Formal Proof-Verification and Mathematical Intuition: the Case of Univalent Foundations

The idea of formal proof verification in mathematics and elsewhere, which dates back to Leibniz, made important advances since the beginning of the last century when David Hilbert launched the research program called today after his name. In the beginning of the present century an original contribution into this area of research has been made by Vladimir Voevodsky (b. 1966 Moscow - 2017 Princeton) who proposed a novel foundations of mathematics that he called Univalent Foundations (UF). UF involves an interpretation of Constructive Type theory with dependent types due to Per Martin-Lof by means of Homotopy theory (called Homotopy Type theory or HoTT for short) and is designed to support an automated, viz. computer-assisted, verification of non-trivial mathematical proofs [1].

The present paper analyses the epistemic role of mathematical intuition in UF. The received view on the role of intuition in formalized mathematical theories stems from Hilbert. Hilbert stresses the importance of human capacity to distinguish between different symbol types, identify different tokens of the same symbol type and manipulate with symbols in various ways. He qualifies this cognitive capacity as a form of intuition and claims that it plays a major epistemic role in formal logic and formalized mathematical theories. All other forms of mathematical intuition, in Hilbert's mature view, have significant heuristic and pedagogical values but play no role in the formal representation and justification of ready-made mathematical theories (see [2], section 3.4 and further references therein).

Unlike Hilbert Voevodsky didn't write a philosophical prose but he expressed his vision of mathematics and explained motivations of his projects in many public talks, which are now available in record via his memorial web page maintained by Daniel Grayson in Princeton Institute of Advanced Studies [3]. In a number of these talks Voevodsky stresses the importance of preserving in the framework of formalized computer-assisted mathematics an "intimate connection between mathematics and the world of human intuition" . Using a simple example of classical theorem in the Algebraic Topology formalized in the UF setting [4], I show how a spatial intuition related to Homotopy theory serves here as an effective interface between the human mind and the computer, and argue that it plays in UF not only heuristic but also a justificatory role.

Bibliography:

1. D.R. Grayson, An Introduction of Univalent Foundations for Mathematicians, arXiv: 1711.01477

2. A. Rodin, Axiomatic Method and Category Theory (Synthese Library vol. 364), Springer, 2014

3. http://www.math.ias.edu/Voevodsky/ (Voevodsky’s memorial webpage)

4. M. Shulman and D. Licata, Calculating the fundamental group of the circle in homotopy type theory. arXiv:1301.3443 (The accompanying Agda code is available from Dan Licata's web site at http://dlicata.web.wesleyan.edu)