Directed Homotopy Type Theory and the (In)vertibility of Mathematics.

Directed Homotopy Type theory (DHTT) is a generalization of Homotopy Type theory (HoTT) where fundamental groupoids of spaces are replaced by more general (higher) categories. Along with type formers for identity types which admit the standard HoTT interpretation in terms of invertible paths and their homotopies, DHTT comprises type formers for non-invertable homomorphisms of all levels which admit an interpretation in terms of non-invertable paths in appropriate spaces. The choice between DHTT and HoTT as foundational formal frameworks for building mathematical theories has an epistemological dimension, which concerns the epistemic significance of the invertibility condition. While HoTT and the related notion of Univalent Foundations support Mathematical Structuralism DHTT supports a more dynamic conception of Mathematics, which I shall outline in my talk.

Related papers:

Paige North, Towards a Directed Homotopy Type Theory, arXiv:1807.10566

Andrei Rodin, Categories Without Structures, arXiv: 0907.5143 (published in Philosophia Mathematica 19/1 (2011), p. 20-46)

Michael Warren, Directed Type Theory (video of talk in IAS Princeton) https://video.ias.edu/univalent/1213/0410-MichaelWarren