Prove that $\overset{(1)}{x \in L} \longleftrightarrow \overset{(2)}{x \in R}$

$$\overline{A} \cup \overline{B} = \overline{A \cap B}$$

1. Let $a := x \in A$ and $b := x \in B$ where x is an arbitrary element.

(Def.) 2. $x \in \overline{A}$ iff $\neg x \in A = \neg a$

$x \in \overline{B}$ iff $\neg x \in B = \neg b$

left part $x \in (\overline{A} \cup \overline{B})$ iff $= \neg a \vee \neg b$ (1)

right part $x \in (A \cap B)$ iff $a \wedge b$

$x \in \overline{A \cap B}$ iff $\neg(a \wedge b)$ (2)

3.1 left p.

a	b	$\neg a$	$\neg b$	$\neg a \vee \neg b$
t	t			
t	f			
f	t			
f	f			

3.2 r.p.

(1) $\longleftrightarrow$ (2)

4.

$(A \vee B) \wedge C$

$A \vee (B \wedge C)$

$a \times b + c$

$A \vee (B \wedge C)$

$(A \vee B) \vee C$

$\wedge \quad \wedge$

distribution laws

$a \times (b + c) = a \times b + a \times c$

$a + b \times c \neq (a + b) \times (a + c)$

implication / Boolean algebra of propositions

1) binary ops (aka logical connectives)

			Logic	Sets
$p \wedge q = r$	$\wedge$	conjunction		$A \rightsquigarrow (X \in A) \to p$
$p \vee q = s$	$\vee$	disjunction		$\cap \rightsquigarrow \wedge$
				$\cup \rightsquigarrow \vee$
				$\overline{A} \rightsquigarrow \neg(x \in A)$
				$\in \rightsquigarrow \to$

2) unary op:

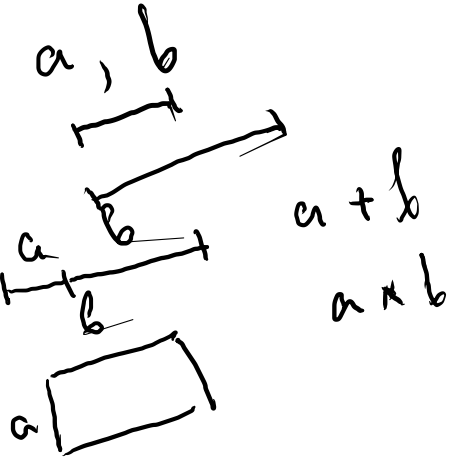
$\neg p = \dots$ $\neg$ - negation

$(p \to q) \leftrightarrow \neg p \vee q$

p	q
t	t

truth-values:
t, f

p	q	p ⇒ q	¬p	¬p ∨ q
t	t	t	f	t
t	f	f	f	f
f	t	t	t	t
f	f	t	t	t



$$a^2 + 2ab + b^2 = (a+b)^2$$

false for X

1. $X \in \emptyset$ — $\emptyset \in \emptyset$ false
2. $X \in U$ — true for all $X \in U$

Sets: (subs of U) ⇝ Props:

$\emptyset$ ⇝ $\bot$ falsum

U ⇝ $\top$ verum

p : Prop

$p \vee \bot = p$
$p \wedge \bot = \bot$
$p \vee \top = \top$
$p \wedge \top = p$

$A \subseteq U$ forall A

if $x \in A$ then $x \in U$

U [P : Set]

Venn logical diagrams $P \subseteq U$

de Morgan Laws

P, Q : Prop.

prop. eq. ($\leftrightarrow$)

$\neg P \vee \neg Q = \neg(P \wedge Q)$

$\neg P \wedge \neg Q = \neg(P \vee Q)$

$P \rightsquigarrow x \in P$

$T \rightsquigarrow U$

"classical logic"

$\neg(\neg P) = P$

not not

$\overline{\overline{P}} = P$

intuitionistic logic

$p : Prop$ $\quad p \vee \neg p = \top$

tautology

$p \wedge \neg p = \bot$

contradiction

$P : Set$ $\quad P \cup \overline{P} = U$

tertium non datur

$P \cap \overline{P} = \emptyset$

Law of Excluded Middle

L.E.M

Example: Problem:

$\sqrt{2}$ is irrational

$\frac{m}{n}$ $\quad m, n$ are integers

$\sqrt{2} \neq \frac{m}{n}$)

for no m, n int. $\left(\frac{m}{n}\right)^2 = 2$

$$m^2 = 2 \times n^2$$

~~Do~~ Does there such a, b is rational such that a^b is rational?

$p = \boxed{c \text{ is rational}}$

Prop. Such a, b exist!

Proof: Consider $a = b = \sqrt{2}$

L.E.M

$a^b = \sqrt{2}^{\sqrt{2}} = c$ Consider cases

$\boxed{p \vee \neg p}$ Case 1: c is rational. We are done!

Case 2: c is irrational

Consider $c^{\sqrt{2}} = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^{(\sqrt{2} \times \sqrt{2})} = (\sqrt{2})^2 = 2$ is rational

$\neg(\neg p) \neq p$

QED

Combinatorics

$n=3$

Vocabulary

a b c

c b a

$a_1, \dots, a_n$

$0! = 1$

$n = 1$

$1! = 1$

$2! = 2$

Permutations

$n = 2$

a b

b a

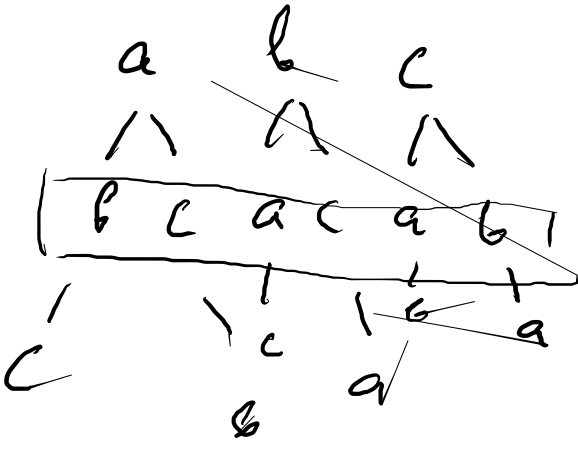
$P_2 = 2$

$$\boxed{P_n = n!}$$

1 2 3

a, b, c

# of options $3 \times 2 \times 1 = 6$



$n = 4$

a b c d

$4 \times 3 \times 2 \times 1 = 24$

factorial

$n! := n \times (n-1) \times (n-2) \times \dots \times 1$

Arrangements

$$A_k^n = \frac{n!}{(n-k)!}$$

$a, b, c, \dots, z$ — n symbols

words of (w/out repitions) | lengths $k \le n$

cut off the tail of length $n-k$

$n \times (n-1) \times \dots \times 2 \times 1$

head of length k | tail

A_3^4 | $4 \times 3 \times 2 \times 1 = 24$

$\{a, b, c, d\}$

$k = 3$ $\quad n = 4$

$0!$ $\quad (n-k)! = 0! = 1$

Homework

# of permutation with repetitions.

a b c ... z

Combinations

$\{a, b, c\} = \{c\ b\ a\}$

$$C_k^{n} = \frac{A_k^n}{P_k} = \frac{n!}{(n-k)!\,k!}$$

I have n balls

I want to take out $m \le n$ balls | How many ways to do it?