# Knowledge Representation with HoTT

Andrei Rodin (IPRAS/HSE), jww Serge Kovalyov (ICSRAS)

Trends in Logic 2019, Moscow, October 2-4, 2019

# JTB

Knowledge is Justified True Belief:

Remark: JTB identifies knowledge with knowledge of certain proposition or propositions. This type of knowledge is conventionally referred to as propositional knowledge aka knowledge-that.

# Gettier 1963

$$\frac{16}{64} = \frac{1\cancel{6}}{\cancel{6}4} = \frac{1}{4}$$

$$\frac{26}{65} = \frac{2\cancel{6}}{\cancel{6}5} = \frac{2}{5}$$

No satisfactory formal account of justification was available until recently...

Justification Logic (Artemov&Fitting 2019), Proof-Theoretic Semantics (since 1990-ies), MLTT/HoTT

The constructive approach to knowledge gives justification *even more* importance. Thus justification is a key element of knowledge both under the JTB and the constructive conceptions of knowledge.

# KR conceptions of knowledge

Ex.: Knowledge is the "whole body of data and information that people bring to bear to practical use in action" (after Jukus et al. 2013)

# KR conceptions of knowledge

Ex.: Knowledge is the "whole body of data and information that people bring to bear to practical use in action" (after Jukus et al. 2013)

No mention of justification in the studied CS literature!

# Ontology vs. Epistemology

Since late 1980-ies the concept of ontology has been widely used in KR as a formal semantic tool.

# Ontology vs. Epistemology

Since late 1980-ies the concept of ontology has been widely used in KR as a formal semantic tool.

In philosophy *ontology* is a discipline that accounts for being/existence.

# Ontology vs. Epistemology

Since late 1980-ies the concept of ontology has been widely used in KR as a formal semantic tool.

In philosophy *ontology* is a discipline that accounts for being/existence.

*Epistemology* accounts for knowledge.

# Ontology vs. Epistemology

Since late 1980-ies the concept of ontology has been widely used in KR as a formal semantic tool.

In philosophy *ontology* is a discipline that accounts for being/existence.

*Epistemology* accounts for knowledge.

Puzzle: Why KR makes use of (formal) ontology but not of (formal) epistemology?

# Neglect of Epistemic Considerations

Semantic of Logical Inference: Model-theoretic (Tarski's logical consequence relation) vs. Proof-theoretic semantis

# Justification as a practical issue

A regular user of KR system has no means to verify/justify an information obtained via the system provided by this very system. The existing verification technologies are not designed to be available to regular users.

# Desiderata for KR:

# Desiderata for KR:

- Support justification/verification in form of routine procedure available to all users;

# Desiderata for KR:

- Support justification/verification in form of routine procedure available to all users;
- Support justification/verification in form, which is specific w.r.t. obtained information: specific evidences rather than general assurances.

# A Solution:

Use epistemically-laden alternative conceptions of logic along with their formal implementations for developing theoretical prototypes of KR systems.

# Why HoTT? (1)

HoTT admits the constructive epistemically-laden proof-theoretic semantics intended by Martin-Löf's Type for MLTT (in a slightly modified form).

# Why HoTT? (2)

The cumulative *h*-hierarchy of types made explicit via the homotopical interpretation supports the distinction between propositional, set-level and higher-level types.

This distinctive feature of HoTT supports formal constructive representation of objects (of various levels) and propositions "about" these objects within the same framework. Each such object serves as a witness/truthmaker for proposition obtained via the propositional truncation of type where the given object belongs.

# Higher Identity Types in MLTT

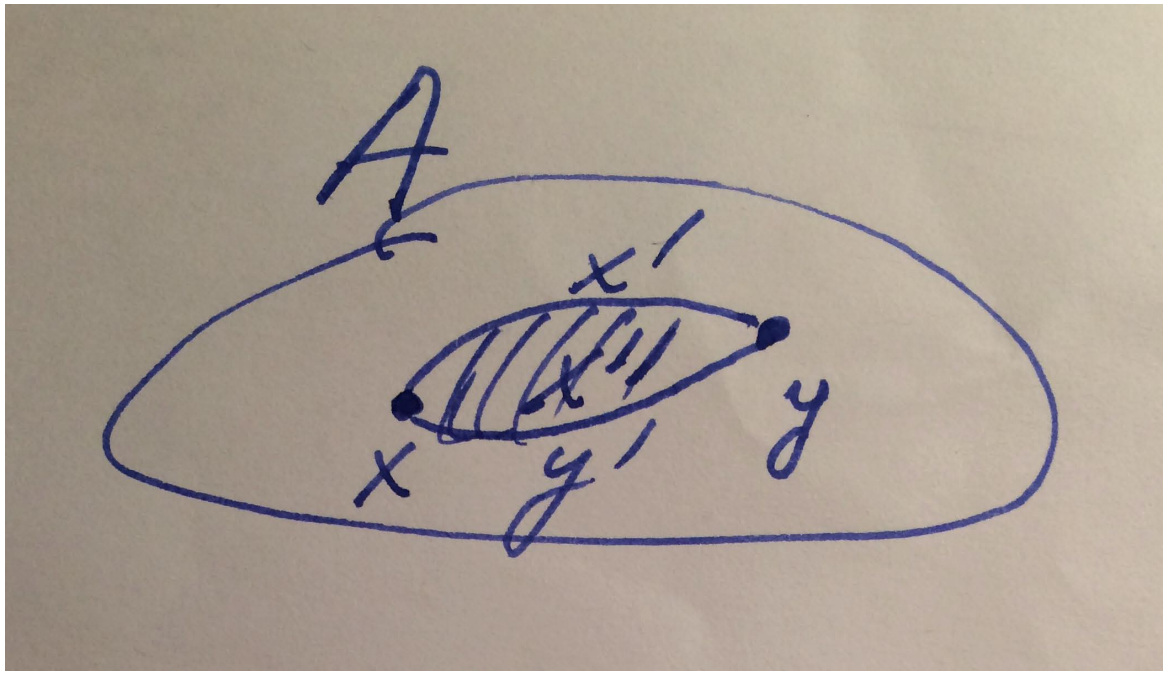
- $x', y' : x =_A y$
- $x'', y'' : x' =_{x=_A y} y'$
- $\ldots$

# Homotopical interpretation of Intensional MLTT

- $x, y : A$
  $x, y$ are points in space $A$
- $x', y' : x =_A y$
  $x', y'$ are paths between points $x, y$; $x =_A y$ is the space of all such paths
- $x'', y'' : x' =_{x=_A y} y'$
  $x'', y''$ are homotopies between paths $x', y'$; $x' =_{x=_A y} y'$ is the space of all such homotopies
- ...

# Homotopical intuition

helps to identify a higher-level syntactic structure.



# Homotopy Levels

## Definition

*We say that* $S$ *is a space of h-level* $n+1$ *if for all its points* $x, y$ *path spaces* $x =_S y$ *are of h-level* $n$.

# Cummulative Hierarchy of Homotopy Types

- -2-type: single point *pt*;
- -1-type: the empty space $\emptyset$ and the point *pt*: truth-values aka (mere) propositions
- 0-type: sets: points in space with no (non-trivial) paths
- 1-type: flat groupoids: points and paths in space with no (non-trivial) homotopies
- 2-type: 2-groupoids: points and paths and homotopies of paths in space with no (non-trivial) 2-homotopies
- ...

# Why HoTT? (3)

HoTT comprises a system of formal rules, which are interpreted as logical rules at the propositional *h*-level and as rules for object-construction at all higher levels.

This feature of HoTT supports representation various extra-logical procedures (such as material technological procedures, i.e., *knowledge-how*) keeping track of the corresponding logical procedures at the propositional level of representation.

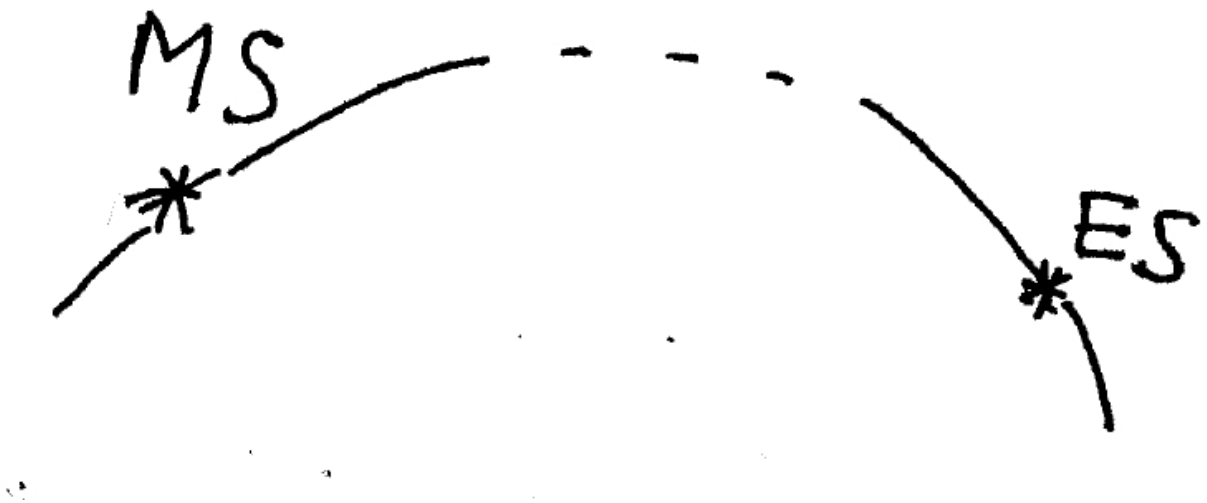
# Why HoTT? (4)

HoTT/MLTT is computationally implementable. Fragments of HoTT/MLTT have been implemented in proof-assistant Coq, program languages AGDA, LEAN and some other products.

# Why HoTT? (5)

HoTT-constructions admit intuitive spatial (homotopical) interpretations that may be used for facilitating human-computer interactions.

# The Morning Star is The Evening Star

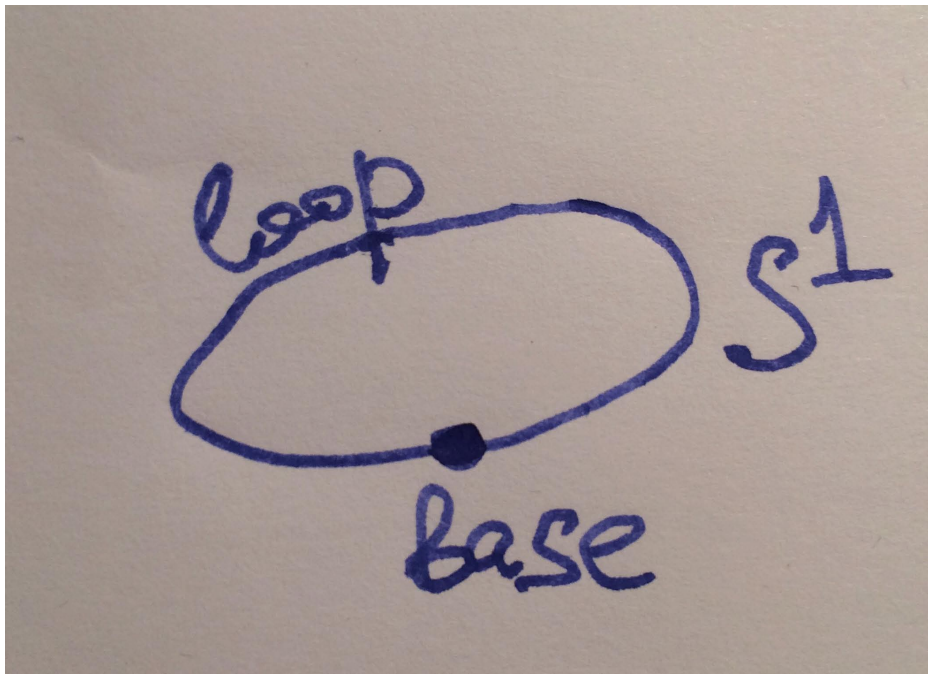



*Venus Homotopically* http://philsci-archive.pitt.edu/12116/

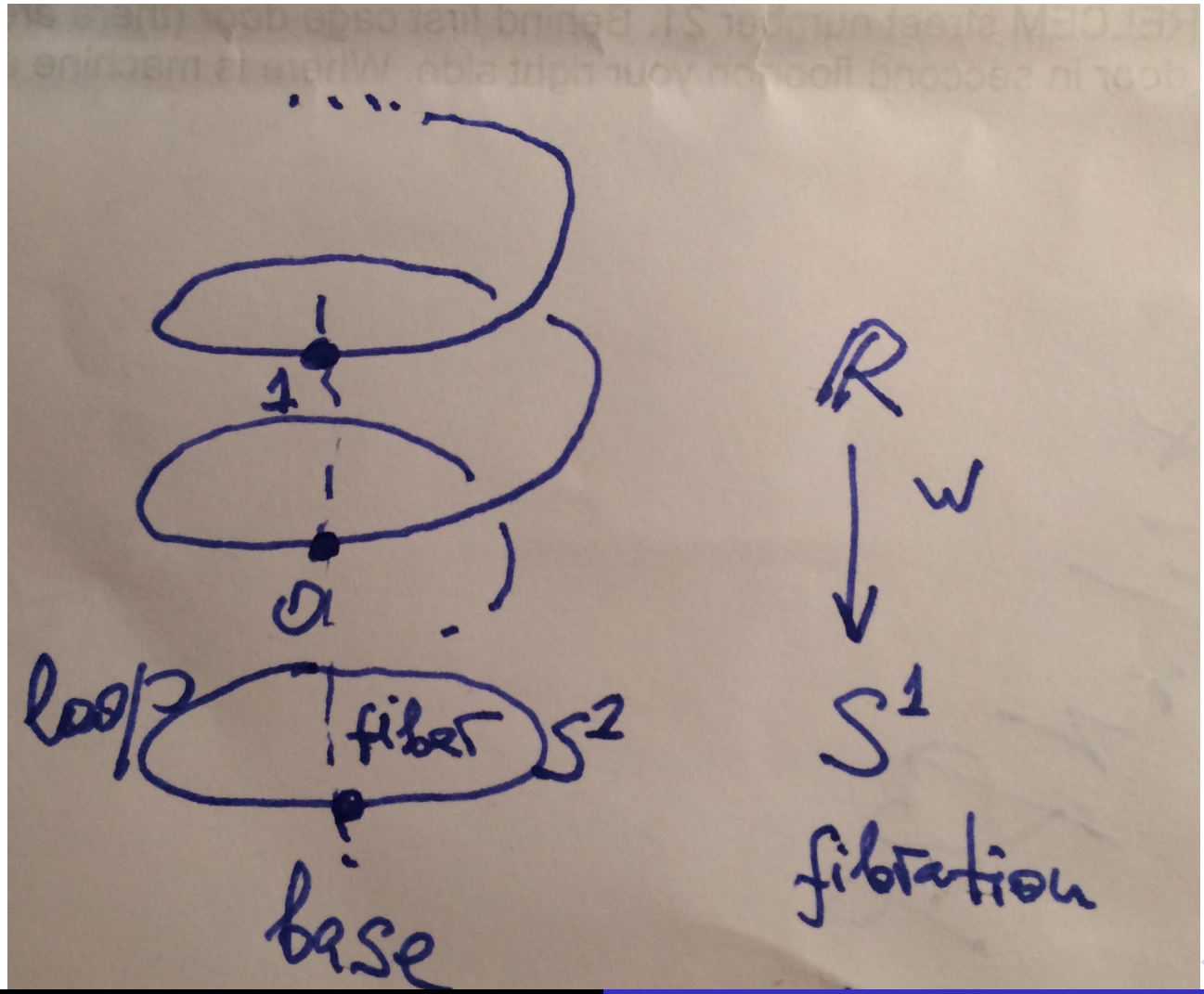
# circle as higher inductive type (after Shulman)

$$b : S^1$$

$$loop : b =_{S^1} b$$



# $\pi_1(S^1) \simeq \mathbb{Z}$ (after Shulman)



# Finitary Symbolic Intuition

FSI facilitates semantic interpretation of the microscopic syntactic structures (meaning of logical and non-logical constants, meaning of short symbolic expressions such as axioms of ZFC) but not semantic interpretation of larger syntactic structures, which typically are present in non-trivial mathematical proofs.

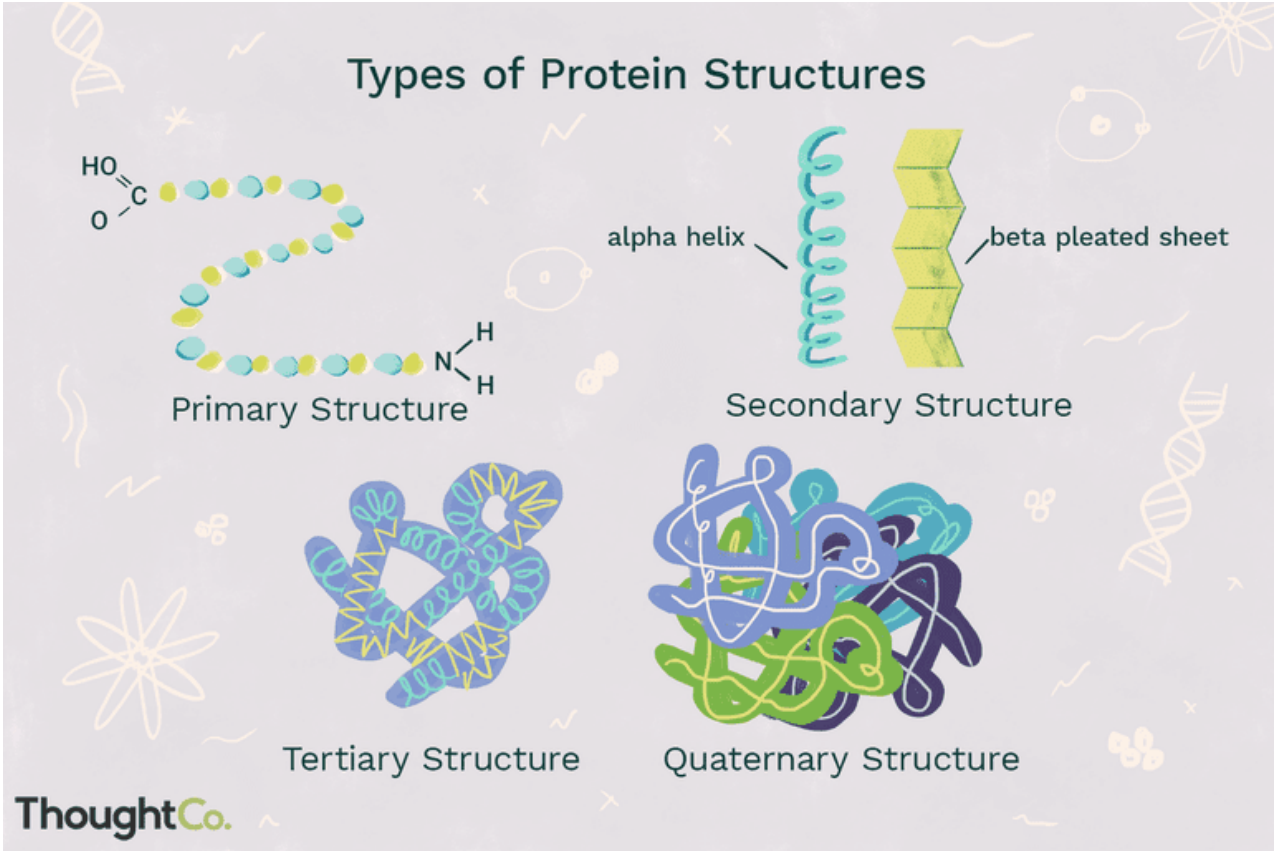
# Proof structures

The macroscopic proof structures can be made evident via other kinds of mathematical intuition including geometrical/homotopical intuition (HoTT example above).
The macroscopic structure of a given proof usually contributes essentially to its total evidential force. Whether or not the microscopic proof structure fully determines the macroscopic one is not relevant to our argument.

# Biochemical analogy: proteins

- Primary structure : the linear sequence of amino acids;
- Secondary structure : the three-dimensional form of local fragments of proteins;
- Tertiary structure : the global spatial shape;
- Quaternary structure ...

# Biochemical analogy: proteins



# Two kinds of Automated Proof Verification

# Two kinds of Automated Proof Verification

- Computer as a magic box : Hilbert-style deductive systems. Only assumptions (including axioms) and conclusions express a mathematical meaning. No meaningful proof. No meaningful reasoning.

# Two kinds of Automated Proof Verification

- Computer as a magic box : Hilbert-style deductive systems. Only assumptions (including axioms) and conclusions express a mathematical meaning. No meaningful proof. No meaningful reasoning.
- Computer as a tool extending human intuitive constructive capacities on all levels of structure (imagery, VR, ...). Meaningful proofs and reasoning. UF/HoTT supports APV of this latter sort.

# Conclusion

Standard logical architectures and formal ontologies, which are presently used in KR, don't provide a sufficient theoretical background for KR because they have no epistemological content. In this paper we explained the relevance of epistemological considerations in logic and KR and then pointed to some recent advances in mathematical logic, more specifically discussing the Homotopy Type theory, that may allow to use logical approaches in KR more effectively.

Thank You; Спасибо.