Model-Based Knowledge and Its Axiomatic Representation

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IP RAS, 25 апреля 2016 г.

Received view

A theory is a set of propositions. It can be possibly (but not generally) represented through a list of *axioms* and described as the deductive closure of these axioms.

Non-statement view

A theory is a class of models but not an axiom system, nor its deductive closure. (Suppes, Sneed, Stegmüller, Balzer, Moulines, van Fraassen)

Suppes 2002

Term “model” is used in logic (after Tarski) and science similarly. (A very doubtful thesis!)

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Tarski-style set-theoretic models. Alleged advantages of such models:

- generic character (after Tarski and the mainstream Model theory)
- abstract character: it allows for modeling physical and other natural phenomena, which cannot be directly perceived or even imagined (ex. Quantum Physics)

Outcome

Appears to be useful for a logical and "structural" analysis but certainly not for general research and educational purposes. Perspectives of computer realization are unclear.

Model-based reasoning in AI

Ex. (Russell&Norvig 2010):

Stroke(patient) $\rightarrow$ Confused(patient) $\wedge$ UnequalPupils(patient)

rather than

Confused(patient) $\wedge$ UnequalPupils(patient) $\rightarrow$ Stroke(patient)

(causal rather than non-causal rules; Cf. Aristotle *An. Post*)

In spite of the fact that digital modeling techniques are highly developed and highly successful (e.g. modeling climate etc.) making such techniques a part of computer-based Knowledge Representation systems remains a widely open problem.

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Example

Flow Dynamics and Navier-Stokes Equations:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial[\rho u_i u_j]}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + \rho f_i \tag{2}$$

$$\frac{\partial(\rho e)}{\partial t} + (\rho e + p)\frac{\partial u_i}{\partial x_i} = \frac{\partial(\tau_{ij} u_j)}{\partial x_i} + \rho f_i u_i + \frac{\partial(\dot{q}_i)}{\partial x_i} + r \tag{3}$$

Example (continued)

Question: do Navier-Stokes equations belong to the theory (= its syntax) or rather to its model?

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- systems of thought-things (Gedankendinge)

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- use of Set-theory or a weaker theory of classes for building structures

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- The intensional version of MLTT contains propositional types but can not be fully interpreted as a system of propositions. (Restricted Curry-Howard: Propositions as *Some* Types). Hence nothing like the usual satisfaction relation $\models$ based on T-Convention may provide it with a full model.

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- This fact has strong epistemological consequences, which I stress elsewhere (Constructive Axiomatic Method); presentation of Knowledge-How as a basis for Knowledge-That.

Functorial Semantics

Model m of theory T is a functor $m : T \longrightarrow Set$ from the given theory (presented as a syntactic category) to the category of sets, which preserves finite products (Lawvere).

A more general notion of functorial model can extends to functors into categories other than Set.

Tarski-style models of 1-st order theories can be always described as functorial models but not, generally, the other way round.

Theories as Generic Models

Functorial models of a given theory, generally, form a category of the form $M = [T, C]$. If $C = Set$ then there exist *Yoneda embedding* $T \hookrightarrow M$, under which T becomes initial in M.

Voevodsky Conjecture (open): MLTT is the initial object in a certain category of its models.

(The corresponding property of *Claculus of Construction*, which is weaker than MLTT, has been proved by Thomas Streicher back in 1991!)

The technical notion(s) of functorial model still wait(s) to be understood from an epistemological viewpoint and introduced into a wider logical and philosophical discussion.

Model Theory for MLTT/HoTT

In which sense, if any, HoTT qualifies as of *model* of MLTT?

MLTT is a syntactic (rather than specifically logical) skeleton of HoTT. HoTT is its geometrical interpretation. To make such an interpretation formal one needs to use a special presentation with simplicial sets, cubical sets or the like. The "pure" HoTT (without Univalence) is MLTT informally recasted into a homotopical language.

Geometrical Intuition Strikes Back?

Shall we reassess the early 20th century rejection of the idea of standard intuitive support?

It appears promising to develop today this idea with a support of Cognitive Science and practical attempts towards an effective computer-based Knowledge Representation including methods of Computer Vision.

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- A Model theory of MLTT-HoTT is a good place for looking for it;
- The notion of theory as a generic model, which belongs to the functorial semantics, applies to MLTT and at the same time apparently better squares with the colloquial theory-model relation in science...

THE END