Mario Bunge's Views on Mathematical Logic and Axiomatic Method

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- today: LRB2021 (July) (Hungarian school)

Sources:

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- Foundations of Physics (1967): FP

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- Why Axiomatize (2017): WA

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- To axiomatise a body of knowledge is just to exhibit its main ideas in an orderly fashion. (PP134)

Logic:

There is a single theory that starts from scratch: mathematical logic (which is actually a set of theories). Indeed, the truths of logic or tautologies — such as $A \rightarrow (B \rightarrow A)$ — are those that can be proved without resorting to assumptions other than the rules of logic. All other theories presuppose at least logic and usually a lot more.

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Compare: Tarski, Introduction to Logic and to the Methodology of Deductive Science 1941 (translated to English from German from Polish 1937 versions).

Logic (contd.):

More precisely, the least a mathematical or a scientific theory takes for granted is the so-called ordinary (two-valued) predicate calculus enriched with the microtheory of identity [in symbols (PC=)]. This theory is necessary and sufficient to analyse the concepts, formulas, and reasonings occurring in mathematics and in science - or rather to analyse their form. In fact, every statement in mathematics or in science is, as far as its form is concerned, a formula of that calculus; and every valid reasoning is an instance of an inference pattern consecrated by that same theory. (PP136)

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Compare: Suppes on the "standard formalisation", Quine

Alternative Logics

There are a number of alternative logics, as many as the logicians have cared to invent; the best known are intuitionistic logic, modal logics, and many-valued logics. Although interesting in themselves as mathematical frameworks, none of these exotic logics is used in contemporary science and mathematics either to analyze conceptual systems or to justify inferences.

Ultimately there is no escape from ordinary logic [(PC =)] save irrationalism. (FP18)

Logic: Argument?

If a logic other than (PC=) were to underlie one scientific theory, all other theories would have to be reformulated on the basis of the same exotic logic, for otherwise it would be impossible to apply them jointly to the explanation of facts and the design and interpretation of experiments, as each of these procedures summons a number of different theories. In other words, the theory with an extraordinary logic would remain isolated: inapplicable and untestable. Therefore the claim that QM has its own logic is a joke. (FP52)

I cannot fully understand how and why so many otherwise very open-minded people (such as Bunge, Suppes, Tarski, Quine — to name only a few) took such a dogmatic stance towards Logic at this point of the recent intellectual history.

Jóseph Bochénski (1902-1995, the founder of Sovetology among other things) did the same for a clear and explicit ideological and political reason trying to revive the Scholastic tradition in philosophy and human thought after what he regarded as a long period of Modern intellectual corruption that was started with Galileo, Descartes and other founders of Modern Science.

However such motivations were most certainly wholly alien to Mario Bunge and others. Suppes and Quine justify their choice of (PC=) pragmatically rather than metaphysically. I don't know if this is also Mario Bunge's case. Was it rather a fear of 'irrationalism' (evidenced all around in the everyday life, political events, and also in the academia, particularly in the philosophical circles)?

Physicist' view on Logic:

It is not that actual scientific reasoning needs logic in order to keep going: logic is not supposed to construct anything outside itself, but to control the validity of whatever is being built with concepts. [..] What is implied is that no theory can be expected to validate nondeductive [?] reasonings, even though their conclusions may be true. (PP136)

Scientific Realism

[A]n empiricist theory [as in the Copenhagen interpretation of QM] is a contradiction in terms, and in particular the reduction of mechanics to obervables ("operationally defined quantities") is hopeless: experience is not that generous, and the aim of science is not to summarize sense data but to explain reality, which task requires the invention of transempirical concepts.

Scientific Realism and Logic

Bunge's uncompromised Scientific Realism is related to his commitment to (PC=) and its Tarskian semantics. This logical framework requires ontological commitments (Quine) but does not require epistemological commitments.

Scientific Realism and Logic

Consider Tarski's notion of *logical consequence*: $A_1, \ldots A_n \models B$ just in case B is true under all interpretation where $A_1, \ldots A_n$ are true. Logical consequence $A_1, \ldots A_n \models B$ is implied by the syntactic derivability $A_1, \ldots A_n \vdash B$ (soundness).

Truthmakers of $A_1, \ldots A_n, B$ may not have any epistemic load whatsoever (cf. Prawitz on 'general proof theory').

Since QM or any other physical theory is (PC=)-formalized and axiomatized it naturally admits a realistic interpretation.

Scientific Realism and Logic

Bunge's Scientific Realism does more justice to the Hilbert-style axiomatic method (in its Tarskian semantic version) than Logical Empiricism, which interprets logical procedure in epistemic terms.

The inadequacy of the standard logical tool to its intended applications is clearly seen, for example, in Ernest Nagel's works.

Axiomatics of physicists vs. axiomatics of logicians

In his epoch-making book (von Neumann,1932, Mathematische Grundlagen der Quantenmechanik), which enriched the mathematical framework of the theory, von Neumann is wrongly supposed to have laid down the axiomatic foundations of quantum mechanics. As a matter of fact his exposition lacks all the characteristics of modern axiomatics: it does not disclose the presuppositions, it does not identify the basic concepts of the theory, it does not list all the initial assumptions (axioms), it fails to propose a consistent physical interpretation of the formalism, and it is ridden with inconsistencies and philosophical naivetés. Yet for some strange reason it passes for a model of physical axiomatics. (PP132)

Axiomatics of physicists vs. axiomatics of logicians

In appearance, at least, Suppes-type axiomatizations of physical theories are much closer to those of working physicists. [...] These axiomatizations fail, to differing degrees depending upon the specific example, to meet modern standards of logical rigor. Primitive concepts and axioms are sometimes not clearly identified; questions of independence of primitives and axioms are not carefully raised; the epistemological status of the axioms is often fuzzy; "physical intuition"is sometimes employed as an inference rule in obtaining theorems. (Moulines&Sneed 1979)

Axiomatics of physicists vs. axiomatics of logicians

Non-trivial physical theories are formulated within portions of mathematical language which, in spite of being reasonably well defined, are not easy to formalize in first-order logic, and indeed have not yet been formalized in any formal language. Outstanding examples of those portions are: partial differential equations, matrix theory, tensor analysis. Before dealing with even such a simple physical theory as classical particle mechanics, we should have to assume that an enormous part of mathematics has already been formally axiomatized — which is obviously not the case; so we should have to do it ourselves or *wait*. (Moulines&Sneed 1979)

Axiomatic Particle Mechanics (Bunge, FP196730ff)

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- (Specific) Axioms

Formal Background

(a) elementary logic (PC=); (b) elementary set theory; (c) elementary topology; (d) vector spaces theory; (e) analysis and the algebraic and number-theoretical fragments underlying it.

Protophysical Background

(a) chronology, in particular the theory of universal time (UT); (b) Physical Euclidean geometry; (c) the general systems theory

Physical Euclidean geometry

Axioms of (Hilbert-style axiomatic) Euclidean geometry interpreted in terms of material physical objects rather than numbers or other mathematical entities.

Example: physically interpreted Boolean lattice:

field + (cream $\times$ coffee) = (field + cream) x (field + coffee)
field x (air + needles) = (field x air) + (field x needles)
body x empty space = empty space
electron + all electrons = all electrons

+ is juxtaposition; $\times$ is superposition

Chronogeometrical axioms:

PM 1.1. (a) T is an interval of the real line. (b) Every $t \in T$ represents (refers to) an instant of time, (c) The relation *leq* that orders T means "earlier than or simultaneous with".

...

Remark

"By virtue of this axiom time is endowed with all the formal properties of the real number continuum. In other words, the theory of universal time underlying PM is a model or interpretation of the mathematical theory of real numbers. "

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- PM 2.3. a) K is a nonempty denumerable set. (b) Every $k \in K$ is a rigid system of corpuscles at least four of which lie on the vertices of a regular trihedral, (c) For every $k \in K$ there exists a Cartesian system of axes $e = \langle e_1, e_2, e_3 \rangle$. (d) No $k \in K$ interacts [?] with any $\sigma \in \Sigma$ that is not a part of k.

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- PM 2.4. (a) For every $\sigma \in \Sigma$ and every $k \in K$, every $X \in \{X\}$ is a continuous and real valued function from T to E^3. [...] (c) $X(a, k, t)$ represents the position of a, relative to the frame k, at the instant t. (d) Every quintuple $\langle \sigma, X_1, X_2, X_3, t \rangle$ represents an event.

Sample of a theorem:

Thm. 1 (absoluteness of simultaneity). If $k, k' \in K$ and two events are simultaneous in k then they are simultaneous in $k' \neq k$.

Proof: From the frame-independence of time asserted by PM 1 and the characterization of "event" supplied by PM 2.4d. ■

On "semantic axioms"

In contrast to mathematics, in physics we must characterize not only the structure and the interrelations of the primitives but also their meaning. This is the function semantical axioms. [...] Semantical hypotheses lay down concept-physical object relations, i.e. reference relations. (FP66)

On "semantic axioms"

(Me) The notion of semantic axiom is absent from the standard logical toolbox. I cannot see how it can be admissible (= to be a conservative extension of the standard axiomatic method). Relations between symbols and their referents are established by interpretations (modelling) of axioms and theorems of a given formal theory but not by additional axioms.

Bunge's axiomatic PM involves the standard mathematical interpretation of E^3 along with its physical interpretation (formal and protophysical layers). How the two sorts of interpretations relate to each other?

Bunge's 2017 reply to Suppesians: physical and mathematical models

[A]ll model theorists, such as Tarski, know that their models are examples or interpretations of abstract theories (or formal systems) such as those of sets, graphs, lattices, and groups hence totally unrelated to the theoretical models devised by scientists and technologists, which are special theories, such as that of the simple pendulum. Thus the entire model-theoretic (or structuralist) approach to theoretical physics, adopted [after Suppes!]ṙby Joseph Sneed (1971) and his followers, such as Moulines and Stegmüller, is the fruit of an equivocation, as would be regarding ring theory as dealing with fried onion rings and the like. (WA)

Bunge's 2017 reply to Suppesians: physical and mathematical semantics

The formalist school started by the McKinsey et al. (1953) paper on the axiomatization of classical particle mechanics overlooks semantics. [WA, SIC!]

Given the fact that McKinsey, Suppes and all their followers describe their formal method as 'semantic' (the semantic view of theories contra the 'syntactic' view) this Bunge's argument needs a qualification. Apparently Bunge's notion of semantics / meaning differs drastically from Suppes'!

Bunge's 2017 reply to Suppesians: physical and mathematical semantics

In sum, we reiterate the axiomatization strategy proposed in earlier publications (Bunge 1967a, b, d), which differs from the structuralist one defended by Suppes, Sneed, Stegmüller, Moulines, and other philosophers. This formalist stance, which ignores the semantic side of scientific theories [. . .] been the target of a couple of criticisms (Bunge 1976; Truesdell 1984) that I regard as decisive. But these criticisms have been ignored by nearly all philosophers of science.[WA]

Frege vs.Hilbert

Bunge's proposal appears as Frege-style traditional axiomatics that does not allow for non-interpreted formulas and for multiple interpretations of these formulas. The problem is that Hilbert-style axiomatic theories cannot be simply recast as Frege-style theories; consider the notion of of "metamathematical replacement" (of a traditional Frege-style mathematical proof), which belongs to the core of Hilbert-style axiomatic method. This method cannot be recast in Frege style without a significant modification of its symbolic and semantic machinery.

Conclusions and further research

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The Hilbert-style axiomatic method provided with Tarski's logical and extra-logical model-theoretic semantics (Burbaki-Suppes) fails to perform as an effective formal representational technique for physics.

- The claim is evidenced by the works of Supessians, who follow the method more accurately and get results that are not satisfactory from a physicist's point of view.
- It is also evidenced by works Mario Bunge, Clifford Truesdell, Bill Lawvere and some other people, who are not satisfied with this method in its received form.

Conclusions and further research

Mario Bunge's goal to renew/reinvent the received Hilbert-style axiomatic method in order to make it more effective in physics can hardly be achieved without a deeper revision of its logical and philosophical foundations and modification of its technical features.

Conclusions and further research

(PC=) does not qualify as a universal logic of all scientific reasoning and thought. Alternative logical approaches should no, generally, be thought of as "exotic". No system of logic and moreover no logical calculus can be simply taken for granted (whether or not it looks "scientific").

Conclusions and further research

For a (very preliminary) proposal of how to reform the received Hilbert-style axiomatic method see my

On Constructive Axiomatic Method :
`https://arxiv.org/abs/1408.3591v3`

Axiomatic Architecture of Scientific Theories :
`http://philsci-archive.pitt.edu/17600/`

THANK YOU!