. We don't mean here a criticism resulted from a competition of the rival scientific theories, for example, matrix and wave formulations of quantum mechanics. We rather mean a soft criticism which improves the structure of the given theory.

Such a criticism is close to what Edmund Husserl calls the *phenomenological reduction*. The phenomenological reduction means bracketing (or clothing in inverted commas) of current beliefs or notions. This means that we suspend these beliefs and notions in order to analyze and clarify them.

To illustrate our version of scientific realism we provide a rational reconstruction of the history of mathematical foundations of quantum mechanics: from Dirack's *Principles* to Neumann-Birkhoff's logic of quantum mechanics.

3.6 Andrei Rodin (Institute of Philosophy RAS) Constructive Axiomatic Method and Modern Physics

In 1900 David Hilbert announced his famous list of then-opened mathematical problems; the problem number 6 in this list is the axiomatization of physical theories. Since then a lot of systematic efforts has ben invested into this project. However the results of these century-long efforts (including Hilberts own work) turned to be less successful than the proponents of using the axiomatic method in hoped for. The existing axiomatizations of physical theories arguably provide a valuable logical analysis but they do not constitute anything like a standard presentation of these theories, which can be used for transmission, evaluation and justification of physical knowledge.

This state of the art in the axiomatization of physics is a strong evidence that the standard notion of axiomatic theory stemming from Hilbert and Tarski is not quite appropriate for the task. However in the recent years in mathematics there emerged a new axiomatic approach best represented by the Homotopy Type theory. I shall argue that this new axiomatic approach, which I shall call *constructive* (see arXiv: 1408.3591), better fits the needs of modern physics, and review attempts to use it for axiomatizing Quantum Field theory by Urs Schreiber and co-authors (arXiv: 1109.0955, 1408.0054).

3.7 Irina Starikova (University of São Paulo) Models in Applied Mathematics: an Example from the Study of Turbulence

What are the motivations for choosing one mathematical means than another when solving physical problems? Why some mathematical approaches are more efficient, explanatory and powerful than the other? A case study from turbulence demonstrates that imaginary *modeling* of physical phenomena can play an important epistemic role in choosing and developing the mathematical means. In particular, it demonstrates that Richardson's model of a cascading wave motivated both Kolmogorov's statistical theory of turbulence and more recent applications of Riemanian geometry methods (Ricci flows) in the mathematical description of turbulence.

3.8 Vladislav Terekhovich (independent researcher) Explanatory Potential of Mathematics in Quantum Physics

One of amazing properties of the equations of motion is that real systems follow them with unexplained persistence. This is the part of an overall problem of "unreasonable effectiveness of mathematics". In the classical physics equations of motion usually correspond to a physical theory, which offers models and interpretations connecting these equations with reality. However the quantum physics applies the notions of possible, alternative, virtual, or imaginary trajectories and histories that are not related to the physical reality. My report is devoted to the explanatory potential of such notions in some interpretations of quantum mechanics and in the Feynman path integral formalism.

On the one hand, it is widely accepted that possible trajectories and histories are merely formal mathematical tools used for calculation. On the other hand, physicists often unwittingly borrow these notions from mathematics and metaphysics without a proper criticism. Modern authors take very different attitudes to the reality of possible trajectories and histories in the mathematical formalisms of quantum physics. Recently, there has been a growing interest in a realistic interpretation of the Feynman paths and alternative quantum histories in addressing the problem of quantum reality. There are several interesting attempts to combine possible histories in quantum physics and metaphysics. However, scholars have not yet adequately addressed two central questions: Are the possible histories real? What is common and different between the possible histories in mathematics, classical physics, quantum physics, and metaphysics?