AXIOMATIC AND GENETIC METHODS OF CONCEPT- AND THEORY-BUILDING: AN ATTEMPT OF SYNTHESIS

ANDREI RODIN (ANDREI@PHILOMATICA.ORG)

In 1900 David Hilbert distinguishes between the axiomatic method known today after his name and the more traditional *genetic* method of concept- and theory-building in mathematics and science, which involves construction of complex mathematical objects from certain primitive objects [1]. In the introductory part of his 1934 volume co-authored with Paul Bernays Hilbert develops a different perspective on the genetic method and suggests that the axiomatic method in the narrow sense of the word is a part of a more general method of theory-building that Hilbert now calls interchangeably genetic and constructive. According to this mature Hilberts view the constructive method is exemplified in history by Euclids *Elements*, Newtons *Principia* and Clausiuss works in Thermodynamics [2].

Building on Hilberts insight on the constructive axiomatic method I attempt to provide it with a modern formal specification and epistemological foundation. This includes using the Gentzen-style formal syntax along with a proof-theoretic semantics and relaxing the standard rigid distinction between logical and extra-logical semantics of formal theories. More specifically I consider the Homotopy Type theory as a formal tool that helps one to identify the logical part of a given theory internally. Finally I argue that the constructive version of axiomatic method is more apt to represent mathematical and scientific theories than the standard formal axiomatic method. The present paper develops ideas earlier presented in my [3]

REFERENCES

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