Extra-logical proof-theoretic semantics in HoTT

I. Kant famously argued that elementary geometrical statements such as Euclid's Triangle Angle Sum theorem cannot be deduced from the first principles by purely logical means because their proofs require extra-logical geometrical constructions. The discovery of non-Euclidean geometries in the 19-th century made Kant's analysis of geometrical reasoning untenable in its original form, and throughout the following 20th century it was generally viewed as fundamentally mistaken or at least wholly outdated. However the recently emerged Homotopy Type theory (HoTT) and the related program of building new "univalent" foundations of mathematics provide a formal and conceptual basis for revising, once again, the epistemic role and logical function of extra-logical constructions in mathematical (and other) proofs.

The key feature of HoTT, which modifies the intended logical semantics of Martin-Löf Type theory (MLTT), is the homotopical cumulative hierarchy of types, which distinguishes between types of different homotopy levels. This hierarchy suggests a simple (albeit not uncontroversial) criterion of logicality according to which only types with at most one term qualify as propositions, and only applications of MLTT rules to propositions and their terms (proofs), that is, to judgements in the traditional sense of the term, qualify as logical inferences stricto sensu. According to the same criterion, applications of the same schematic rules to types and terms of higher homotopy levels are extra-logical constructions. Such extra-logical applications of deductive rules also have a proof-theoretic impact because the obtained non-propositional constructions serve as witnesses for associated propositions (formally obtained via the propositional truncation of higher types) in a manner similar to which elementary geometrical constructions support the traditional geometrical proofs. For this reason it is justified, in our view, to qualify the standard semantic of HoTT just outlined as proof-theoretic. Using HoTT as a motivating example I would like to discuss further the role of logical and extra-logical elements in formal proofs.